Blind Identification of MA Systems Using a Combinational Method

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Abstract: In blind identifying Single Input / Single Output (SISO) systems, if we assume a minimum phase system, the possibility of identifying it by using autocorrelation function of the received signal does exist. For non-minimum phase systems, owing to absence of phase information in autocorrelation function, its deployment alone is not sufficient. To identify the non-minimum phase systems, it is possible to use methods based on Higher Order Statistics (HOS). But due to high variance of prediction errors, these methods do not yield satisfactory results. To identify the non-minimum phase systems, a new method based on autocorrelation function and the fourth order cumulant for the received signal is presented in this article. In this method, the same level of precision used in minimum phase system through autocorrelation function, the possibility of extracting phase information from HOS, and combining the two methods is used.

Key-Word: Blind Identification, MA system, HOS, SOS, Cumulant, SEMP

1 Introduction

Methods, which use only the signal received to identify a system, without accessing the input, are known as blind identification methods [1,2]. These can be classified generally as parametric and non-parametric methods. Parametric methods work generally by generating a linear or non-linear relation between the statistics of the received signal and the coefficients of a suitable model (AR, MA and ARMA) for the system under study. The autocorrelation function, or the 2nd moment of the received signal, has found many applications in systems identification since its estimation error variance is slight [3,4,5], but methods using the autocorrelation function alone are not able to identify non-minimum phase systems. The HOS can be used for identification of non-minimum phase systems as the phase and amplitude data are contained within it [6,7,8], but these methods are not considered accurate because of their high estimation error variance [9,10]. Hence in this paper a more suitable method for identification of non-minimum phase systems is presented, based on a combination of the 2nd moment and the HOS in system identification

First, the equivalent minimum phase system is identified using the 2nd moment. The equivalent minimum phase system is obtained by reflecting the zeros outside the unit circle inside it. If we suppose the system to have q zeros, then 2^q different systems are obtained by reflecting each zero with respect to the unit circle. Then using the characteristics of the HOS, the appropriate system is identified among them. The advantage of this method in system identification is its use of the accuracy of the 2nd moment and the phase data in the HOS. The extraction of the spectrally equivalent minimum phase system is explained in the 2nd part of this paper. The identification of non-minimum phase systems is described in the 3rd section, and the steps of algorithm execution and the simulation results are given in the 4th part.

2 EXTRACTION OF THE SPECTRALLY EQUILENT MINIMUM PHASE SYSTE

Consider a system with an MA model of order *q*, i.e. with a transfer function as below:

$$H(z) = \sum_{k=0}^{q} b_k z^{-k} \quad (1)$$

We'll consider the overall non-minimum phase case where the zeros of *H(z)* can be located without and/or within the unit circle. Let *w(n)* be the system input, then the form of the received signal will be as:

$$X(n) = h(n) * w(n) = \sum_{k=0}^{q} b_k w(n-k) \quad (2)$$

The blind identification of a system means the determination of the coefficients b_k from the signal received, *x(n)*. Supposing the input to be an i.i.d signal with zero mean, then the power spectrum density of the sensed signal will be as follows:

$$S_x(z) = \sigma_w^2 H(z) H^*(1/z) \ , \quad (3)$$

Wherein σ_w^2 is the system's input variance without reducing the generality of the problem, the input variance can be taken as σ_w^2=1. Supposing *H(z)* to have *q* distinct zeros, then $S_x(z)$ will have 2*q* zeros, such that if z_0 is one of the zeros, $1/z_0^*$ will be also a zero of $S_x(z)$, i.e. the power spectrum density will have q zeros inside and q zeros outside the unit circle (reflected from the zeros within the unit circle).In general it can be said that, for real b_k coefficients, the real zeros of $S_x(z)$ will appear as z_0, $1/z_0$ and its complex zeros as z_0, z_0^*, $1/z_0^*$, $1/z_0$ [5].Equation (3) can also be written as follows:

$$S_x(z) = (\sum_{n=0}^{q} b_n z^{-n})(\sum_{m=0}^{q} b_m z^{m}) = \sum_{m=0}^{q}\sum_{n=0}^{q} b_m b_n z^{-(n-m)} \quad (4)$$
$$= \sum_{m=0}^{q}\sum_{k=-m}^{q-m} b_m b_{m+k} z^{-k} = \sum_{k=-q}^{q} d_k z^{-k}$$

$$d_k = \sum_{i=0}^{q-|k|} b_i b_{i+|k|} \qquad -q \le k \le q \quad (5)$$

Considering the symmetry of the coefficients, $d_k = d_{-k}$, the power spectrum density of the sensed signal will be computable as follows:

$$S_x(e^{j\omega}) = S_x(z)\big|_{z=e^{j\omega}} = d_0 + 2\sum_{k=1}^{q} d_k Cos\, \omega k \quad (6)$$

Equation (6) is a linear relation between the coefficients, d_k ,and the values of the sensed signal's power spectrum density, therefore the values of d_k can be computed using the least-squares method for different values of ω. Considering equation (3), it can be seen that the zeros of $S_x(z)$ will contain the zeros of *H(z)*, thus the zeros of $S_x(z)$ inside the unit circle will include the zeros of *H(z)* within it, and the reflection inside the unit circle of its outside zeros, which is the spectrally equivalent minimum phase of *H(z)*.

3 USING THE HOS

It's possible to differentiate between minimum phase systems and non-minimum phase systems by the HOS, since the phase data for spectra of order greater than two are not lost and these data can be used to find the location of the system's zeros. By definition, the 4th order cumulant of the stationary signal *x(n)* with zero mean is given as:

$$C_4^x(\tau_1,\tau_2,\tau_3) = m_4^x(\tau_1,\tau_2,\tau_3) - C_2^x(\tau_1)C_2^x(\tau_2-\tau_3) \quad ,(7)$$
$$-C_2^x(\tau_2)C_2^x(\tau_3-\tau_1) - C_2^x(\tau_3)C_2^x(\tau_1-\tau_2)$$

Wherein C_2^x is the autocorrelation function and m_4^x is the 4th order moment of signal *x(n)*. For the system defined by equation (1) and (2), it can be shown that the 4th order cumulant of the signal received will have the following form [4]:

$$x(n) = w(n) * h(n) \quad (8)$$

$$C_4^x(\tau_1,\tau_2,\tau_3)=\gamma_4^w\sum_n h(n)h(n+\tau_1)h(n+\tau_2)h(n+\tau_3)$$

Wherein γ_4^w is the kurtosis of the input signal. To estimate the 4th order cumulant of signal x(n), equation (7) can be applied as follows:

$$\hat{C}_4^x(\tau_1,\tau_2,\tau_3)=\frac{1}{N}\sum_{n=0}^{N-1}x(n)x(n+\tau_1)x(n+\tau_2)x(n+\tau_3)$$
$$-\frac{1}{N}\sum_{n=0}^{N-1}x(n)x(n+\tau_1)\frac{1}{N}\sum_{n=0}^{N-1}x(n+\tau_2)x(n+\tau_3)$$
$$-\frac{1}{N}\sum_{n=0}^{N-1}x(n)x(n+\tau_2)\frac{1}{N}\sum_{n=0}^{N-1}x(n+\tau_3)x(n+\tau_1)$$
$$-\frac{1}{N}\sum_{n=0}^{N-1}x(n)x(n+\tau_3)\frac{1}{N}\sum_{n=0}^{N-1}x(n+\tau_1)x(n+\tau_2) \quad (9)$$

Equation (9) shows the estimation technique of the 4th order cumulant of the sensed signal for different values of τ_1, τ_2, and τ_3. Considering the subjects discussed in the previous section, after identifying the spectrally equivalent minimum phase system, 2^q different systems can be presented as the final solution. Among these, we choose the solution whose computed 4th order cumulant (from equation (8)) is closer to the 4th order cumulant estimated by equation (9). The following criterion can be used to this end:

$$E=\sum_{\tau_1,\tau_2,\tau_3}\left[\frac{C_4^x(\tau_1,\tau_2,\tau_3)}{C_4^x(0,0,0)}-\frac{\hat{C}_4^x(\tau_1,\tau_2,\tau_3)}{\hat{C}_4^x(0,0,0)}\right]^2 \quad (10)$$

That is to say, the solution is chosen to result in the least value of E.

4 SIMULATION ALGORITHM AND RESULTS

The realization of the method presented in this paper is done as follows: The first sensed signal's power spectrum density is computed by one of the common methods (walch) [5] and the equivalent minimum phase system is identified using equation (4). By identifying q zeros in this step, 2^q different systems are obtained for the non-minimum phase instance by reflecting each zero with respect to the unit circle. Then the 4th order cumulant of the sensed signal is computed using equation (9). In addition, using equations (8), (10) and the values of the 4th order cumulant of the sensed signal, the values of E are computed for the systems introduced in the previous step, and the appropriate system is the one resulting in the minimum E.

In this section, the method presented in this paper is checked using values for an i.i.d signal *w(n)* of length 1024 ,was passed through an MA(5) system with the following parameters. The signal *x(n)* was formed and shown in Fig.1., and the method was applied upon it. Spectrally equilent minimum phases system estimated. this result as shown in Fig.2,by reflected zeros within the unit circle 8 different system estimated. The E value computed from equation (10) for all systems. This value is 12.69, 12.38, 16.19, 16.30, 11.77, 10.72, 11.21, and 13.09.then the system that E is minimum value accepted (E=10.72).this estimated system as shown in Fig.3

Table.1 system parameter for real an estimated system

Simulated sys.		Estimated sys.	
b_k	z_k	b_k	z_k
1	-	-	1.21
0.2	1.2	0.11	1.23
-0.63	0.3+0.4J	-0.69	0.31+0.39J
-1.96	0.3-0.4J	-1.86	0.31-0.39J
1.34	-1+j	1.34	0.98+0.97J
-0.6	-1-j	-0.59	-0.98-0.97J

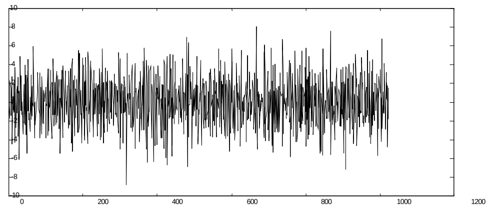

Fig.1 received sequence *x(k)*

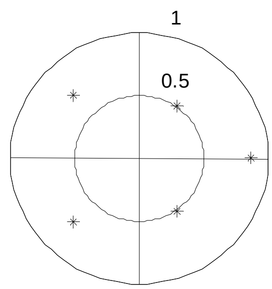

Fig.2 zeros location of minimum phase system

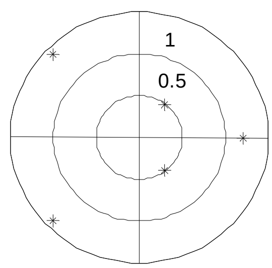

Fig.3 zeros location of estimated system.

5 CONCLUSION

The method presented in this paper is based on a combination of methods using the autocorrelation function and the HOS. the autocorrelation function is used to identify the minimum phase model of the system. In view of the good accuracy of this method, the relative location of the system's zeros are identified. A comparison of the 4^{th} order cumulant of the sensed signal and that of the systems introduced from the system's minimum phase model is used to find the exact location of the system's zeros. This combinational method, because of the high accuracy of the autocorrelation function in system identification, and extraction of phase data by the HOS, gives relatively accurate results.

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