Volume integral method application for the computer modelling of electromagnetic wave propagation in periodic optical structures

ALEXANDER LERER and GALINA KALINCHENKO
Applied Electrodynamics and Computer Modelling Department
Rostov State University
Sorge St 5, 344090 Rostov-on-Don
RUSSIA

Abstract: - The method of volume integral equations (VIE) has been developed for a simulation of two-dimensional waveguide dielectric structure containing a diffraction grating. The efficient numerical codes are created. Complex dispersion curves, transmitting and reflection coefficients have been obtained.

Key-Words: - volume integral equation, Green function, numerical guadratura, Bragg grating, dispersion curve.

1 Introduction

The investigation proposed is an example of lucky application of modern electrodynamics analytical methods to optical waveguide structures simulation. VIE gives an ability to represent the optical electromagnetic wave as a vector and treat the propagation problem analytically. The method under consideration is not only one applied for rigorous theoretical analysis of complex diffraction gratings, the methods of surface [1] and diagram [2] integral might be used as well. But in our opinion VIE has the number of advantages: 1) they are simple; 2) the electric field might be obtained as the direct result of solution; 4) there is a possibility to simulate dielectric structures with nonlinear permittivity.

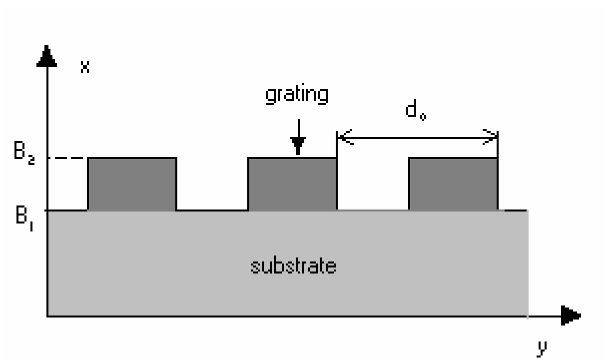

Fig.1 The structure under consideration. A dielectric Bragg grating placed over a substrate. The substrate might be multi-layer.

The object to simulate has been chosen because a great number of diffraction grating types is widely used nowadays in various fields of optoelectronics. Since emergence, this component has found many applications due to its properties, versatility and variety of controllable parameters that can shape in various manners its spectral properties. Due to its possibilities, applications are still being discovered as well as new grating types surge everyday [3]. Here we have considered well-known type of dielectric grating placed over a substrate as it is shown in Fig.1

2 Problem Formulation

The solution of Helmholtz equation has been obtained in the form of following integral equations:

1) for E –polarisation $\vec{E}(0,0,E)$-

$$E(x,y) = E^i(x,y) + k^2 \int_S \psi(x',y') E(x',y') G(x,x',y,y') ds' \qquad x,y \in S \tag{1}$$

where E^i is external field, $G()$ - Green function (GF), $\psi(x,y) = \varepsilon_+(x,y) - \varepsilon_-$, $\varepsilon_+, \varepsilon_-$ - permittivites of a body (here the body is the grating) and ambient space correspondingly S - body cross-section, x – observation point, x' – source point;

2) for H – polarization $\vec{H}(0,0,H)$

$$H(x,y)=\frac{\varepsilon_+}{\varepsilon_-}H^i(x,y)+k^2\int_S \psi(x',y')H(x',y')G(x,x',y,y')ds'+$$
$$\int_L \nu_- \frac{\partial G(x,x',y,y')}{\partial n'}H(x',y')dl'; \quad x,y\in S$$

where L- is the contour of area S, $\nu_\pm = \psi/\varepsilon_\pm$, $\frac{\partial}{\partial n'}$- normal derivation.

2.1 Green function

Finding of the GF is the crucial point of the whole study, and it is interesting to consider the procedure in detail, for example, for E-polarised wave. It is convenient to take GF for free space. the appropriate equation is

$$\Delta G + k^2 G = -\delta(x-x')\delta(y-y').$$

Note, that in the frameworks of our problem we supposed the free space to be vacuum $\varepsilon_- = 1$.

The grating is unlimited periodical structure and therefore we have an ability to expand the GF into the series on space harmonics

$$G(s,s') = \frac{1}{2d}\sum_{l=-\infty}^{\infty} e^{i\alpha_l(y-y')}\tilde{G}_l(x,x'\alpha_l) \quad (2)$$

where l is the number of a space cell, $\alpha_l = n\pi/d_0$. Obviously, for the each space harmonic $\tilde{G}_l$ we have following equation

$$\left(\frac{d^2}{dx^2}-\chi^2\right)\tilde{G}_l = -\delta(x-x'), \quad (3)$$

where $\chi_l = \sqrt{\alpha_l^2 - k^2\varepsilon}$. It is possible to consider the structure as multi-layer one and to find a general solution of inhomogeneous equation (3) separately for every layer. It might be taken in the form:

$$\tilde{G}_{sub} = A_{sub}e^{i\chi_- x} \qquad \text{for} \quad x<0;$$

$$\tilde{G}_m = A_m(\alpha)\sin\chi_m(B_m - x) + A_{m+1}(\alpha)\sin\chi_m(x-B_{m-1}) \qquad \text{for} \quad 0<x<B_2.$$

$$\tilde{G}_{over} = A_{over}\, e^{-i\chi_-(x-B_2)} \qquad \text{for } x>B_2;$$

where B_2 is the upper layer border coordinate, as it is shown in Fig.1, m - the current number of layer inside the structure (if the substrate is homogeneous, we have two layers – grating and substrate).

Full solution for $B_1<x<B_2$ is a sum of two terms, general and particular solutions. The particular solution might be represented as

$$\tilde{G}_{nreg} = \frac{1}{2i\chi_{grat}}e^{-i\chi_{grat}|x-B_1|}.$$

The unknown coefficients A are to be found using the continuity of $\tilde{G}_l$ and $\frac{\partial \tilde{G}_l}{\partial x}$ on the boundary of layers.

3 Problem Solution

It is known that Green function has logarithmic singularity when $s'\to s$, where $s\ (x,y)$ Therefore to solve IE it is necessary to extract this singularity. Here we propose an effective method of singularity extraction. After the identical transformation of equation (1) we have

$$E(s) = E_0(s)+\tau(s)E(s)\int_S G_0(s,s')ds'+$$
$$\int_S [\tau(s')E(s')G(s,s')-\tau(s)E(s)G_0(s,s')]ds' \quad (4)$$

where $G_0(s,s')=\log(r)/2\pi$ (r- the distance between s and s') is the function with the same singularity as G. The first integral in (4) can be calculated analytically when S is of circle, rectangular or ring shape. Thus, for general case it is possible to transform this integral into one without singularities. The second integral in (4) has no singularities and could be simulated using numerical quadratura. Satisfying to condition (4) in the knots of quadratura we arrive at the set of algebraic equations

$$E(s_p) = E_0(s_p)+\tau(s_p)E(s_p)T(s_p)+$$
$$\sum_{q=1}^{M} A_q\left[\begin{array}{l}\tau(s_q)E(s_q)G(s_p,s_q)-\\ \qquad \tau(s_p)E(s_p)G_0(s_p,s_q)\end{array}\right] \quad (3)$$

where M – total number of quadratura's knots, s_q – quadratura's points, A_q - quadratura's coefficients and $T(s_p)=\int_S G_0(s_p,s')ds'$.

For simulation of two-fold integrals, the quadrate formulae of Gauss type have been used. Using such formulae will give a highest algebraic accuracy rate.

To reach the deviation of about 0.01% by using our method, it is required to satisfy the condition $K_j\lambda_\varepsilon>12D_j$, where D_j and K_j are the bar dimensions and points number in Gauss formula correspondingly, λ_ε - wavelength in the dielectric. Note, that when calculating the characteristics of bars with dimensions about $D_j \cong 2\lambda_\varepsilon$ we should solve a set of 36-th order linear algebraic equations.

4 Conclusion

According to the method elaborated the computer modeling of the structures under consideration have been carried out. In the Fig.2 the dispersion characteristic of dielectric waveguide with periodical rectangular cut is shown.

Attempt to give adequate physical interpretation to the obtained result leads us to the following conclusion. The picture of electromagnetic waves in the structure represents a great number of propagating modes and ones reflected from the grating borders. The modes interact, and it is possible to reduce the situation to two waves propagating different directions. In the terms of our method the waves correspond with zero and minus first spatial harmonics. Zero harmonics has positive dispersion, its graph represented on the Fig.2 as curves with negative incline. Minus first one has negative dispersion and its graph is increasing function.

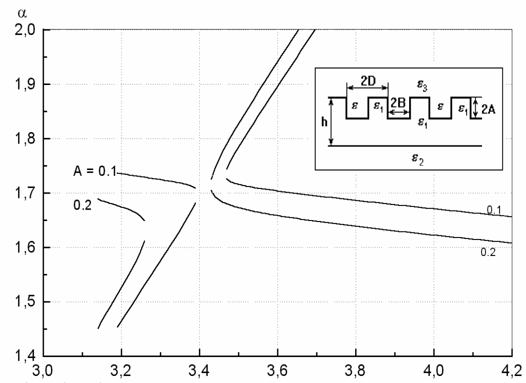

Fig.2 Dispersion curves
Grating parametres h = 1.0 D = 0.5 B = 0.25 ε_1= 4 ε_2= 2.1 ε_3= 1 ε= 1

Thus, unlimited dielectric structure consisting of a grating placed on a substrate has been simulated with a volume integral method. Complex dispersion curves are obtained.

Present investigation is the first step of complex study devoted to the propagation in different types of dielectric diffraction gratings.

References:

[1] Boriskina S., Nosich A. Numerical analysis of surface-wave filters based on a whispering-galery-mode dielectric resonator and slitted metal cavity *Radiophysics and Radioastronomy.*. Vol. 2. No.3. 1997 p.333.

[2] Kurkchan A.G., Manenkov S.A. Electromagnetic wave scattering by inhomogeneity placed next to flat border of two dielectrics *Radiotechnika i Electronika*, , Vol.43, No.1, 1998 pp 8 – 15, (in Russian)

[3] Lee K. S., Erdogan T. Mode coupling in spiral fiber *gratings Electronic Letters*, Vol. 37, 2001, pp.156-157.