Synchronous Stability of Multi-Machine Systems

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Abstract: -In this paper the concept of synchronous stability is introduced to study the transient stability problem of an operating process for a multi-machine power system. Necessary and sufficient conditions for an orbit of the system to be synchronously stable are obtained. Condition of non-synchrony and control of the synchronous stability are discussed.

Key-Words: - Synchronous stability, multi-machine system, many degree of freedom

1. Introduction

Power system engineering deals with the large-scale production, transmission and distribution of electrical energy. Deregulation and an increasing exchange of electricity between countries has led to greater demands being imposed on system stability, system protection and security, see for instance [1]. The synchronous stability of multi-machine systems describes synchronism of an operating process for power-angle of a multi-machine system, which would play an important rule in the study of transient stability of power systems. Xue in [2] proposed a quantification theory in the establishment of quantitative criterion which can be used to identify and to control the synchronous stability of multi machine power systems. Zou and Xue in [3] gave a further explanation for it. This article will describe a simplified mathematical foundation for the quantitative criterion.

Consider the multi-machine system

$$M_k \ddot{\delta}_k = P_{mk}(x,y,\mu) - P_{ek}(x,y,\mu), \quad k = 1,\cdots.n, \tag{1a}$$

$$0 = g_j(x,y,\mu), \quad j = 1,\cdots,m, \tag{1b}$$

where $x = (\delta^T, \omega^T)^T \in R^{2n}$ denote the state variables consisting of the generalized position $\delta = (\delta_1,\cdots,\delta_n)^T \in R^n$ and the generalized velocity $\omega = (\omega_1,\cdots,\omega_n)^T \in R^n$, $\dot{\delta} = \omega$, $y = (y_1,\cdots,y_m)^T \in R^m$ are other non-motion state variables, $\mu \in R^p$ are the parameter, $P_m = (P_{m1},\cdots,P_{nm})^T \in R^n$ the generalized drive force , $P_e = (P_{1e},\cdots,P_{en})^T \in R^n$ the generalized brake force, and $g = (g_1,\cdots,g_m)^T$ =0 is the algebraic restriction. Moreover, the *n* by *n* diagonal matrix *M*

$$M = diag(M_1,\cdots,M_n),\ M_i > 0, \forall i,$$

is the generalized inertia. Sometimes the system (1) can be also written as a general differential-algebraic (DAE) in the power systems form

$$\dot{\delta} = \omega, \quad \dot{\xi} = M^{-1}[P_m(x,y,\mu) - P_e(x,y,\mu)], \tag{2a}$$

$$0 = g(x,y,\mu). \tag{2b}$$

In particular, if we ignore the algebraic part (1b), the system (1a) is then just a many degree of freedom system.

In this article first the concept of synchronous stability is introduced for the study of an operating process in multi-machine power systems. Then necessary and sufficient conditions for an operating orbit of the system to be synchronously stable are established. Finally the condition of non-synchrony and the control problem of synchronous stability are discussed.

2. Synchronous Stability

A (position) orbit of the system (1)

$$\delta(t) = (\delta_1(t),\cdots,\delta_n(t))^T \in R^n$$

is called (orbit) synchronously stable, or simply called synchronous, if the distances $|\delta_i(t)-\delta_j(t)|, \forall i,j,i \neq j,$ between each two components of $\{\delta_i(t)\}$ are uniformly bounded for $t \geq 0$, i.e., there exists $L > 0$ such that

$$|\delta_i(t)-\delta_j(t)| \leq L, \forall i,j, \forall t \geq 0;$$

if we further consider the generalized velocities $\{\omega_i(t)\}$, then it can be obtained the conception of velocity synchrony. However, in the following if no particularly pointed, we will always consider the orbit synchrony. Note that if all components of an orbit $\{q_i(t)\}$ are uniformly bounded, then the orbit is always synchronous. If an orbit is not synchronous, i.e., for a sufficient large *t,* there exists a pair of components, say $\delta_i(t)$ and $\delta_i(t)$, such that the difference $|\delta_i(t)-\delta_j(t)|$ can become large enough, then the orbit is called non-synchronous.

Proposition 1 A sufficient and necessary condition for an orbit $\delta(t)$ to be synchronous is that for any vector $a = (a_1,\cdots,a_n)^T \in R^n$ satisfying $\Sigma a_i = 1, a_i \geq 0,$ the functions

$$|\delta_i(t)-\sum_{j=1}^n a_j\delta_j(t)|, \forall i,j,i=1,\cdots,n$$

are uniformly bounded.

Proof Necessary. Let

$$|\delta_i(t)-\delta_j(t)| \leq L, \forall i,j.$$

From $\Sigma a_i = 1$ it follows

$$|\delta_i(t)-\sum_{j=1}^n a_j\delta_j(t)|$$
$$=|\sum_{j=1}^n a_j\ [\delta_i(t)-\delta_j(t)]|$$
$$\leq \sum_{j=1}^n a_j\ |\delta_i(t)-\delta_j(t)| \leq L.$$

Sufficiency. For each pair *i, j,* taking

$$a = e_j = (0,\cdots,0,\overset{j}{1},0,\cdots,0)^T \in R^n,$$

then from the necessary condition $|\delta_i(t)-\delta_j(t)|$ is uniformly bounded, which means the orbit is synchronous.

Now we consider a complementary cluster $\sigma = \{\sigma_1,\sigma_2\}$ of the index set

$$S = \{1,2,\cdots,n\},$$

i.e., two disjoint nonempty subsets of *S*, σ_1 and σ_2, such that $\sigma_1 \cup \sigma_2 = S$. Let $\delta(t) = (\delta_1(t),\cdots,\delta_n(t))^T$ be an orbit of (1). For a complementary cluster $\sigma = \{\sigma_1,\sigma_2\}$, write

$$\delta_\sigma(t) = \sum_{i\in\sigma_1}\frac{M_i}{\sum_{k\in\sigma_1}M_k}\delta_i(t) - \sum_{j\in\sigma_2}\frac{M_j}{\sum_{k\in\sigma_2}M_k}\delta_j(t). \quad (3)$$

Recall that a continuous function $c(t) \in R$ is uniformly bounded for $t \to \infty$ means there exists a constant $\eta > 0$ such that

$$|c(t)| \leq \eta, \forall t \geq 0.$$

Remark 1 Suppose two qualities $\alpha(t)$ and $\beta(t)$ approach infinity as $t \to \infty$. They called infinitely large of the same order if $|\alpha(t)-\beta(t)|$ uniformly bounded. It is easy to see that an orbit $\delta(t)$ is synchronous if and only if each component $\{\delta_i(t)\}$ is either bounded, or infinitely large of the same order.

Proposition 2 An orbit $\delta(t)$ is synchronous if and only if for each complementary cluster σ of *S*, $\delta_\sigma(t)$ is uniformly bounded.

Proof Necessary. Suppose $\delta(t)$ is synchronous. For any complementary cluster $\sigma = \{\sigma_1,\sigma_2\}$ of *S*, write

$$\sigma_1 = \{i_1,\cdots,i_k\}, \sigma_2 = \{j_1,\cdots,j_l\},$$

$(k+l=n).$ Then from Proposition 1

$$|\delta_\sigma(t)|$$

$$= \left|\sum_{m=1}^{k} \frac{M_{i_m}}{\sum_{r=1}^{k} M_{i_r}} \delta_{i_m} - \sum_{m=1}^{l} \frac{M_{j_m}}{\sum_{r=1}^{l} M_{j_r}} \delta_{j_m}\right|$$

$$\le \sum_{m=2}^{k} \frac{M_{i_m}}{\sum_{r=1}^{k} M_{i_r}} |\delta_{i_m} - \delta_{i_1}|$$

$$+ \sum_{m=2}^{l} \frac{M_{j_m}}{\sum_{r=1}^{l} M_{j_r}} |\delta_{j_m} - \delta_{j_1}| + |\delta_{i_1} - \delta_{j_1}|.$$

So $\delta_\sigma(t)$ is uniformly bounded.

Sufficiency. Suppose $\delta(t) = (\delta_1(t), \cdots, \delta_n(t))^T$ is not synchronous. Then there exist two components of $\delta(t)$, say $\delta_1(t)$ and $\delta_2(t)$, such that

$$\sup_{t \ge t_0} |\delta_2(t) - \delta_1(t)| = +\infty.$$

Consider a complementary cluster $\sigma = \{\sigma_1, \sigma_2\}$ of S, such that $i \in \sigma_1$ if and only if $\sup_{t \ge t_0}[\delta_i(t) - \delta_1(t)] < +\infty$. It is easy to see $1 \in \sigma_1$, $2 \in \sigma_2$. Only need to prove $\delta_\sigma(t)$ is not uniformly bounded. If not, $\delta_\sigma(t)$ is uniformly bounded. Then from (3),

$$\sum_{j \in \sigma_2} \frac{M_j}{\sum_{k \in \sigma_2} M_k} [\delta_j(t) - \delta_1(t)] =$$

$$\sum_{i \in \sigma_1} \frac{M_i}{\sum_{k \in \sigma_1} M_k} [\delta_i(t) - \delta_1(t)] - \delta_\sigma(t),$$

of which the right hand side is uniformly bounded. But for the left hand side for each $j \in \sigma_2$, one has $\sup_{t \ge t_0}[\delta_j(t) - \delta_1(t)] = +\infty$, which induce a contradiction. The proposition is now proved.

3. Condition of Non-Synchrony

For simplicity we consider a "pure" multi-machine system without the algebraic restriction (1b). Consider an orbit $\delta(t)$ of the multi-machine system. Based on the condition of synchrony in Proposition 2 it is easy to see that the orbit $\delta(t)$ is non-synchronous if and only if there exists a complementary cluster σ such that the single orbit determined by (3) is not uniformly bounded. Since

$$\ddot{\delta}_\sigma(t) = \sum_{i \in \sigma_1} \frac{M_i}{\sum_{k \in \sigma_1} M_k} \ddot{\delta}_i(t)$$

$$- \sum_{j \in \sigma_2} \frac{M_j}{\sum_{k \in \sigma_2} M_k} \ddot{\delta}_j(t)$$

$$= \sum_{i \in \sigma_1} \frac{1}{\sum_{k \in \sigma_1} M_k} [P_{mi}(t) - P_{eki}(t)]$$

$$- \sum_{j \in \sigma_2} \frac{1}{\sum_{k \in \sigma_2} M_k} [P_{mj}(t) - P_{ej}(t)]$$

$$\equiv F_\sigma(x, t, \mu),$$

the non-uniform boundedness of the single orbit δ_σ then can be discussed through solutions for the single machine system which satisfies

$$\ddot{\delta}_\sigma = F_\sigma. \qquad (4)$$

Although the system (4) is different from a general single machine system due to its acting force depend on the whole orbit $\delta(t)$, however, this trouble can be removed by introducing numerical integration.

Let the orbit $\delta(t)$ take the value $\delta(t_0) = \delta_0$ and $\dot{\delta}(t_0) = \omega_0$ at time t_0. From this it naturally follows $\delta_\sigma(t_0) = \delta_{\sigma 0}$, $\dot{\delta}_\sigma(t_0) = \omega_{\sigma 0}$, and a curve on the plan $(\delta_\sigma, F_\sigma)$ with parameter $t (\ge t_0)$,

$$\gamma(t) = (\delta_\sigma(t), F_\sigma(x(t), t, \mu)) \in R^2.$$

Since

$$\frac{1}{2}\frac{d}{dt}(\omega_\sigma(t))^2 = \frac{1}{2}\frac{d}{dt}(\dot{\delta}_\sigma)^2$$

$$= \dot{\delta}_\sigma \ddot{\delta}_\sigma = \dot{\delta}_\sigma F_\sigma,$$

we have

$$\frac{1}{2}(\omega_\sigma(t))^2 = \int_{q_{\sigma 0}}^{q_\sigma} F_\sigma d\delta_\sigma + \frac{1}{2}\omega_\sigma^2.$$

And $S(\delta_\sigma(t)) = \int_{q_{\sigma 0}}^{q_\sigma} F_\sigma d\delta_\sigma$ is just the (direction) area between the curve $\gamma(t)$ and the axis δ_σ.

Noticing $\omega_\sigma = \dot{\delta}_\sigma$, the positive or negative area would result in non-uniformly bounded for the system (4). Thus in order to preserve the original system in synchrony, the total area should be zero, which may realize by introducing certain control factor.

4. Control to the Synchronous Stability

In the above we reduced the synchronous stability into the uniform boundedness of the solution of the corresponding one machine system as (4). Moreover, the latter may be evaluated by observing whether the area or integration of the corresponding force with respect to the generalized coordinates approach to zero. For the control problem of synchronous stability it is then necessary to control the parameter such that the area become zero. Let us now analyze this problem.

Consider the plane $(\delta, F) \in R^2$ described by the (generalized) position δ and the force F, from which we may obtain the γ curve determined by F. Based on the above result to preserve synchronous stability of the system operating the (algebraic) sum of the area between the curve and the δ axes should become zero. By using the equal area criterion (EAC), if the area is positive (negative) then we may adjust with a negative (positive) control such that the sum become zero, which is just the clearance process of the fault.

After the clearance of fault there may be two cases: when the (positive) area is too small one need increase a brake force, on the other hand, if the area is too large, then one need decrease the drive force. By this way the fault may effectively cleared so that the system preserve synchronous stability.

References

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