About universal modification of the method of discrete sources

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Abstract: - A new universal modification of the method of discrete sources (MMDS) was applied for the scattering of an E and H polarized plane wave by dielectric and perfect conducting cylinders with complicated cross section contour (multifoil). It was shown that utilizing an analytical transformation of the cylinder's cross section contour allows to construct the auxiliary contour in the procedure of the method discrete sources and it leads to stable results with high accuracy.

Key-Words: - discrete sources, analytic transformation, 2D metal and dielectric scatterers

1 Introduction

When numerically simulating wave fields of various physical origins, one naturally wants to obtain a universal method for solving problem with a given accuracy and numerical computational cost. Present numerical methods (for example, the method of integral equations (MIE) [1]) usually provide only a certain optimal trade-off between the accuracy and numerical computation time. It is important to stress that the key element in achieving this optimal trade-off is utilizing of efficient numerical methods, which is valid in a sufficiently wide range of parameters of the problem and are stable and fast.

In recent years one can see that such methods as the method of discrete sources (MDS) [2], the method of auxiliary currents (MAC) [3] and its spline modification [4] are widely practiced due to their simplicity and algorithmic. MDS is based on the theorem of V. D. Kupradze [2] in which was proved the completeness for a set of fundamental solutions to the Helmholtz equation that have singularities on a certain closed surface (contour) Σ inside scatterer. The theorem proved by V. D. Kupradze places almost no constraints on the geometry of Σ. Because of this, it has long been unknown why the algorithm becomes unstable under certain circumstances. In [3], the origin of this effect was found and constraints on the geometry of Σ were formulated in form of the theorem (see also [5]): Let a simple closed Lyapunov curve Σ be such that k is not an eigenvalue of the interior homogeneous Dirichlet problem for the region inside Σ. Then the Fredholm integral equation

$$\int_{\Sigma} H_0^{(2)}(k\,|\,\vec{r}_S - \vec{r}_\Sigma\,|)I(\vec{r}_\Sigma)d\sigma = -U_0(\vec{r}_S), \quad \vec{r}_S \in S$$

of the first kind, in which $U_0(\vec{r})$ is a incident wave, $I(\vec{r}_\Sigma)$ is a auxiliary current, has a solution if and only if Σ encloses all of the singularities of the diffracted field inside of scatter's contour S. Using this theorem a simple and universal method for constructing Σ was proposed in [6] for a case of Dirichlet or Neuman boundary problems.

In this paper we has developed MMDS for a case of dielectric scatterers and E or H polarized incident plane waves. The results of it's utilizing are illustrated by solving of the scattering problem by dielectric and metal elliptic cylinders, perfect conducting and dielectric cylinder with complicated cross section contour (multifoil contour) in case of plane incident wave when the ordinary methods MDS and MAC are failed. The problems accuracy and stable are investigated when the extent of haul a contour Σ taut closer at singularities and interpretation of the results is made.

2 Diffraction of a plane wave by cylindrical structures

First of all let consider two-dimensional problem for scattering of a plane wave $U_0(r,\varphi)$:

$$U_0(r,\varphi) = \exp\{-ikr\cos(\varphi - \varphi_0)\} \qquad (1)$$

by dielectric cylinder in cylindrical coordinate system (r,φ,z). We suppose that *z* axis is the axis of cylinder; in

(1) φ_0, k are the angle of incident and the wave number of the plane wave respectively. Let the cross section contour S of the cylinder's boundary has equation in the polar coordinates as follows

$$r|_S = \rho(\varphi). \qquad (2)$$

According to MDS, the diffracted field $U\ (r,\varphi)$ outside of S and field $U^i(r,\varphi)$ inside of S could be presented in the next form

$$U(r,\varphi)=\sum_{m=1}^{N} A_m H_0^{(2)}(k|\vec{r}-\vec{r}_m|),$$
$$U^i(r,\varphi)=\sum_{m=1}^{N} B_m H_0^{(2)}(k_i|\vec{r}-\vec{r}_m^{\,'}|) \qquad (3)$$

Here, $H_0^{(2)}(k|\vec{r}-\vec{r}_m|)$ is the fundamental solution to the Helmholz equation; A_m, B_m are the coefficients to be determinate; $|\vec{r}-\vec{r}_m| = [r^2+r_m^2-2rr_m\cos(\varphi-\theta_m)]^{1/2}$ -is the distance between points with radius vectors $\vec{r}$, $\vec{r}_m$ in the polar coordinates; $\vec{r}_m, \vec{r}_m'$ - are the radius vectors position's of the sources on Σ within the boundary S and Σ' outside of S, k_i - the wave number of the medium inside of S. The system of algebraic equations for coefficients A_m, B_m is obtained by placing (3) under boundary condition

$$U^i(r,\varphi)|_S - U(r,\varphi)|_S = U_0(r,\varphi)|_S;$$
$$\frac{\partial}{\partial n}\{\chi U^i(r,\varphi)|_S - -U(r,\varphi)|_S\} = \frac{\partial}{\partial n}U_0(r,\varphi)|_S; \qquad (4)$$

where $\chi = 1$ in the case of E polarized incident wave and $\chi = 1/\varepsilon_r$ in case of H polarized incident wave; ε_r is the relative dielectric penetrability of cylinder. When the scattering problem by perfect conducting cylinder is under consideration then parameter $\chi = 0$, $U^i(r,\varphi) = 0$, contour Σ' is absent and (4) could be reduced at the Dirichlet or Neumann boundary problems.

The main arising problem in MDS is known how to detect the location of Σ and Σ' contours. Almost all published works had choused this contours as like as S one. In [6] was shown that this way has a limitation for accuracy and leads to unstable result. To avoid this effects we have to find all singularities of the diffracted field and make an analytical transformation of the contour S to enclose its [6]. As was indicated in [7] for plane incident wave and S as ellipse

$$\rho(\varphi) = \frac{a}{\sqrt{1-\varepsilon^2\cos^2(\varphi)}};$$
$$\varepsilon^2 = 1 - \frac{a^2}{b^2}; \qquad (5)$$

or multifoil cylinder

$$\rho(\varphi) = a + b\cos(q\varphi), q = 1,2,3,...; \qquad (6)$$

the singularities can be found in analytical form. Using the results [6,7] one can write for Σ, and Σ' contours next equations in case of ellipse (5)

$$r_\Sigma =, \varphi_\Sigma = \arg(\zeta),$$
$$\zeta = \rho(\varphi + i\varphi_1)\exp(\varphi + i\varphi_1); \qquad (7)$$
$$\varphi_1 = -\ln[\varepsilon/\{2-\varepsilon^2\}^{1/2}] + \delta;$$
$$r_{\Sigma'} = |\zeta|, \varphi_{\Sigma'} = \arg(\zeta),$$
$$\zeta = \rho(\varphi - i\varphi_1)\exp(\varphi - i\varphi_1); \qquad (8)$$

and for multifoil cylinder (6)

$$r_\Sigma = |\zeta|, \varphi_\Sigma = \arg(\zeta),$$
$$\zeta = \rho(\varphi + i\varphi_1)\exp(\varphi + i\varphi_1); \qquad (9)$$
$$\varphi_1 = \delta - \ln[b(q-1)/aG];$$
$$r_{\Sigma'} = |\zeta|, \varphi_{\Sigma'} = \arg(\zeta),$$
$$\zeta = \rho(\varphi + i\varphi_2)\exp(\varphi + i\varphi_2); \qquad (10)$$
$$\varphi_2 = \delta + \ln[b(q-1)/aG_1]$$

where δ - specifies the extent of nearness of Σ, Σ' at singularities,

$$G = [1 + b^2(q-1)/a^2]^{1/q},$$
$$G_1 = [b^2(q-1)^2/a^2 - 1]^{1/q}.$$

It was detected that applying of these contours (7)-(10) leads to stable results and the accuracy (we estimate the accuracy Δ by fulfillment of the boundary condition on S) becomes much better when contours approach to singularities (δ is decreasing) and N is fixed or N is increasing and δ is fixed. So we have a possibility to achieve high accuracy and stable by changing of δ, N parameters.

The result of calculations of the scattering pattern g(φ) for dielectric ellipse (6), H polarized plane incident wave, N=512, ka=1, kb=9, $\varepsilon_r = 4$, $\varphi_0 = 0$, max(Δ)=0.012, δ=0.000001 and E polarized plane incident wave, max(Δ)=0.0238 are depicted in Fig. 1 and Fig. 2 respectively. This situation is close at scattering problem by dielectric band.

The scattering pattern $g(\varphi)$ for E polarized plane incident wave, dialectical multifoil (6) when *kb=1*, *ka=10*, *q*=3, $\varphi_0 = \pi$, N=256, δ=0.000001, max(Δ)=1.05*10^{-7}, $\varepsilon_r = 4$ (in this case contour S is close at triangular one) is shown at Fig.3 . The result of calculation $g(\varphi)$ for dielectrical multifoil with *ka*=5, *kb*=0.5, *q*=4, N=256, $\varphi_0 = 0$, δ=0.000001, $\varepsilon_r = 4$, max(Δ)=$1.2*10^{-12}$ (in this case contour S is close at square one) is presented at Figure 4.

At last, we had a graphical agreement between scattering pattern $g(\varphi)$ for perfect conducting cylinder with ellipse cross section, ka=0.1, kb=5.3, N=512, δ=0.000001, $\varphi_0 = 0$, E and H polarized incident plane wave and the $g(\varphi)$ for metal band [8].

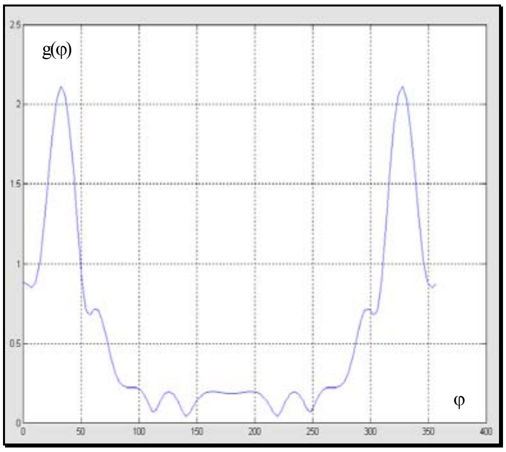

Fig.1

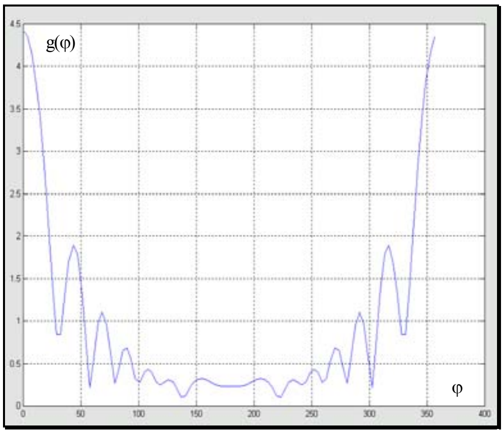

Fig.2

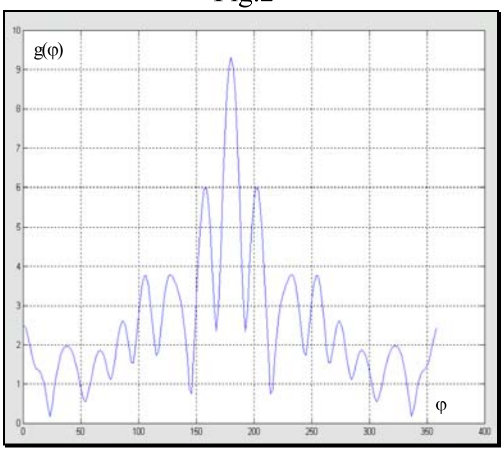

Fig.3

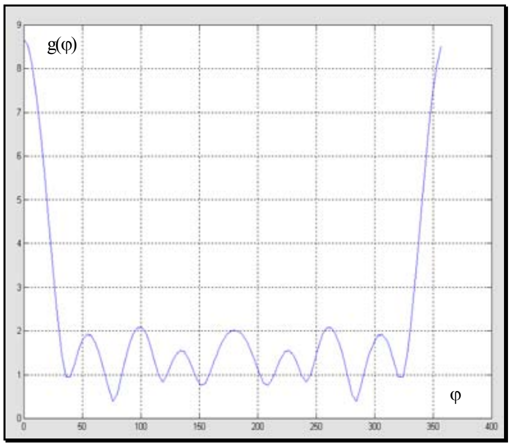

Fig.4

3 Conclusion

The numerical solutions of a number of model diffraction problems provided above allow us to conclude that the approach proposed in this paper has significant advantages over the traditional techniques for constructing the optimal auxiliary contour. This modification of the MDS largely extends the class of admissible scatterer's shapes and dimensions, to a great extensive improves the accuracy, and, simultaneously, retains the simplicity of realization and versatility typical of the MDS. It is promising to use this idea for developing a modified procedure of the method of auxiliary currents based on the spline approximation [4]. The method can easily be extended to three-dimensional problems, to plane-layered media and various types of incident waves.

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References:

[1] D.Colton and R.Kress, *Integral Equation Methods in Scattering Theory* Wiley, New York, 1983.

[2] V.D.Kupradze, "On the approximate solutions of problems of the mathematical physics", Success Mathematical Sci., vol. 22, 1967, pp.58-108 .

[3] A.G.Kyurkchan, "On the method of auxiliary currents and sources in problems of waves diffraction", *Radio Eng. Electron. Phys*., vol. 28, No 8, 1984, pp. 2129-2139.

[4] A.G.Kyurkchan and A.I.Sukov "Method of auxiliary currents and the numerical simulation of wave fields", *4th Conf. on Electromagn. and Light Scatt. by Nonspher. Part.: Theory and Appl.*, Vigo, Spain., Sept,1999, pp. 265-267.

[5] A.G.Kyurkchan, “Representation of diffraction fields by wave potentials and the methods of auxiliary currents in problems of the diffraction of electromagnetic waves”, *Soviet Journal of Commun. Techn. And Electron.* , vol. 31, No 5, 1986, pp. 20-27.
[6] A.G.Kyurkchan, S.A.Minaev and A.L.Soloveichik “A modification of the method of discrete sources based on a priori information about the singularities of the diffracted field”, *Journ. of Commun. Techn. and Electron.*, vol .46, No 6, 2001, pp. 615-621.
[7] A.G.Kyurkchan, “Analytical continuation of wave fields”, *Soviet Journ. of Commun. Techn. and Electron.* , vol. 31, No 11,1986, pp. 59-69 .
[8] P.Ya.Ufimtsev, *The method of edge waves in physical diffraction theory* , Soviet Radio, Moscow, 1962.