HIGH-ACCURATE FINITE-DIFFIRENCE SCHEME FOR COMPUTATIONAL AEROACOUSTIC SIMULATION

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***Abstract*:-** Aeroacoustic is defined as the acoustic propagation of sound generated by fluid flow. Different aeroacoustic problems are used to test the computational schemes performance. A new high-accuracy numerical scheme was modified from Runge-Kutta method with more operators and improved factors. The scheme is tested and modified in the present work for benchmark computational aeroacoustic problems. Test of the numerical dissipation and dispersion properties of the computational scheme is developed to obtain the scheme accuracy with the running time. It is found that the new scheme is more stable under different initial conditions and with long time computation. The calculated speed of sound is in close agreement with the exact solution. Also, the nonlinear wave propagation of the scheme is considered for the initial value and shock tube cases with very accepted results comparing with the exact ones. The third category is a convergent-divergent nozzle with incident a small amplitude sound wave. This problem is used to test the suitability of the numerical scheme for direct numerical simulation of very small amplitude acoustic waves superimposed on a non-uniform mean flow in a semi-infinite duct. It can be concluded that the numerical scheme is robust and accurate for the tests considered.

***Key-words*:** Computational aeroacoustics, dissipation, dispersion, wave propagation, CFD

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1. Introduction

Computation of steady-state solutions to fluid flow problems is now well established. A problem with these CFD-type algorithms is dispersive errors that distort propagating waves. Dispersion caused by numerical artifacts that selectively alter phase shifts among the component wavelengths at each time step. In complex problems, it is not possible to separate algorithmic dispersion from true physical dispersion. These effects are subtler than amplitude related artifacts (dissipation) that cause either catastrophic failure or excessive smoothing.
A new numerical approach to dissipation-less finite difference schemes was reported by Jurgen and Zingg (1) which is considered as a modification of explicit fourth order Runge-Kutta method, it is fifth order solution. In general, this scheme is quite dispersive near discontinuities. In the recent studies it has appeared that high-order schemes would be more suitable for computational acoustics than the lower-order schemes since the former are usually less dispersive and less dissipative. In addition, the artificial dissipation, while having good shock resolving capabilities, can significantly contaminate the acoustic solution.

The use of computational methods in the analysis of unsteady flow and the resultant far field acoustic radiation requires care in the application of any numerical scheme. In particular, the dissipation and dispersion characteristics of the numerical scheme are critical to the accuracy of the solution. Dissipative schemes tend to natural unsteady or oscillatory disturbances while dispersive schemes generate non-physical oscillations. Either can degrade or contaminate the numerical solution of acoustic propagation to the point that it is unreliable.

In general, there are three types of waves present in unsteady flows. These include acoustic waves, which are isotropic, non-dispersive, non-dissipative, and propagate at the speed of sound, as well as entropy and vorticity waves, which are non-dispersive, non-dissipative and highly directional and propagate at the mean convection speed of the flow. The acoustic phenomenon is generally considered to be governed by the Linearized Euler equations.

$$\frac{du}{dt} = f(u,t)$$

Regarding the work that well reported in the state of art, Atkins (2) used a finite-difference essentially non-oscillatory (ENO) method to model adequately dissipation and dispersion problems and the shock tube as well using 8 points per wavelength. Also, Davis (3) derived a compact high order three-spatial point, two-time level dissipationless scheme to solve the planar and spherical acoustic waves. Accordingly Lee et al. (4) compared between different schemes to solve the problems considered with relatively adequate results. Fung et al. (5), Hu et al. (6) and Huynh (7) introduced the same results with using different schemes. Koprivea and Kolias (8) performed close agreement between the exact and computed solutions of the convergent-divergent nozzle. Many authors in the ICASE/LaRC workshop on benchmark problems in computational aeroacoustics (9) developed different schemes for the same purpose, the reader can find different models of the art. In these

$$(\partial_x^s u)_j = \frac{1}{\Delta x}[d0 u_j + d1(u_{j+1} + u_{j-1}) + d2(u_{j+2} + u_{j-2}) + d3(u_{j+3} + u_{j-3})]$$

studies, some of the schemes give adequate solution and a percentage of error in others. Therefore a new finite-difference scheme is used in the present study to test and modify its accuracy and performance under different conditions of dissipation, dispersion and wave propagation.

2. METHODOLOGY

High accuracy finite-difference method was modified from Runge-Kutta method. It consists of five operators in five steps. The method details can be found in ref. (1). The time-marching method, when applied to the ordinary differential equation and can be written as

And

$$\begin{aligned}
u^{(1)}_{n+\alpha 1} &= u_n + h\alpha 1 f_n, \\
u^{(2)}_{n+\alpha 2} &= u_n + h\alpha 2 f^{(1)}_{n+\alpha 1}, \\
u^{(3)}_{n+\alpha 3} &= u_n + h\alpha 3 f^{(2)}_{n+\alpha 2}, \\
u^{(4)}_{n+\alpha 4} &= u_n + h\alpha 4 f^{(3)}_{n+\alpha 3}, \\
u^{(5)}_{n+\alpha 5} &= u_n + h\alpha 5 f^{(4)}_{n+\alpha 4}, \\
u_{n+5} &= u_n + h f^{(5)}_{n+\alpha 5}
\end{aligned}$$

Where α1=1/6, α2=1/5, α3=1/4, α4=1/3, α5=1/2 and h is the time step and n is the iteration number. This method is applied into the following Euler equation.

$$\frac{\partial Q}{\partial t} + A\frac{\partial Q}{\partial x} = 0$$

Where

$$A\left(\frac{\partial Q}{\partial x}\right)_j = A(\partial_x^a Q)_j + |A|(\partial_x^s Q)_j$$

The antisymmetic or central-difference operator is

$$(\partial_x^a u)_j = \frac{1}{\Delta x}[a1(u_{j+1} - u_{j-1}) + a2(u_{j+2} - u_{j-2}) + a3(u_{j+3} - u_{j-3})$$

and a symmetic operator is

Where

$$|A| = X|\Lambda| X^{-1}$$

And X is the matrix of right eigenvectors of A, and Λ is the matrix of eigenvalues of A. The factor a1=3/4, a2=-3/20, a3=1/60, d0=1/10, d1=(-3/4)d0, d2=(3/10)d0, d3=(-1/20)d0.
This numerical method is tested and modified for the following computational aeroacoustics categories:

2.1 problems to test the numerical dispersion and dissipation properties of the computation scheme (linear waves)

This case includes two problems:

2.1.1 initial value problem

The non-dimensional form considered of the convection equation is

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0$$

Where –20≤ x ≥450 and the initial condition at t=0 is

$$u = 0.5\exp\left[-(\ln 2)\left(\frac{x}{3}\right)^2\right]$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) = -\frac{\partial p}{\partial x}$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + \gamma p\frac{\partial u}{\partial x} = 0$$

2.1.2 spherical wave problem

The above convection equation in the spherical form is

$$\frac{\partial u}{\partial t} + \frac{u}{r} + \frac{\partial u}{\partial r} = 0$$

With boundary velocity u = sin ωt where ω = π/4, π/3 and 5≤ r ≥450.

2.2 Problems to Test the Nonlinear Wave Propagation Properties of The Computational Scheme

This case can be expressed by one-dimensional Euler equations:

Two problems considered in this category; the initial value and shock tube problems. The x range considered in the two problems are –50≤ x ≥350 and –100≤ x ≥100 respectively.

2.3 problems to test the suitability of the numerical scheme for direct numerical simulation of very small amplitude acoustic waves superimposed on a non-uniform mean flow in a semi-infinite duct

The scheme is applied in this category for a convergent-divergent nozzle with the following governing equations:

Where

$$A(x) = \begin{cases} 134 & x \le -100 \\ 117 - 17\cos(x/100) & -100 \le x \ge 19 \\ 97.2 + 0.3x & 19 \le x \ge 80, \end{cases}$$

and γ=1.4

$$\frac{\partial \rho A}{\partial t} + \frac{\partial \rho u A}{\partial x} = 0$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) + \frac{\partial p}{\partial x} = 0$$

$$\frac{\partial p A}{\partial t} + \frac{\partial p u A}{\partial x} + (\gamma - 1)p\frac{\partial u A}{\partial x} = 0$$

presented in Fig.2 and Fig. 3 computed a long sinusoidal wavetrain. Excellent agreement between

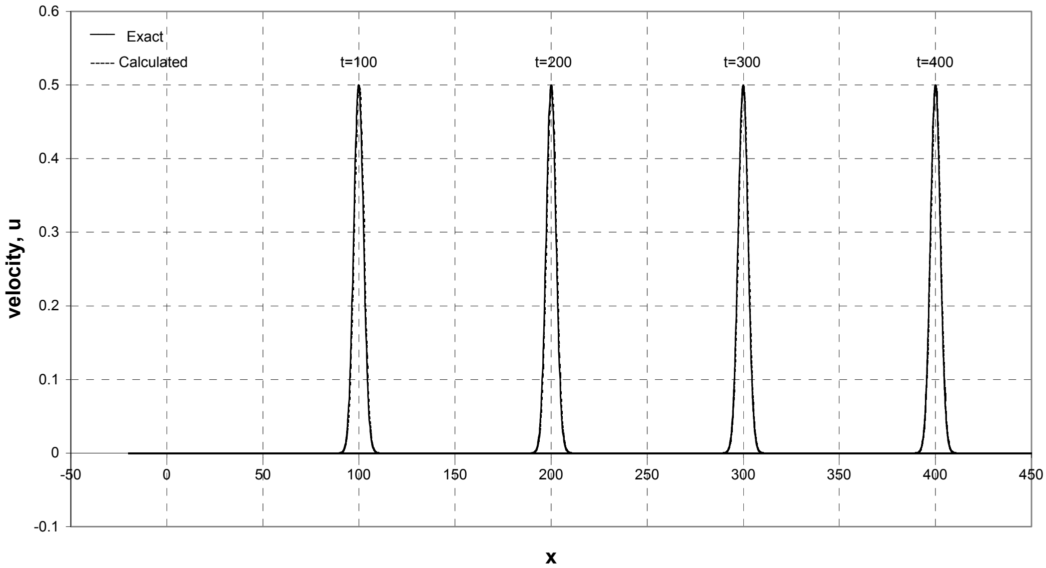

Fig. 1: Dispersion and Dissipation properties of the scheme (Initial value problem)

3. Results and Discussions

The first category is to test the dissipation and dispersion properties of the scheme for the linear advection and spherical waves. The initial condition for the first problem is a Gaussian distribution. As shown in Fig. 1 the wave is predicted exactly according to the exact wave. There are no dispersive and no dissipative errors in the wave. The grid spacing of 6 per wavelength is used with a courant number of 1 and 0.1 time step. Different time periods of computation are considered to explain the time effect on the scheme performance.

The second problem in the first category as the exact and computed solutions is obtained for both wave numbers, with no evidence to graphical precision of phase and dissipation errors. The errors are from the discontinuity in the first derivative at the front of the wave. It is possible to filter the oscillations at the front of the wave not in the back. Although the conclusions are applied for the two cases $\omega=\pi/4$ and $\pi/3$ it is more clear for the second case and after long time calculation. It is found that one point per wavelength is enough to obtain a fine solution with courant number of 0.1 and 0.1 time step.

For the second category, the first problem (initial value problem) starts with a Gaussian pressure distribution and velocity and density distributions

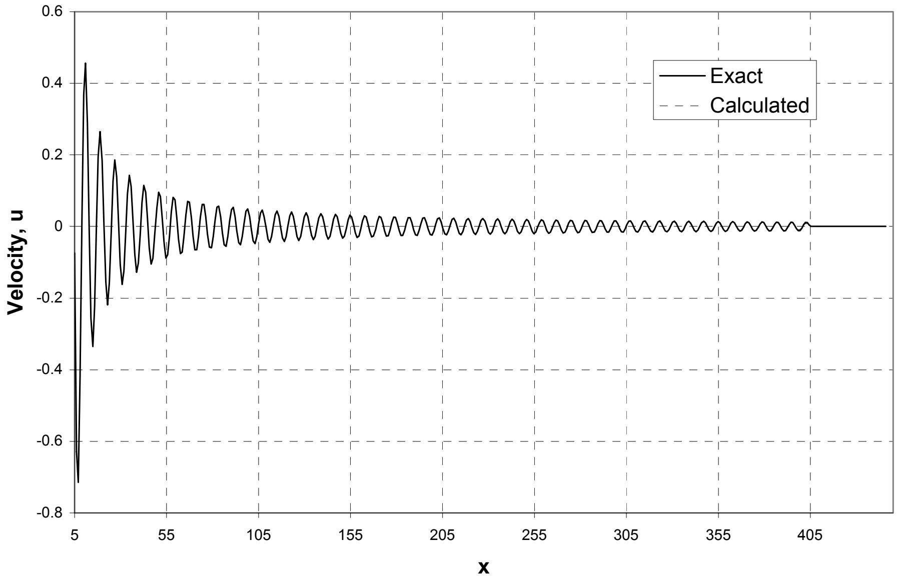

Fig. 2: Numerical dispersion and dissipation properties of the computation schem at t=400 (spherical wave problem, $\omega=\pi/4$)

that are appropriate for a right-traveling acoustic wave. The wave quickly steepens into a shock wave that propagates to the right and weakens. Five points per wavelength are used in this problem and 0.5-courant number. The properties are calculated at different time periods of 0, 50, 100, 200, 300 and 400. Fig. 3 shows that the calculated and exact solutions are completely the same.

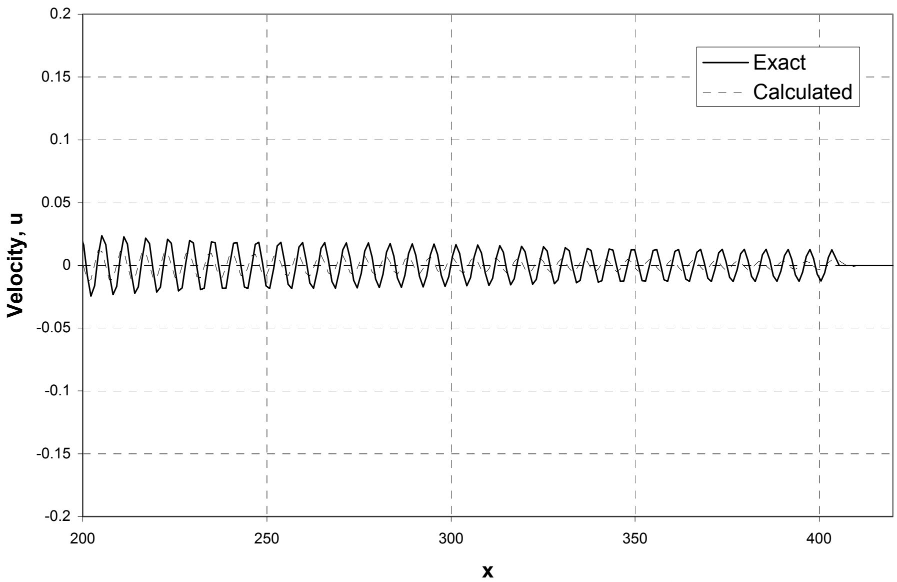

Fig. 3: Numerical dispersion and dissipation properties of the computation schem at t=400 (spherical wave problem, ω=π/3)

The shock tube problem is the second problem in the category, the shock wave is exited the domain by the final time. To avoid the oscillation around the shock five points per wavelength are used through the calculation. The factor α5 is modified from 0.5 into 1.6 to eliminate the oscillations around the shock occurring. The time step used is 0.1 where the courant number is 0.5. It is obtained that there is a good agreement between the exact and computed results along the tube with different times as presented in Fig. 5. Also it is seen that the shock is propagated with the time to the end of the tube.

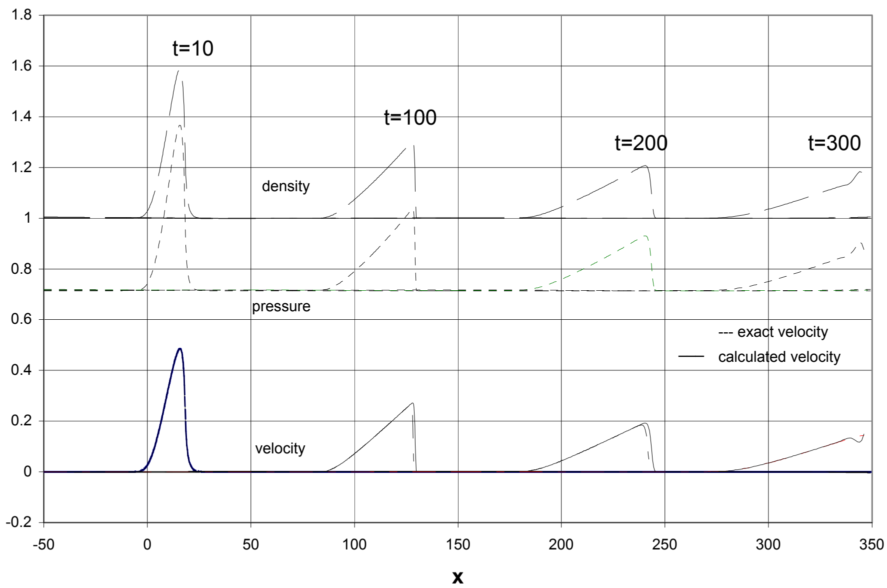

Fig. 4: Nonlinear wave propagation properties of the computation scheme (Initial value problem)

The last category that considered in this study is the third one that studies the flow in the semi-infinite duct to test the scheme for the small amplitude of

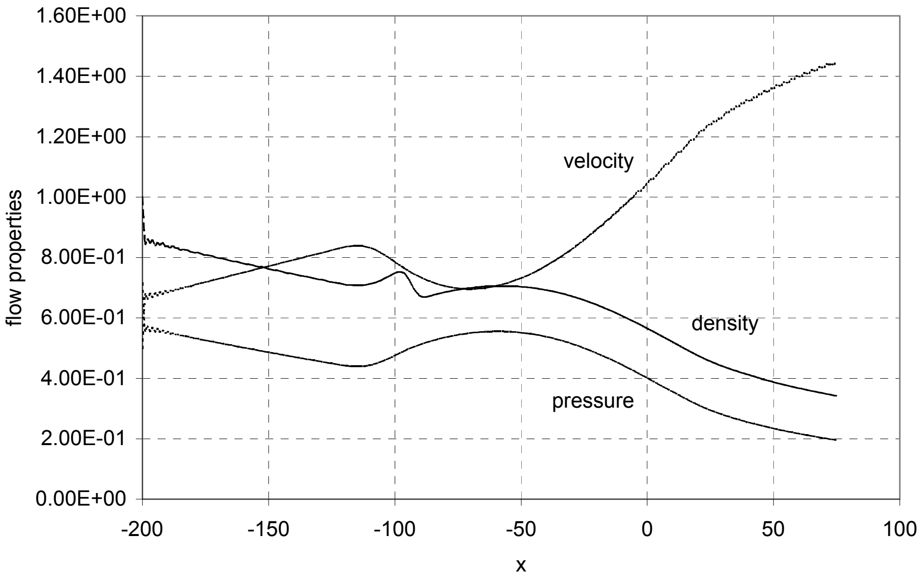

Fig. 6: Density, velocity and pressure of the convergent-divergent nozzle (steady state condition)

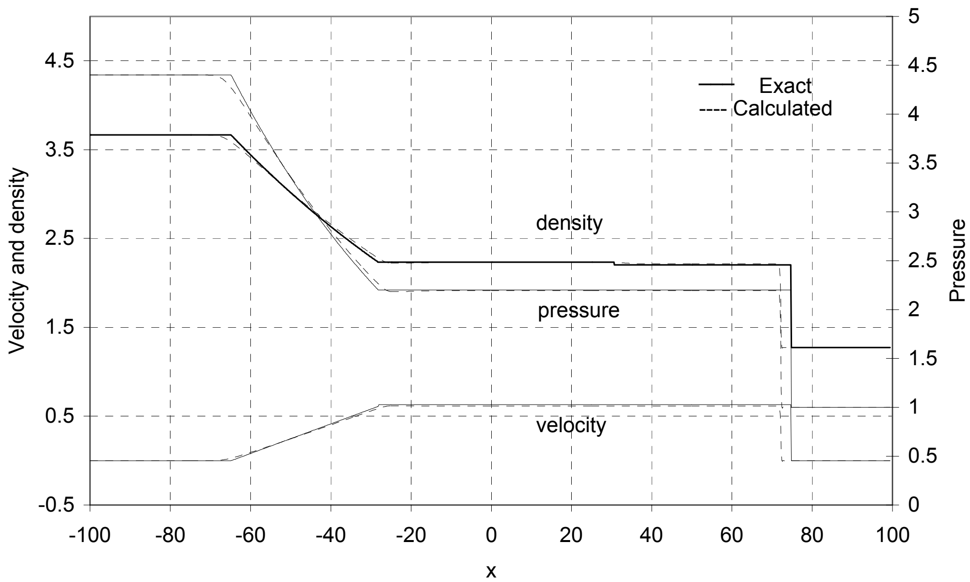

Fig. 5: Nonlinear wave propagation properties of the computation schem at t=50 (Shock tube problem)

acoustic non-uniform flow. The velocity (u), pressure (p) and density (ρ) of the flow are obtained for the convergent-divergent nozzle as presented in Fig. 6 after reaching steady state. In this problem there is an incoming acoustic wave at x=-200 and this wave is propagating to the right. This incoming wave is expressed as:

$$u = M + \varepsilon \sin\left[\omega\left(\frac{x}{1+M} - t\right)\right]$$

$$p = \frac{1}{\gamma} + \varepsilon \sin\left[\omega\left(\frac{x}{1+M} - t\right)\right]$$

$$\rho = 1 + \varepsilon \sin\left[\omega\left(\frac{x}{1+M} - t\right)\right]$$

Where M (Mach number) = 0.5, $\varepsilon = 10^{-6}$, $\omega = 0.1\pi$, γ=1.4 and t is the running time.

In comparison these results with the results reported by Lee et al. (4) it is obtained that there is an agreement between them and the scheme is stable and robust in these cases of modeling. Also Hu et al. (6) explained the same results that can be obtained in this category. Fig. 7 shows the convergence history of the calculation. Although the convergence history is in the fourth order only but the resulted values are expected. It can be concluded that the scheme needs a fourth-order dissipation treatment to fast the convergence history and to eliminate the oscillations at the inlet flow.

4. Conclusions

A new high-accuracy finite-difference scheme is presented in this study and different categories were carried out to test the scheme under different conditions. The dispersion and dissipation properties as well as the nonlinear wave propagation are simulated by the scheme. It is obtained that the scheme is non-dispersive and non-dissipative and there are no oscillations around the shock applications. The scheme was modified to avoid the oscillations around the shock. Also it is found that the scheme is suitable for the direct numerical simulation of very small amplitude acoustic waves superimposed on a non-uniform mean flows. Therefore, it can be concluded that although the scheme is robust and accurate for different numerical simulations it is costly in calculations.

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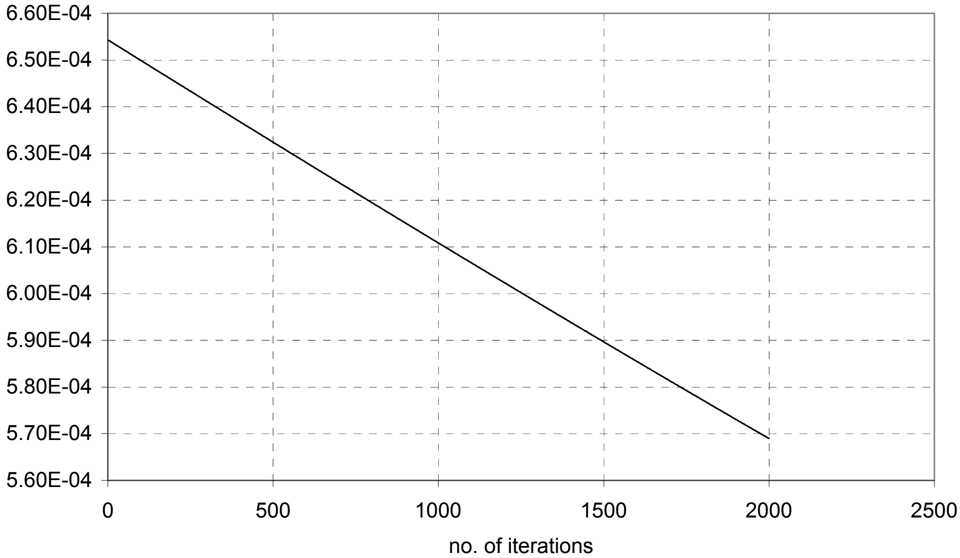

Fig. 7: Convergence history (Residuals) of the convergent-divergent nozzle after the steady state condition

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