Effective Method of Solving the Waves Diffraction Problems by Impedance Scatterers with Angular Boundary

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Abstract: - In this paper, we present the results of solution of three-dimensional electromagnetic scattering problems by impedance bodies of revolution, using the Pattern Equations Method (PEM). The good convergence this results is shown on the example of a prolate spheroid and a cylinder with spherical rounding.

Key-Words: - Scattering pattern, scatterer, impedance, prolate spheroid, cylinder

1 Introduction

The three-dimensional (3D) scattering problem by body is the classical problem of the diffraction theory [1]. It is known that the solving of this problem has some difficulties in resonance domain when the size of the body is about wavelength [2]. The body's boundary has some angels, the difficulties becomes much more. For example, in this case the well known and attractive method of the discrete sources [3] could not be applied as due to the theorem was proved in [4] the support of the auxiliary currents have to enclose all singularities of the diffracted field. It is evident that in case of mention above some of the singularities are located at the boundary of the body. It is clear too that situation is not corrected by smoothing the angels.

In addition to this the size of the linear system of equation arising by the method of integral equations is known to be about 10-20 *kd* (*k* – is the wave number, d is the characteristic size of the body). It is evident that when 3D problem is under consideration the order of that system of linear equations will be much increased.

The developed pattern equations method (PEM) [5,6] is shown to have a large velocity of convergence and its dependence of body's geometry is very weak. By our opinion this effect cold be explained as follows. The scattering pattern is the solving of the integral equation in the PEM. So we do not need in surface field's harmonics with high numbers and it leads to reducing of calculation's volume.

In this work we have developed the method PEM for a case of impedance boundary problem and body with angles. Besides is shown, that the method remains high effective and in the case, when the scatterer boundary has breaks.

2 Formulation of the Pattern Equations Method

Let consider the wave scattering problem for incident monochromatic electromagnetic field $\vec{E}^0, \vec{H}^0$ on a compact obstacle. Let the scattering body has a surface S and we have the boundary condition on S as follows

$$(\vec{n}\times\vec{E})\big|_S = -Z\left[\vec{n}\times(\vec{n}\times\vec{H})\right]_S,$$

where $\vec{E}=\vec{E}^0+\vec{E}^1, \vec{H}=\vec{H}^0+\vec{H}^1$ - complete field, and $\vec{E}^1, \vec{H}^1$ - secondary (diffracted) field, which satisfies the system of Maxwell's homogeneous equations everywhere outside of S

$$\nabla\times\vec{E}^1 = -ik\varsigma\vec{H}^1,\ \nabla\times\vec{H}^1 = \frac{ik}{\varsigma}\vec{E}^1,$$

where $k=\omega\sqrt{\varepsilon\mu}$ is the wave number, $\varsigma=\sqrt{\mu/\varepsilon}$ is the medium impedance and also the radiation condition at infinity, for example, of the form

$$\left(\vec{E}^1 \times \frac{\vec{r}}{r}\right) + \varsigma \vec{H} = o\left(\frac{1}{r}\right),$$

$$\left(\vec{H}^1 \times \frac{\vec{r}}{r}\right) - \frac{1}{\varsigma}\vec{E} = o\left(\frac{1}{r}\right), \quad r \equiv |\vec{r}| \to \infty.$$

In accordance with the standard PEM scheme[5,6], let us look for the scattering pattern (wave field pattern), i.e. function determining diffracted field dependence from angels (θ,φ) in spherical system of coordinates (r,θ,φ) in a so-called far zone (at $kr >> 1$), where asymptotic relation are implemented of kind:

$$\vec{E}^1 = \frac{\exp(-ikr)}{r}\vec{F}^E(\theta,\varphi) + O\left(\frac{1}{(kr)^2}\right),$$

$$\vec{H}^1 = \frac{\exp(-ikr)}{r}\vec{F}^H(\theta,\varphi) + O\left(\frac{1}{(kr)^2}\right),$$

in which $\vec{F}^E$, $\vec{F}^H$ - electrical and magnetic fields patterns, respectively.

It is known (see, for example, [7]), that:

$$\vec{F}^E(\theta,\varphi) = -\sum_{n=1}^{\infty}\sum_{m=-n}^{n} a_{nm} i^n \left(\vec{i}_r \times \Phi_n^m(\theta,\varphi)\right) - $$
$$-\sum_{n=1}^{\infty}\sum_{m=-n}^{n} b_{nm} i^n \varsigma \Phi_n^m(\theta,\varphi),$$

$$\vec{F}^H(\theta,\varphi) = \sum_{n=1}^{\infty}\sum_{m=-n}^{n} a_{nm} i^n \frac{1}{\varsigma}\Phi_n^m(\theta,\varphi) - $$
$$-\sum_{n=1}^{\infty}\sum_{m=-n}^{n} b_{nm} i^n \left(\vec{i}_r \times \Phi_n^m(\theta,\varphi)\right),$$

where

$$\Phi_n^m(\theta,\varphi) = \vec{r} \times \nabla P_n^m(\cos\theta)\cdot\exp(im\varphi).$$

Operating in accordance with the scheme, proposed in [6], one could get the linear system of algebraic equations with coefficients a_{nm}, b_{nm}

$$\begin{cases} a_{nm} = a_{nm}^0 + \sum_{q=1}^{\infty}\sum_{p=-q}^{q}\left(G_{nm,qp}^{11} a_{qp} + G_{nm,qp}^{12} b_{qp}\right), \\ b_{nm} = b_{nm}^0 + \sum_{q=1}^{\infty}\sum_{p=-q}^{q}\left(G_{nm,qp}^{21} a_{qp} + G_{nm,qp}^{22} b_{qp}\right), \end{cases} \quad (1)$$

$$n = 1,2,..., \quad |m| \le n.$$

In this system

$$a_{nm}^0 = a_{nm}^{00} + Za_{nm}^{z0}; \quad b_{nm}^0 = b_{nm}^{00} + Zb_{nm}^{z0};$$

$$G_{nm,qp}^{ij} = G_{nm,qp}^{0ij} + ZG_{nm,qp}^{zij},$$

where values of the appropriate variables is denoted by sign "0" at Z = 0, given in [6], and additional terms – by sign "z", due to impedance quantity difference from zero. For these terms, we have

$$a_{nm}^{z0} = \frac{-\varsigma}{4\pi} N_{nm} \int_S \left[\vec{n} \times (\vec{n} \times \vec{H}^0)\right]\Big|_S \cdot \overline{\vec{h}_{nm}^e} \; ds ,$$

$$b_{nm}^{z0} = \frac{1}{4\pi\varsigma} N_{nm} \int_S \left[\vec{n} \times (\vec{n} \times \vec{H}^0)\right]\Big|_S \cdot \overline{\vec{e}_{nm}^e} \; ds ,$$

where $N_{nm} = \frac{2n+1}{n(n+1)}\frac{(n-m)!}{(n+m)!}$, $\vec{n}$ – the unite vector of external normal to S,

$$G_{nm,qp}^{z11} = \frac{-\varsigma}{4\pi} N_{nm} \int_S \left[\vec{n} \times (\vec{n} \times \vec{H}_{qp}^e)\right]\Big|_S \cdot \overline{\vec{h}_{nm}^e} \; ds ,$$

$$G_{nm,qp}^{z12} = \frac{-\varsigma}{4\pi} N_{nm} \int_S \left[\vec{n} \times (\vec{n} \times \vec{H}_{qp}^h)\right]\Big|_S \cdot \overline{\vec{h}_{nm}^e} \; ds ,$$

$$G_{nm,qp}^{z21} = \frac{1}{4\pi\varsigma} N_{nm} \int_S \left[\vec{n} \times (\vec{n} \times \vec{H}_{qp}^e)\right]\Big|_S \cdot \overline{\vec{e}_{nm}^e} \; ds ,$$

$$G_{nm,qp}^{z22} = \frac{1}{4\pi\varsigma} N_{nm} \int_S \left[\vec{n} \times (\vec{n} \times \vec{H}_{qp}^h)\right]\Big|_S \cdot \overline{\vec{e}_{nm}^e} \; ds ,$$

where

$$\vec{H}_{qp}^e = \frac{ik}{\varsigma}\nabla \times \left(\vec{r}\psi_q^p\right), \quad \vec{H}_{qp}^h = \nabla \times \nabla \times \left(\vec{r}\psi_q^p\right),$$

$$\vec{h}_{nm}^e = \frac{ik}{\varsigma}\nabla \times \left(\vec{r}\chi_n^m\right),$$

$$\vec{e}_{nm}^e = \nabla \times \nabla \times \left(\vec{r}\chi_n^m\right),$$

$$\psi_n^m = h_n^{(2)}(kr)P_n^m(\cos\theta)e^{im\varphi},$$

$$\chi_n^m = j_n(kr)P_n^m(\cos\theta)e^{im\varphi}.$$

In case of a body of revolution, the system (1) could be written as follows

$$a_{nm} = a_{nm}^0 + \sum_{q=m}^{\infty}\left(G_{nm,qm}^{11} a_{qm} + G_{nm,qm}^{12} b_{qm}\right),$$
$$b_{nm} = b_{nm}^0 + \sum_{q=m}^{\infty}\left(G_{nm,qm}^{21} a_{qm} + G_{nm,qm}^{22} b_{qm}\right), \quad (2)$$
$$n = 1,2,..., |m| \le n,$$

where now matrix elements $G_{nm,qm}^{ij}$ are expressed by single integrals [6].

From asymptotic estimation of matrix elements and free terms executed by analogy with [6], it leads that in system (1) it is necessary to make the changing of unknown coefficients [5,6] and assign: $a_{nm} = \frac{\sigma^n}{n!} x_{nm}$, $b_{nm} = \frac{\sigma^n}{n!} y_{nm}$.

As a result of that changing, for example, system (2) take the following form:

$$x_{nm} = x^0_{nm} + \sum_{q=|m|}^{\infty} \left(g^{11}_{nm,qm} x_{qm} + g^{12}_{nm,qm} y_{qm} \right),$$
$$y_{nm} = y^0_{nm} + \sum_{q=|m|}^{\infty} \left(g^{21}_{nm,qm} x_{qm} + g^{22}_{nm,qm} y_{qm} \right), \quad (3)$$

where

$$x^0_{nm} = \frac{n!}{\sigma^n} a^0_{nm}, \qquad y^0_{nm} = \frac{n!}{\sigma^n} b^0_{nm},$$

$$g^{jl}_{nm,qp} = G^{jl}_{nm,qp} \frac{n!}{q!} \sigma^{q-n}.$$

The system (3) could be solved by a method of reduction if it is satisfy the next condition:

$$\sigma_2 > \sigma \quad (4)$$

(the definition of the variables σ, σ_2 see [5,6]). In the case, when the incident field is a plane wave the condition (4) gives the restrictions on the scatterer's geometry only (in this case it have to belong at the class of a so-called weakly non-convex bodies [5]).

3 Applications to Solution of Three-Dimensional Electromagnetic Problems

Let us consider some examples of applying the method given above. The next scattering problems by bodies of revolution were considered: the prolate spheroid with semi-axis *ka* = 2, *kc* = 4 (big axis of the spheroid is directed along an axis OZ) and the circular cylinder of radius *ka* = 2 with spherical roundings of the same radius and height of a cylindrical part $kh = 4$ (axis of the cylinder coincides with an axis OZ). The calculations were made for impedances: $Z = 0$ and $Z = i \cdot 120000 \cdot \pi$.

All calculations were made on the base of solving the system

$$\begin{cases} x_{nm} = x^0_{nm} + \sum_{q=|m|}^{N} \left(g^{11}_{nm,qm} x_{qm} + g^{12}_{nm,qm} y_{qm} \right), \\ y_{nm} = y^0_{nm} + \sum_{q=|m|}^{N} \left(g^{21}_{nm,qm} x_{qm} + g^{22}_{nm,qm} y_{qm} \right), \end{cases} \quad (5)$$

$|m| \le n, \quad n = 1,2,\ldots,N,$

which can be obtained from general system (3) for the bodies of revolution.

The system (5) was solved by the Gauss method of solving the linear algebraic system the of equations with a choice of the main element for columns. During the solving of system for everyone fixed m ($-N \le m \le N$) the systems of the following dimensions are obtained: the minimal system dimension was equal 2×2, and the maximal system dimension was equal $2 \cdot N \times 2 \cdot N$. So we had solved $2 \cdot N + 1$ systems of linear equations that reduced the numerical mistakes of a rounding off at accounts of coefficients x_{nm} and y_{nm}.

The results of calculation for the scattering pattern when incident wave is a plane wave with incident angels $\theta_0 = 0$, $\varphi_0 = 0$, vector $\vec{E}^0$ of the incident wave have direction along the *OX* axis are shown at Fig. 1, Fig. 2, Fig. 3, and Fig. 4. The moduli of a φ-th components of the scattering pattern for cylinder (dashed line) and spheroid (continuos line) are shown at Fig. 1, and the moduli of its a θ-th components are shown at Fig. 2 (dashed line for cylinder and continuos line for spheroid).

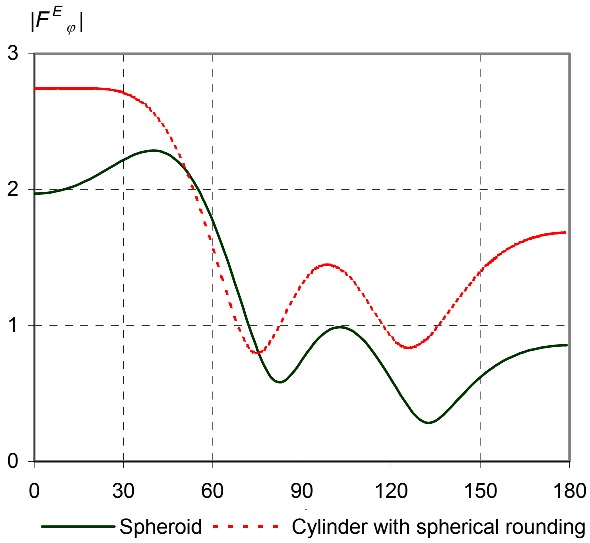

Fig.1: $\theta_0 = 0$ and $Z = 0$

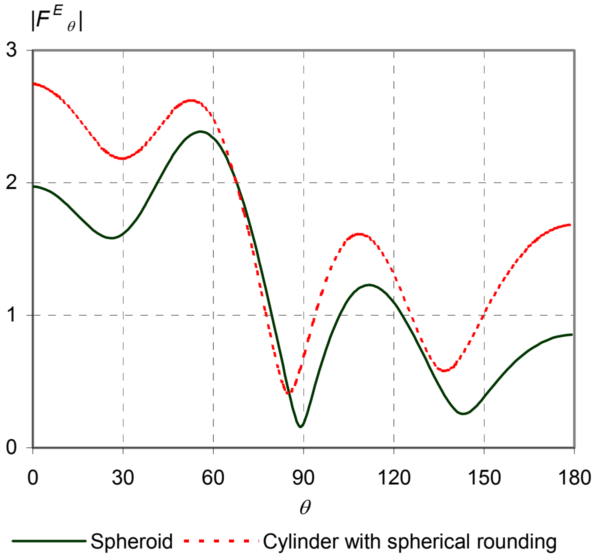

Fig.2: $\theta_0 = 0$ and $Z = 0$

The impedance here is taken equal to zero. The φ-th component of the scattering pattern was calculated in a half-plane $\varphi = \pi/2$ whereas its θ-th component in a half-plane $\varphi = 0$. It is visible that the appropriate components of the scattering pattern are enough close to each other as it also should be by virtue of geometrical affinity of the scattering bodies.

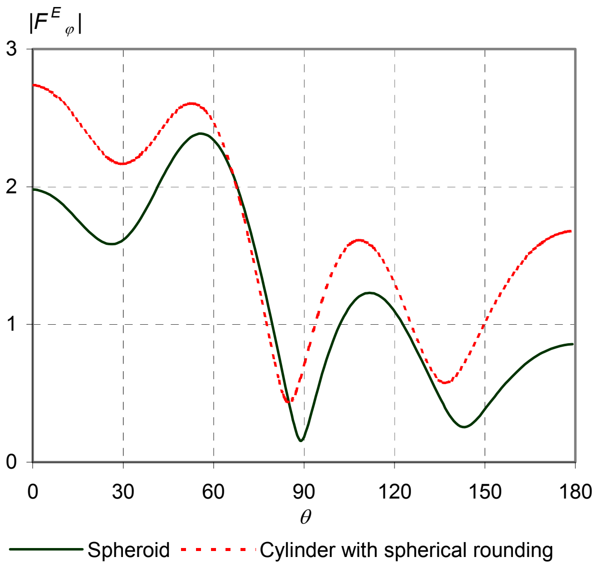

Fig.3: $\theta_0 = 0$ and $Z = i \cdot 120000 \cdot \pi$

Similar picture is observed at impedance $Z = i120000\pi$ Om. The moduli of a φ-th components (the half-plane $\varphi = \pi/2$) of the scattering pattern for cylinder and spheroid is shown at Fig. 3, and the moduli of its a θ-th components (the half-plane $\varphi = 0$) is shown at Fig. 4.

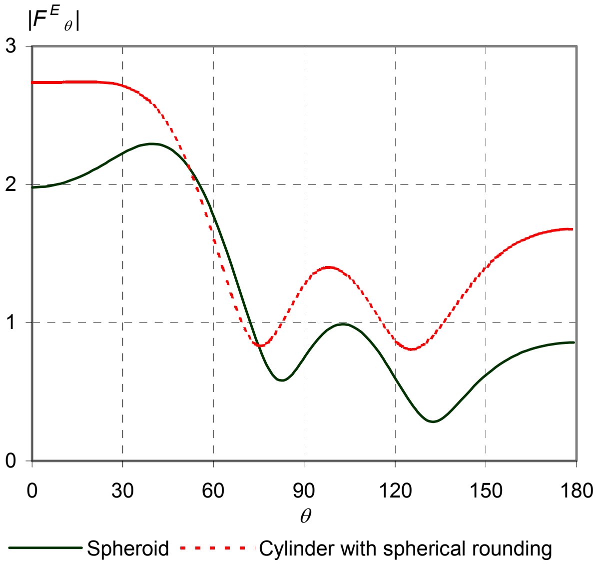

Fig.4: $\theta_0 = 0$ and $Z = i \cdot 120000 \cdot \pi$

The perpendicular incidence of the unit plane wave with angels of incidence $\theta_0 = 90$ and $\varphi_0 = 0$ also was considered. Thus the vector $\vec{E}^0$ was directed along the axis *OZ*. The moduli of a φ-th and θ-th components of the scattering pattern for cylinder (dashed line) and spheroid (continuos line) are shown at Fig. 5 and Fig. 6 respectively at impedance $Z = 0$. All φ-th components were considered in the plane $\varphi = \pi/2$, $\varphi = 3\pi/2$ All θ-th components were considered in the plane $\varphi = 0$ and $\varphi = \pi$ ($\theta = 0...180$) corresponding to range $\theta = 180...360$ at Fig. 6. The moduli of the same components of the scattering pattern for cylinder (dashed line) and spheroid (continuous line) and in the same planes are shown at Fig. 7 and Fig. 8 but already at impedance $Z = i \cdot 120000 \cdot \pi$ Om.

All given figures of the scattering patterns for spheroid and cylinder with spherical rounding were calculated at a value $N = 17$ (see equation (5)) that is $N \approx 2 \cdot kd$.

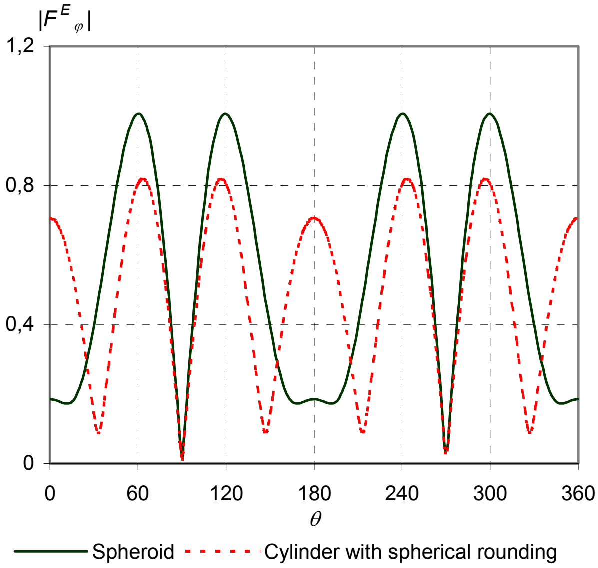

Fig. 5: $\theta_0 = 90$ и $Z = 0$

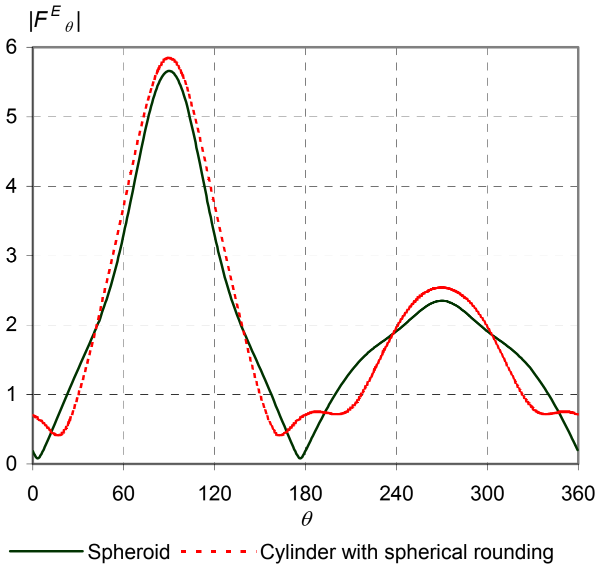

Fig. 6: $\theta_0 = 90$ и $Z = 0$

In Table the date are given that illustrate the velocity of convergence of computational algorithm for spheroid and cylinder discussed above. It is visible that in a case of scatterer with analytical boundary (spheroid) the five correct signs after a separatrix are established in the scattering pattern already at $N = 11$ (where N - upper limit of summation in system (5)), that is at $N \approx 1,5\,kd$, where d is greatest body size. In the case of the body with non-analytical boundary (cylinder with spherical rounding) the two correct signs after a separatrix are established at $N \approx 2 \cdot kd$ only.

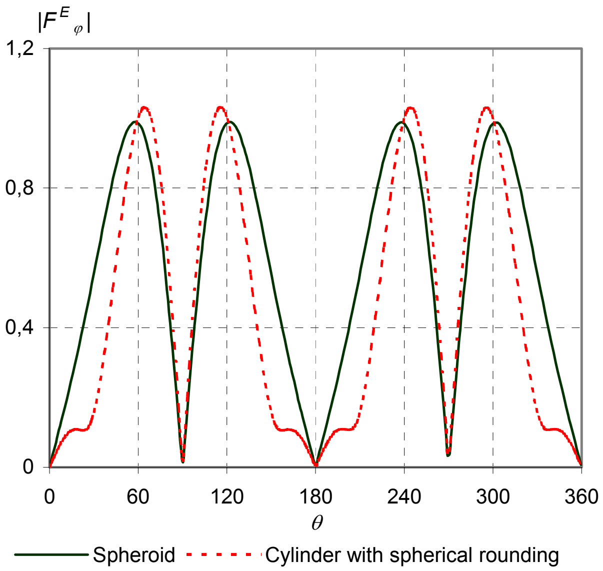

Fig.7: $\theta_0 = 90$ and $Z = i \cdot 120000 \cdot \pi$

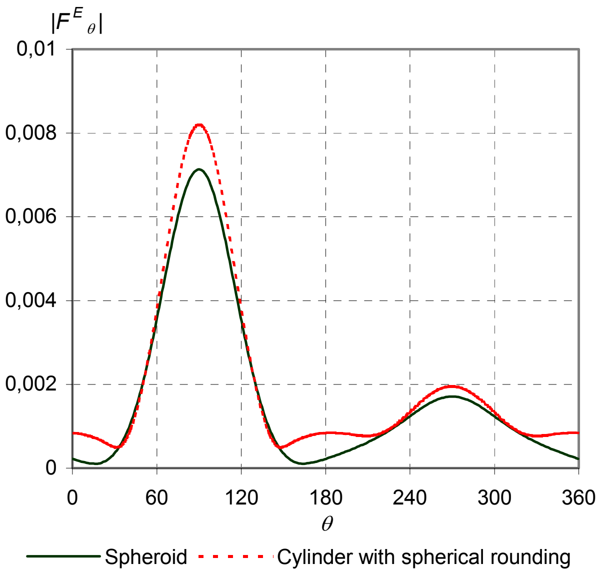

Fig.8: $\theta_0 = 90$ and $Z = i \cdot 120000 \cdot \pi$

The proposed approach can be easily generalized on the wave scattering problems by a group of dielectric bodies.

Table: Check of the convergence of pattern value modulus $\left|F_{\theta}^{E}(\theta,\varphi)\right|$ at θ =180 and φ=0.

N		11	13	15
axis incidence				
spheroid	Z=0	0,854868822498156	0,85486414208137	0,854864228215656
	Z=i120000π	0,856924333229293	0,856863730513405	0,85685273260578
cylinder	Z=0	1,65621143098733	1,6868449964265	1,69019882112764
	Z=i120000π	1,70859547869609	1,68945906039784	1,678856238035
N		17	19	21
axis incidence				
spheroid	Z=0	0,854864229507943	0,854864226174644	0,854864219279615
	Z=i120000π	0,856849417403774	0,856848453777103	0,856848171390379
cylinder	Z=0	1,68377344549195	1,68188432930541	1,68298761989378
	Z=i120000π	1,6810970676356	1,68893225887373	1,690169349337

N		11	13	15
perpendicular incidence				
spheroid	Z=0	0,182612146761232	0,182614203051027	0,182614158434439
	Z=i120000π	0,000219664894677348	0,000219778329397654	0,000219811348324129
cylinder	Z=0	0,717823417686801	0,695218125678592	0,697992743074454
	Z=i120000π	0,000819809217106535	0,000833177342415702	0,000846700713742607
N		17	19	21
perpendicular incidence				
spheroid	Z=0	0,182614148988725	0,182614139523385	0,182614130667818
	Z=i120000π	0,000219821297770531	0,000219824355709049	0,000219825313774006
cylinder	Z=0	0,705599779834765	0,706118765687515	0,703785361782326
	Z=i120000π	0,000841787328038978	0,000836647322054094	0,000838459518518361

4 Conlusion

For reason of obtained scattering patterns for impedance bodies we can lead to a conclusion that the suggested PEM allows to solve such problems with rather high efficiency. The computational algorithm constructed on the base of PEM appears extremely fast-acting and do not require additional actions for elimination of boundary singularities of scatterer.

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