Wavelets and 2D method of auxiliary currents

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Abstract: - Wavelets technique is applied for solving of a two-dimension Dirichlet and Neumann problems when scattering body are elliptical cylinder or cylinder with complicated cross section contour (multifloil). The developed method utilizes Haar functions and method of moments for solving the Fredholm integral equation of first kind with smooth kernel. The results of the wavelet technique's using are illustrated by calculations of the scattering pattern for dielectric and metal elliptical cylinder or cylinder with complicated cross section contour. The problems of accuracy, choosing auxiliary contour Σ and stable results are discussed

Key-Words: - integral equations, wavelet technique, 2D scattering problems

1 Introduction

It is known that wavelet technique is widely practiced in such domains as time- or frequency analyses of the signals and image process. One of the most attractive ideas were appeared in last years was deal with applying of wavelets as basis functions in method of the moments (see, for example [1,2]).It is clear that wavelets technique allows to create a faster algorithms then ordinary one due to using a specific attributes of its as a basis functions. It is well known too that many of diffraction problems could be reduced to integral equations and method of moments was widely used for its solving.

This paper is concerned the extending wavelets technique for solving an integral equation (or a system of the integral equations) of auxiliary currents [3] which could be presented as a Fredholm integral equations of the first kind. In this case the unknown function is an auxiliary current on a contour Σ located within (outside) the scattering contour S. The kernel of these integral equations is a smooth function therefore applying of wavelets technique allows to create more effective and fast algorithms then ordinary one. The results of utilizing wavelets technique are illustrated by solving of a Dirichlet and Neumann boundary scattering problems or scattering problem by dielectric cylinders with cross section contour as ellipse or multifoil for a case of the plane incident wave. The problem of choosing the auxiliary surface was considered too.

2 Diffraction of a plane wave

Let consider a scattering of E and H polarized wave $u_0(\vec{r})$ by cylinder with cross section contour $\rho(\varphi)$ in cylindrical system of coordinates (z,r,φ). Diffracted field $u^1(\vec{r})$ is a solution of the Helmholts equation

$$\Delta u^1 + k^2 u^1 = 0 \tag{1}$$

and satisfies the Dirichlet boundary condition on S

$$u(\vec{r})|_S \equiv [u_0(\vec{r}) + u^1(\vec{r})]|_S = 0. \tag{2}$$

or the Neunamm boundary condition

$$\frac{\partial}{\partial n_S}[u_0(\vec{r}) + u^1(\vec{r})]|_S = 0 \tag{3}$$

and Sommerfeld's radiation condition

$$\frac{\partial u^1(\vec{r})}{\partial r} + iku^1(\vec{r}) = o(r^{-1/2}), |r| \to \infty \tag{4}$$

in case of perfect conducting body and for dielectric body (2),(3) have to be replaced by

$$\frac{\partial}{\partial n_S}[u_0(\vec{r}) + u^1(\vec{r})]|_S = \chi \frac{\partial}{\partial n_S}[u^i(\vec{r})] \tag{5}$$

where k is a wave number, $u^i(\vec{r})$ - is a field inside of the dielectric body satisfies (1) with $k = k_i$, $\chi = 1$ for E polarized incident wave and $\chi = 1/\varepsilon_i$ for H polarized incident wave, ε_i is a relative dielectric penetrability of dielectric.

In accordance with the method of an auxiliary current [3] the boundary problems mention above could be reduced to the Fredholm integral equations of the first kind

$$u_0(\vec{r}_S) = \int_{\Sigma} \mu\ (\vec{r}_\Sigma) H_0^{(2)}(k \,|\, \vec{r}_S - \vec{r}_\Sigma \,|) d\sigma, \qquad (6)$$

$$\frac{\partial}{\partial n_S} u_0(\vec{r}_S) = \int_{\Sigma} \mu(\vec{r}_\Sigma) \frac{\partial}{\partial n_S} H_0^{(2)}(k \,|\, \vec{r}_S - \vec{r}_\Sigma \,|) d\sigma, (7)$$

or system of the integral equations of the first kind

$$u_0(\vec{r}_S) + \int_{\Sigma} \mu\ (\vec{r}_\Sigma) H_0^{(2)}(k \,|\, \vec{r}_S - \vec{r}_\Sigma \,|) d\sigma \quad =$$

$$= \int_{\Sigma 1} \mu\ (\vec{r}_{\Sigma 1}) H_0^{(2)}(k_i \,|\, \vec{r}_S - \vec{r}_{\Sigma 1} \,|) d\sigma, \qquad (8)$$

$$\frac{\partial}{\partial n_S} u_0(\vec{r}_S) + \int_{\Sigma} \mu(\vec{r}_\Sigma) \frac{\partial}{\partial n_S} H_0^{(2)}(k \,|\, \vec{r}_S - \vec{r}_\Sigma \,|) d\sigma =$$

$$= \chi \int_{\Sigma 1} \mu_1(\vec{r}_{\Sigma 1}) \frac{\partial}{\partial n_S} H_0^{(2)}(k_i \,|\, \vec{r}_S - \vec{r}_{\Sigma 1} \,|) d\sigma,$$

Here, $\mu(\vec{r}_\Sigma), \mu_1(\vec{r}_{\Sigma 1})$ - allocated on Σ and $\Sigma 1$ an auxiliary currents; $\Sigma, \Sigma 1$ - auxiliary closed contours within S and outside of S ; $\vec{r}_\Sigma$ - is the radius-vector of the integration's points on Σ, $\vec{r}_S$ - is the radius-vector of the points on S; $d\sigma$ - is the element's length of shaft-bow on Σ or $\Sigma 1$; $|\, \vec{r}_S - \vec{r}_\Sigma \,| = [r^2(\alpha) + \rho^2(\theta) + 2r(\alpha)\rho(\theta)\cos(\alpha - \theta)]^{1/2}$;
$r(\alpha)$ - is the equation of the contour S and $\rho(\theta)$ - is the equation of the contour Σ inside S in cylindrical coordinate system, $\rho_1(\theta)$ - is the equation of the contour $\Sigma 1$ outside of S;
$|\, \vec{r}_S - \vec{r}_{\Sigma 1} \,| = [r^2(\alpha) + \rho_1^2(\theta) + 2r(\alpha)\rho_1(\theta)\cos(\alpha - \theta)]^{1/2}$.

Using the wavelet technique [4] one can obtain the presentation for $\mu(\theta)$ in form of set

$$\mu(\theta) = c_0 \phi_0(\theta) + \sum_{j=0}^{\infty} \sum_{k=0}^{2^j - 1} d_{jk} \psi_{jk}(\theta), \qquad (9)$$

where

$$\sqrt{\pi}\ \ \phi_0(\theta) = \begin{cases} 1, & 0 \le \theta < 2\pi, \\ 0, & \theta \notin [0, 2\pi[, \end{cases}$$

is the scaling function and

$$\sqrt{\pi}\ \ \psi_{jk}(\theta) = 2^{j/2} \psi_H(2^j \theta - 2k\pi),$$

$$\psi_H(t) = \begin{cases} 1, & 0 \le t < \pi, \\ -1, & \pi \le t < 2\pi, \\ 0, & t \notin [0, 2\pi[. \end{cases}$$

is the Haar function [4], d_0, d_{jk} are to be found , $M = 2^j - 1$.
Placing (9) into (6) (or (7),(8))) one could get

$$c_0 L\phi_0 + \sum_{j=0}^{p} \sum_{k-0}^{2^j - 1} d_{jk} L\psi_{jk} \approx \psi(\alpha), \qquad (10)$$

were L is a integral operator in (3). Making a procedure of description with basis functions $\xi_m(\tau), m = 1, \ldots, M1$, we get a system of linear equations

$$c_0 \langle \xi_m, L\phi_0 \rangle + \sum_{j=0}^{p} \sum_{k=0}^{2^j - 1} d_{jk} \langle \xi_m, L\psi_{jk} \rangle = \langle \xi_m, \psi \rangle . \quad (11)$$

which can be written in the matrix form as

$$[A_{mn}][d_n] = [b_m] \ , \qquad (12)$$

where

$$\begin{aligned} &A_{mn} = \langle \xi_m, L\Psi_{ik} \rangle, m = 1, \ldots, M1; \\ &j = 0, \ldots, p; k = 0, \ldots, 2^j - 1; n = p + 2^j \end{aligned} \qquad (13)$$

$$b_m = \langle \xi_m, \Psi \rangle. \qquad (14)$$

Let as take ξ_m in form of the delta functions

$$\xi_m = \delta(\alpha - \alpha_m),$$

where α_m are the points on contour S from the interval $[0,2\pi]$ .In result we obtain a scheme for the method of point matching . When we take the Haar functions as a basis functions ξ_m we get the Galerkin method and we have to make two dimensional integrations.

The $\mu(\theta)$ function having been found from (6), the scattering pattern $g(\varphi)$ could be calculated as follows

$$g(\varphi) = \int_0^{2\pi} \exp[ik\rho_0(\theta)\cos(\theta - \varphi)] \mu(\theta) d\theta \ . \quad (15)$$

We had applied described above method for calculation of the scattering pattern $g(\varphi)$ on the base of (6), (7), (8) and Haar functions as a basis functions .As scatterers we had considered the elliptical cylinder $r(\alpha) = a / \sqrt{1 - \varepsilon^2 \cos^2(\alpha)}$, $\varepsilon^2 = 1 - a^2 / b^{2,}$ with small semi-axis a and big b one, the mulrifoil cylinder $r(\alpha) = a + b\cos(q\alpha)$ and case of the plane $u_0 = \exp(-ikr\cos[\varphi - \varphi_0])$ incident E(or H) polarized wave.

The results of calculation of the $g(\varphi)$ for perfect conducting ellipse with M1=128 (M1 is the total number of the basis function), ka=0.1, kb=5.3, E polarized plane incident wave, $\varphi_0 = \pi/2$ is depicted in Fig.1.In the this case we have a situation which is close at the scattering problem by perfect conducting infinitely thin band [5] in high frequency domain. Our result has agreed with [5] with graphical accuracy.

The scattering pattern $g(\varphi)$ for dielectric ellipse with M1=128 , ka=1, kb=9, $\varepsilon_i = 4$, H polarization, $\varphi_0 = \pi/2$ is shown at Fig2. It is evident that this case is modeling the situation of scattering by thin dielectric band. The accuracy Δ of the fulfillment of boundary condition for different contour's point's m is presented at Fig.3. It is seen that the boundary conditions are executed worse for points which are closed at singularities. But these domains are rather little.

The results of calculation of the $g(\varphi)$ for multifoil with ka=5, kb=0.5, q=4, $\varphi_0 = 0$, $\max(\Delta) = 10^{-12}$, M1=128, H polarization and for E polarization of the plane incident waves are shown at Fig. 3 and Fig.4 respectively.

It was detected that the point-matching method gives the stable results when descriptive points are choosing in the middle of the intervals 2π/M1. At the same time we had the matrix A_{mn} close to diagonal's one when the method of moments was utilized and the stable problem did not arise. It was fixed that accuracy depends from location of Σ.

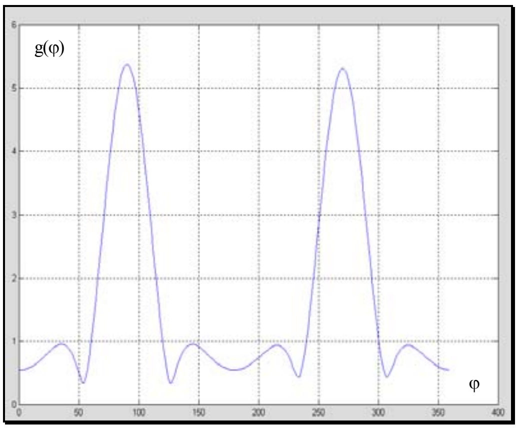

Fig. 1

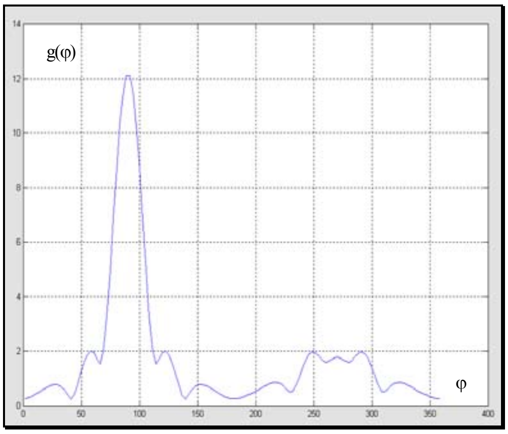

Fig. 2

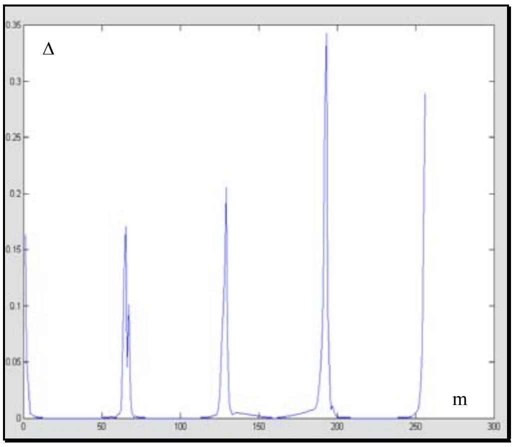

Fig.3

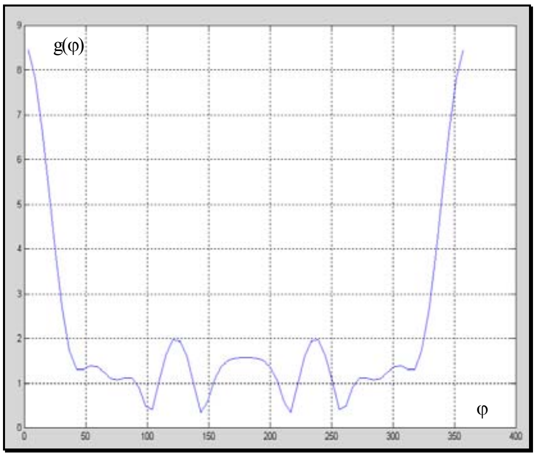

Fig.4

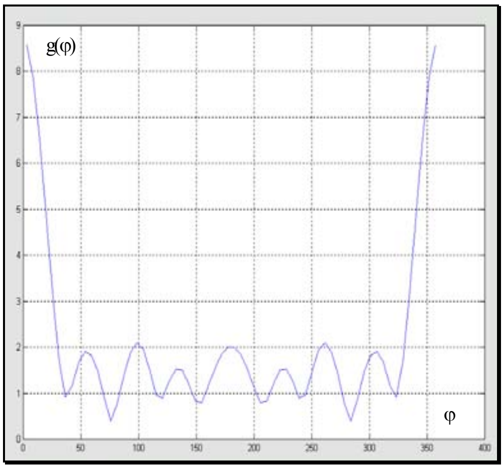

Fig.5

3 Conclusion

The developed method allows making of the essential step into high frequency domain. It decrees the time of calculation and has a good accuracy and stable. It has significant advantages over the traditional techniques and largely extends the class of problems, which could be solved. The method can easily be extended to three-dimensional problems, to plane-layered media and vector case.

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