About universal 3D modification of the method of discrete sources

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Abstract: - A new universal modification of the method of discrete sources (MMDS) was developed for a case of scalar scattering problem by three-dimension impedance convex body. The method MMDS utilize analytical transformation of the body's surface for constructing the auxiliary surface's in the procedure of the method discrete sources. The results of the applying MMDS are illustrated by solving the scattering problems by bodies of revolution with ellipsoid and multifoil forms. The accuracy of the solutions and interpretation of the stable results are made.

Key-Words: - 3D scattering problems, Body of revolution, Discrete sources.

1 Introduction

It is well known that present numerical methods (for example, the method of integral equations (MIE) [1]) usually provide only a certain optimal trade-off between the accuracy and numerical computation time. Also it is evident that the key element in achieving an optimal trade-off is utilizing of efficient numerical methods, which is valid in a sufficiently wide range of parameters of the problem and are stable and fast.

In recent years one can see that 2D or 3D methods of discrete sources (MDS) [2] and some it's modifications - method of auxiliary currents (MAC) [3] and it's spline modification [4] are widely practiced for solving two and three dimensional scattering problems due to their simplicity and algorithmic. MDS is known to base on the theorem of V. D. Kupradze [2] in which was proved the completeness for a set of fundamental solutions to the Helmholtz equation that have singularities on a certain closed surface Σ inside scatter. We would like to emphasize that the theorem proved by V. D. Kupradze places almost no constraints on the geometry of Σ. Because of this, it has long been unknown why the algorithm becomes unstable under certain circumstances. In [3,5], the origin of this effect was found and constraints on the geometry of Σ were formulated in form of the theorem (see also [6]): Let a simple closed Lyapunov surface Σ be such that k is not an eigenvalue of the interior homogeneous Dirichlet problem for the region inside Σ. Then the Fredholm integral equation

$$\int_{\Sigma} \frac{\exp\left(-ik\,|\vec{r}_S-\vec{r}_\Sigma|\right)}{|\vec{r}_S-\vec{r}_\Sigma|} I(\vec{r}_\Sigma)\,d\sigma = -U_0(\vec{r}_S),\ \vec{r}_S \in S$$

of the first kind, in which $U_0(\vec{r})$ is a incident wave, $I(\vec{r}_\Sigma)$ is a auxiliary current, has a solution if and only if Σ encloses all of the singularities of the diffracted field inside of scatter's surface S. Using this theorem a simple and universal method for constructing Σ was proposed in [7] for a 2D case of perfect conducting cylinder.

In this paper we has developed MMDS for a scalar 3Dcase of impedance body and the results of it's utilizing are illustrated by solving a scattering problem for metal body with ellipsoidal form of revolution. The problems accuracy and stable are investigated when the extent of haul a surface Σ taut closer at singularities.

2 Diffraction of a plane wave by a convex body

Let us consider the problem of a plane wave

$$\begin{aligned} &U_0(r,\varphi,\theta) = \\ &= \exp\{-ikr[\sin\theta\sin\theta_0\cos(\varphi-\varphi_0) + \\ &+\cos\theta\cos\theta_0]\}; \end{aligned} \qquad (1)$$

diffraction by impedance body in spherical coordinate system (r,φ,θ). In (1) θ_0,φ_0,k are the angles of incident and the wave number of the plane wave Let the surface S of the body's boundary has equation in the spherical coordinates system as follows

$$\vec{r}\big|_S = \rho_0(\varphi,\theta) \qquad (2)$$

According to MDS, the diffracted field $U(r,\varphi,\theta)$ outside of S could be presented in the next form

$$U(r,\theta,\varphi) = \sum_{n,m=1}^{N,M} A_{nm} \exp(-ik|\vec{r}-\vec{r}_{nm}|) \Big/ |\vec{r}-\vec{r}_{nm}| \qquad (3)$$

Here, $\exp[-ik|\vec{r}-\vec{r}_{nm}|]/|\vec{r}-\vec{r}_{nm}|$ is the fundamental solution to the Helmholz equation; A_{nm} are the coefficients to be determinate; $|\vec{r}-\vec{r}_{nm}| = [(x-x_{nm})^2+(y-y_{nm})^2+(z-z_{nm})^2]^{1/2}$ -are the distances between points with radius vectors $\vec{r},\vec{r}_{nm}$ respectively; $\vec{r}_{nm}$ - are radiuses vectors of the the position of the sources on Σ within the boundary *S*. The system of algebraic equations for coefficients A_{nm} is obtained by placing (3) under boundary condition

$$[U_0(r,\varphi,\theta)+U(r,\varphi,\theta)]|_S = = \frac{W}{k}\frac{\partial}{\partial n}[U_0(r,\varphi,\theta)+U(r,\varphi,\theta)]|_S \qquad (4)$$

where W – is the effective surfaces impedance, impedance and then calculating (4) at points $r=\hat{r}_{nm}$, $\theta=\theta_n, \varphi=\varphi_m$:

$$\hat{r}_{nm} = \rho_0(\theta_n,\varphi_m);$$
$$\theta_n = \pi n/N; n=1,2,...,N;$$
$$\varphi_m = 2\pi m/M; m=1,2,...,M$$

are the position of the matching points on S in which a boundary conditions are satisfies .

The main problem in MDS is known how to detect the location of Σ suffice. In almost all published works this surface were choused as like as S one. But it often leads to unstable and bad accuracy. To avoid this effects we have to find all singularities of the diffracted field and then make an analytical transformation of the surface S to enclose its [6]. As example of this procedure let as consider the body as ellipsoid of revolution

$$\rho_0(\varphi,\theta) = \frac{a}{\sqrt{1-\varepsilon^2\cos^2(\theta)}}; \varepsilon^2 = 1-\frac{a^2}{b^2} \qquad (5)$$

and multifoil of revolution

$$\rho_0(\varphi,\theta) = a + b\cos(q\theta). \qquad (6)$$

In this case the singularities can be found in analytical form [6,7] and one could obtained for Σ the next surface of revolution (when ellipse revolutions round z-axis)

$$r_\Sigma(\theta,\varphi) = |\zeta|, \theta_\Sigma = \arg(\zeta),$$
$$\zeta = \rho(\theta+i\theta_1)\exp(\theta+i\theta_1), \qquad (7)$$
$$\theta_1 = -\ln[\varepsilon/\{2-\varepsilon^2\}^{1/2}] + \delta;$$

where δ - specifies the extent of nearness of Σ at singularities. Another surfaces of revolution could be easy obtained by revolution of ellipse round y-axis or when the multifoil rotates around the z-axis.

It was detected that applying these surface (7) leads to stable results and the accuracy (we estimate the accuracy Δ by fulfillment of the boundary condition on *S*) becomes much better when contour approaches to singularities and *N1* (*N1*- is a total number of the sources) is fixed or *N1* is increasing. So we can assert that by changing *N1* and δ one can obtain any given accuracy.

Some results of calculation for the scattering pattern $g(\theta,\varphi)$ as for perfect conducting body (5) as ellipsoid of revolution (when ellipse is revolving around z-axis) with ka=4; kb=8, N1=1196, W=0, φ_0=π/2, $U_0 = \exp[-iky]$, max(Δ)=0.00023, δ =0.00001 is depicted in fig. 1 and for $U_0 = \exp[-ikz]$, max(Δ)=0.0005 - in fig. 2.

Example of calculation $g(\theta,\varphi)$ for body as ellipse of revolution (when ellipse is revolting round y axis) with w=0, *kb*=5. *ka*=1, *N1*=1413, δ =0.00001, max(Δ)=0.01, W=0, φ_0=π/2, $U_0 = \exp[-iky]$ is shown at Fig.3.

At Fig.4 is shown the scattering pattern $g(\theta,\varphi)$ for multifoil of revolution with q=3, *kb*=1. *ka*= 8, *N1*=1263, δ =0.00001, max(Δ)=0.1, W=0, φ_0=π/2, $U_0 = \exp[-ikz]$.

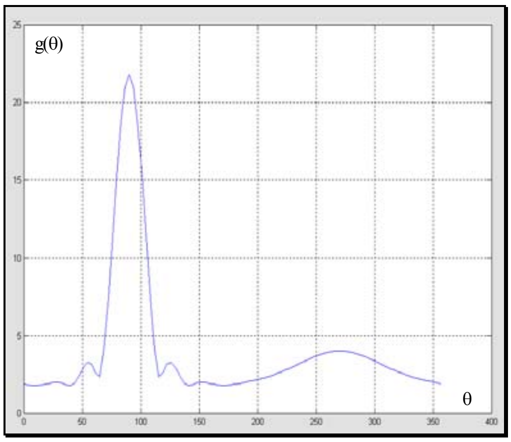

Fig.1

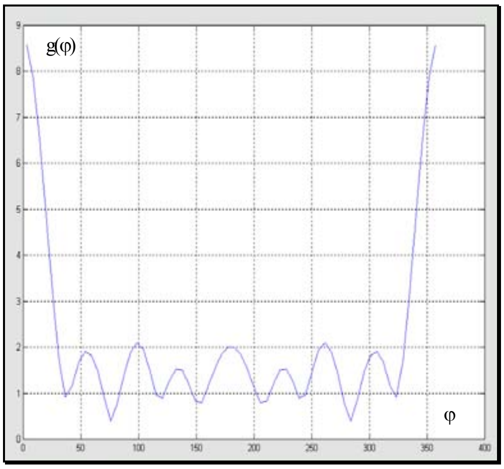

Fig.2

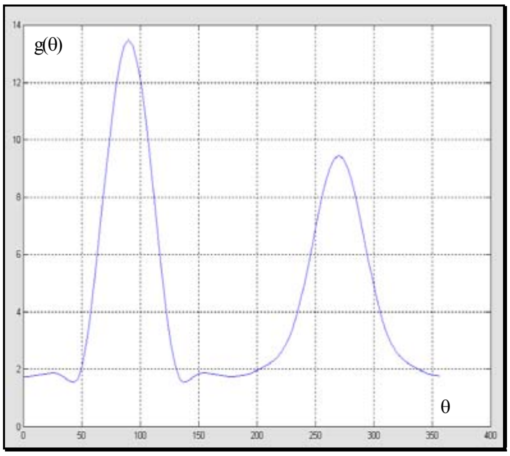

Fig.3

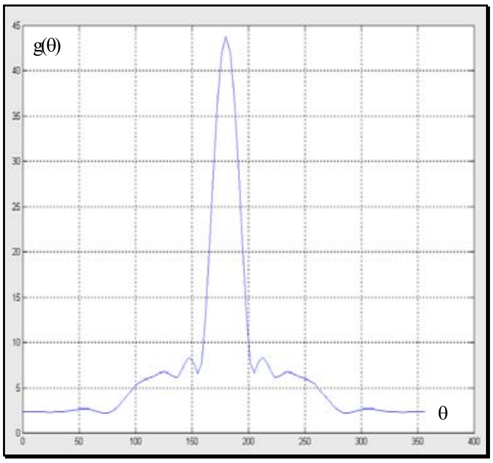

Fig.4

3 Conclusion

The numerical solutions of a number of model diffraction problems provided above allow us to conclude that the approach proposed in this paper has advantages over the traditional techniques for constructing the optimal auxiliary contour and allow to get any accuracy for body with analytical form of the surface. This modification of the MDS largely extends the class of admissible scatter's shapes, and, simultaneously, retains the simplicity of realization and versatility typical of the MDS. The method can easily be extended to plane-layered media, various types of incident waves and vector case.

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