A method to define z-transform of different signals

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*Abstract :*Refering to z-transform tables ,we see many discreate-time signals (or LTI systems) do not have z transfer function .On the other hand several signals and systems are found in nature which are used in signal processing and do not exist in tables because their z transfer function are not defined.(e.g. logarithmic system) In this letter ,we have used fourier series expansion to approximate these signals to basic available signals in z-transform tables and tried to involve these functions in z-transform domain.

1 Introduction

Transform techniques are important tools in the analysis of signals and linear time invariant (LTI) systems .The z-transform plays the main role in the analysis of discrete-time signals and LTI systems as the Laplace transform does in the analysis of continuous-time signals and LTI systems .According to z-transform tables in all signal processing books ,we find out the z transfer function of a few signals are defined (e.g. u(n) ,sin(n)u(n) ,cos(n)u(n)) and all the examples are linear combination of these signals [1].Besides ,the defination of z-transform gives nonlinear functions of z for many of signals and LTI systems (thay are difficult to implement because of being nonlinear) .This limitation decreases ability of z-transform and limits the use of this

$$f(n)u(n) \cong (a_0 + \sum_{k=1}^{r} a_k \cos(kn)$$

$$+ \sum_{k=1}^{m} b_k \sin(kn)$$

transfer function. The purpose of this letter is to remove this limitation .An appraoch is approximation of signals to those which have identifer z transfer function[2].

Approximation way: Refering to basic relations in mathematics ,we can approximate every signals with linear combination of Cosine and Sine functions[3,4]. If g(n) is a signal with defination g(n)=f(n)u(n) then we can write

$$a_0 = \frac{1}{2N} \sum_{k=1}^{2N} f(n)$$

$$a_k = \frac{1}{N} \sum_{k=1}^{2N} f(n)\cos(kn)$$

$$b_k = \frac{1}{N} \sum_{k=1}^{2N} f(n)\sin(kn)$$

where

if we apply z-transform to this equation then we will have

$$Z\{f(n)u(n)\} \cong$$

$$Z\{(a_0 + \sum_{k=1}^{r} a_k \cos(kn) + \sum_{k=1}^{m} b_k \sin(kn))u(n)\}$$

Also we have in z-transform tables

$$Z\{\cos(kn)u(n)\big|_{k=w_0}\} =$$

$$\frac{1-\cos(w_0)z^{-1}}{1-2\cos(w_0)z^{-1}+z^{-2}} \ldots\ldots\ldots\ldots |z| > 1$$

$$Z\{\sin(kn)u(n)\big|_{k=w_0}\} =$$

$$\frac{\sin(w_0)z^{-1}}{1-2\cos(w_0)z^{-1}+z^{-2}} \ldots\ldots\ldots\ldots |z| > 1$$

$$Z\{u(n)\} = \frac{1}{1-z^{-1}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots |z| > 1$$

From combination of the above relations

$$Z\{f(n)u(n)\} \cong$$

$$\frac{a_0}{1-z^{-1}} +$$

$$\sum_{k=1}^{r} a_k \frac{1-\cos(k)z^{-1}}{1-2\cos(k)z^{-1}+z^{-2}} +$$

$$\sum_{k=1}^{m} b_k \frac{\sin(k)z^{-1}}{1-2\cos(k)z^{-1}+z^{-2}} \ldots\ldots\ldots |z| > 1$$

Therefor ,we can convert every signals to an statement in terms of z by this method .Fig .1 shows the approximation of nu(n) signal in z domain (for m =r =10) and Table 1 shows the coefficients of this approximation.we choose nu(n) because of z-transform of it is identical (equal to

$$\frac{z^{-1}}{(1-z^{-1})^2}$$

2 Conclusion

It is clear that the above appraoch does not exactly describe z-transform of the primery signal (because of navigate of approximating) but if we increase the degree of approximation ,the resulting z-transform could be acceptable. By this method we can exand domain of z-transform. also this method can be used for Laplace transform in the continuous-time signals.

References

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Figure Captions :

Fig 1 . Comparison magnitude of approximation signal and primary signal(in dB):

- : primary signal
* : approximated signal

Fig 2 . Comparison Phase of approximation signal and primary signal:

- : primary signal
* : approximated signal

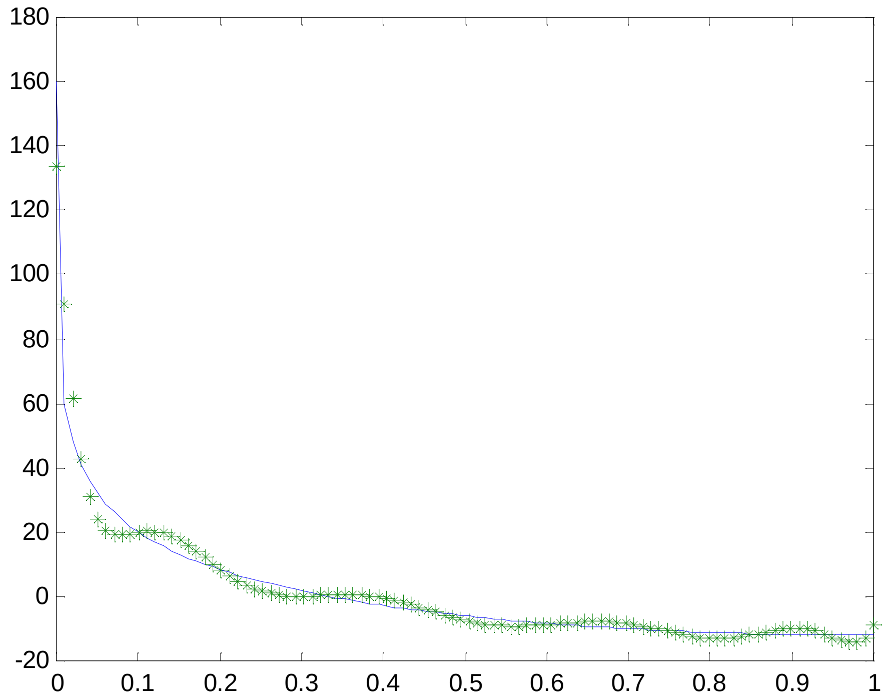
Table Captions :

Table 1 .the coefficients of approximation

Table 1.

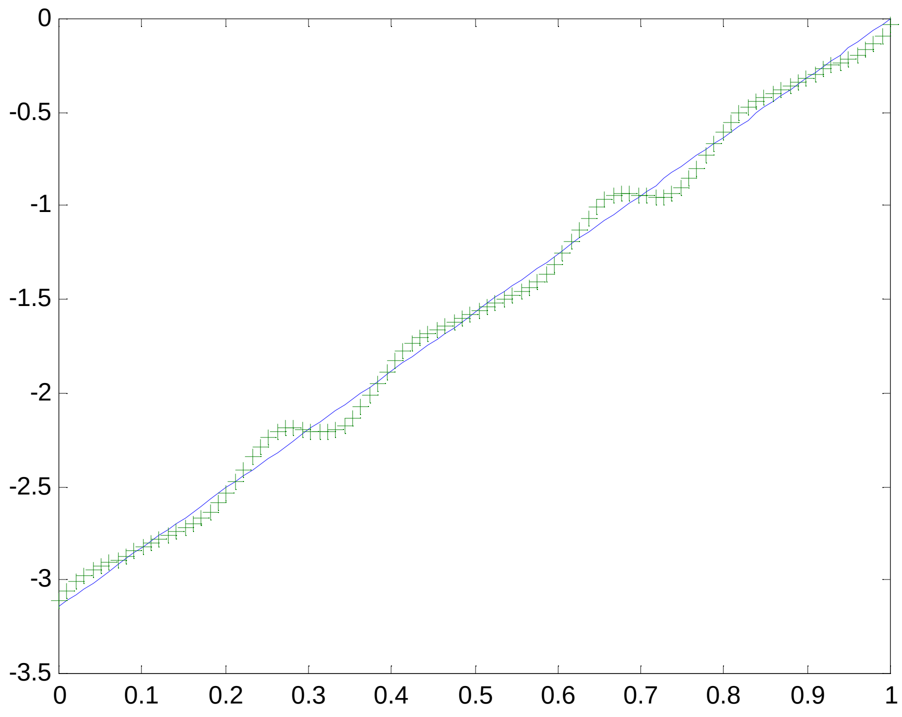
$C_0 = 0.7854$	-------------
$C_1 = -0.6366$	$S_1 = 1$
$C_2 = 0.0000$	$S_2 = -0.500$
$C_3 = -0.0707$	$S_3 = 0.3333$
$C_4 = 0.0000$	$S_4 = -0.250$
$C_5 = -0.0255$	$S_5 = 0.2000$
$C_6 = 0.0000$	$S_6 = -0.1667$
$C_7 = -0.0130$	$S_7 = 0.1429$
$C_8 = 0.0000$	$S_8 = -0.125$
$C_9 = -0.0079$	$S_9 = 0.1111$
$C_{10} = 0.000$	$S_{10} = -0.100$

Fig 1.



180
160
140
120
100
80
60
40
20
0
-20
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Fig .2



0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1