Using least square error approach to approximate $Z^{-p/q}$

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Abstract: One sample of system which has unity magnitude and constant fractional group delay is $z^{-p/q}$ transfer function. but when p/q is not integer we can not describe it in z-transform because of the transfer function in the z-transform only includes integer powers of z. In this letter ,we have used least-square error method for approximation $z^{-p/q}$ with a polynomial of integer powers of z and describing it in z-transform.

Key word : least square error,FIR approximation,mwgnitude,group delay

1 Introduction

Refering to the basic relation in z-transform[1], the z-transform includes integer powers of z. for this reason ,we can not set fractional power of z as a subset of z-transform. If H(z)= $z^{-p/q}$ is the transfer function of a system, by replacing z with e^{jw} then we have :

$$\left|H(e^{jw})\right| = 1$$
$$phase\left[H(e^{jw})\right] = -p/qw$$
$$grd\left[H(e^{jw})\right] = -\frac{d(phase)}{dw} = p/q$$

That gives unity magnitude and fractional group delay. That is important in several applications (e.g.designing digital filter in ITU-T standard).One good idea for describing $z^{-p/q}$ in z-transform is approximation of $z^{-p/q}$ by a polynomial of z[2].Using of approximation is used to design digital filter but they only approximate magnitude at frequency while we use it for approximation group delay at frequency. In the method was described in [2], group delay only approximated accuracy in a limited range of frequency and also this method has considerable affection on magnitude. In this letter we have chosen least-square error method because of low degree of it (in this method, it is possible the number of variables more than the number of equations) and considerable precision. And show this method gives good approximation of group delay in the total band without destruction in magnitude.

2 Approximation approach

In least-square error method, we postulate a model for system and determine the parameters of the model that minimize. In this method sense, the error between the actual system response and the response of the model[1]. We shall describe technique based on an FIR model for the system. On the other hand, the target is to find c_i coefficients as $z^{-p/q} \cong \sum_{k=0}^{m} c_k z^{-k}$. The block diagram of this method is shown in Fig 1. We define an error matrix as form as $\mathbf{e}$ =**Y-B.C** (1) (where $\hat{Y} = B.C$)and try minimize the length of it[3,4]. For applicating this approximation in this letter we need under definations

$\mathbf{?} = [w_0 .. w_1 .... w_N]$ $w_0 = 0, w_N = p$ and $w_0 < w_1 < ... < w_n < .. w_N$.

$$\mathbf{Y} = z^{-p/q} |_{z=e^{jw}} = [e^{-jp/qw_0} .... e^{-jp/qw_N}]^T .$$

$$\mathbf{C} = [c_0 ..... c_M]^T$$

$$\mathbf{e} = [e_0 ..... e_N]^T$$

$$\mathbf{B} = \begin{bmatrix} 1..e^{-jw_0} ....e^{-jMw_0} \\ 1..e^{-jw_1} ....e^{-jMw_1} \\ \\ 1..e^{-jw_N} ....e^{-jMw_N} \end{bmatrix}$$

By replacing the above relations in equation (1), we find a set equation as form as

$$\begin{cases} e_0 = e^{-jp/qw_0} - (c_0 + c_1 e^{-jw_0} + ... + c_M e^{-jMw_0}) \\ e_1 = e^{-jp/qw_1} - (c_0 + c_1 e^{-jw_1} + ... + c_M e^{-jMw_1}) \\ .... \\ e_N = e^{-jp/qw_N} - (c_0 + c_1 e^{-jw_N} + ... + c_M e^{-jMw_N}) \end{cases}$$

when the length of the error vector is be minimum, we have $\mathbf{B}^T$ $\mathbf{e}$ =0 then **B**^{T}**Y= B**^{T}**BC** [3]. That by it, we can find c_i coefficients. we can see of the above set equations, increasing the number points of ? increases decision of approximation without any change in the value of c_i coefficients.(the degree of polynomial)

It is clear that the above approach does not exactly describe z-transform of $z^{-p/q}$ (because of navigate of approximating) but if we increase the degree of approximation, the resulting z-transform will be accuracy.

3 Experimental results

the magnitude and group delay of $z^{-13/2}$ with m=12 and m=30 is shown in Fig 1.

(run by MATLAB 4.2c.1)From the above figure we can see the magnitude is constant and group delay is near to 6.5 at the all of range. Also, increasing the degree of approximation gives better approximating group delay.

4 Conclusion

An efficient approximation of $z^{-p/q}$ by least square error algorithm was proposed. The method makes high disicion for approximating group delay with unity magnitude. This system gives fractional group delay. By cascading a system with constant group delay and desired magnitude with this system, we find a system with desired magnitude and very low group delay[5].

References :

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Figure Captions :

Fig .1 system identification based on FIR model and least-square criterion.

Fig .2 the approximation of $Z^{-(13/2)}$ in magnitude and group delay

- : m=12

* : m=30

Fig .1

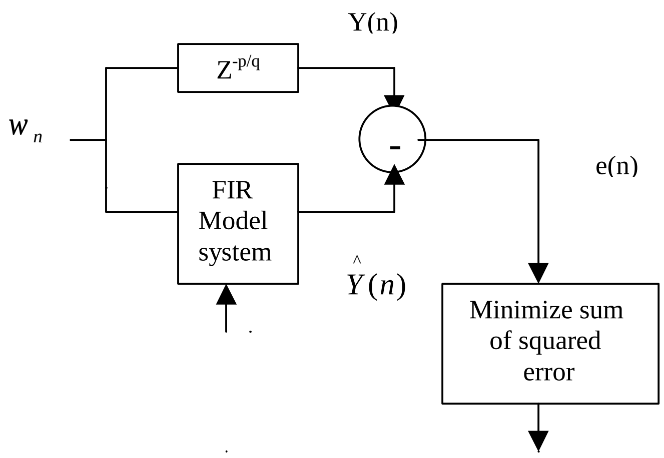
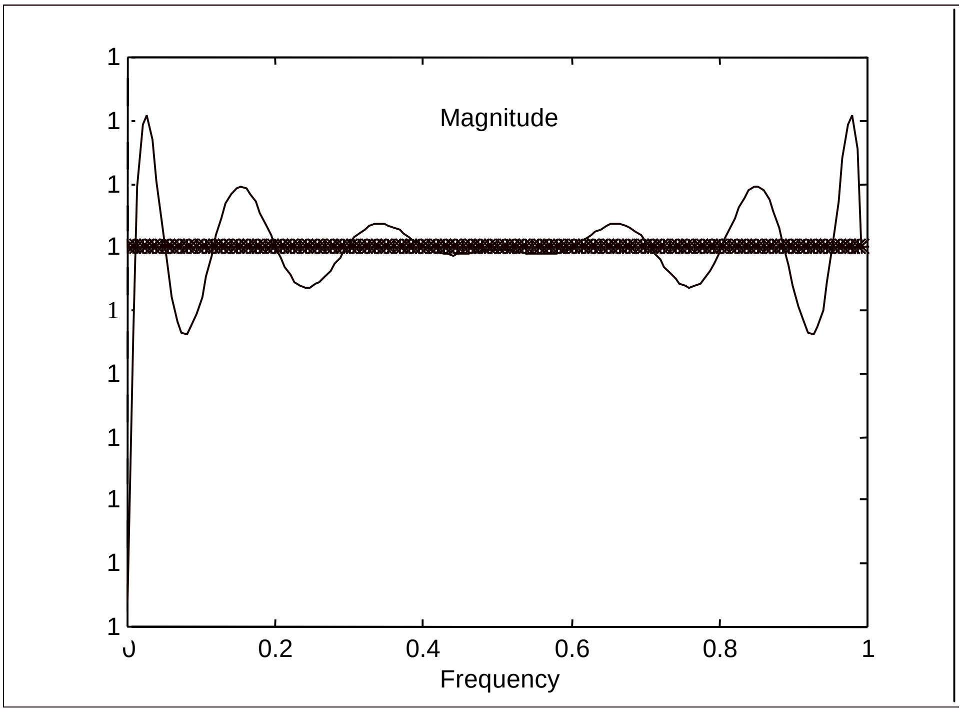
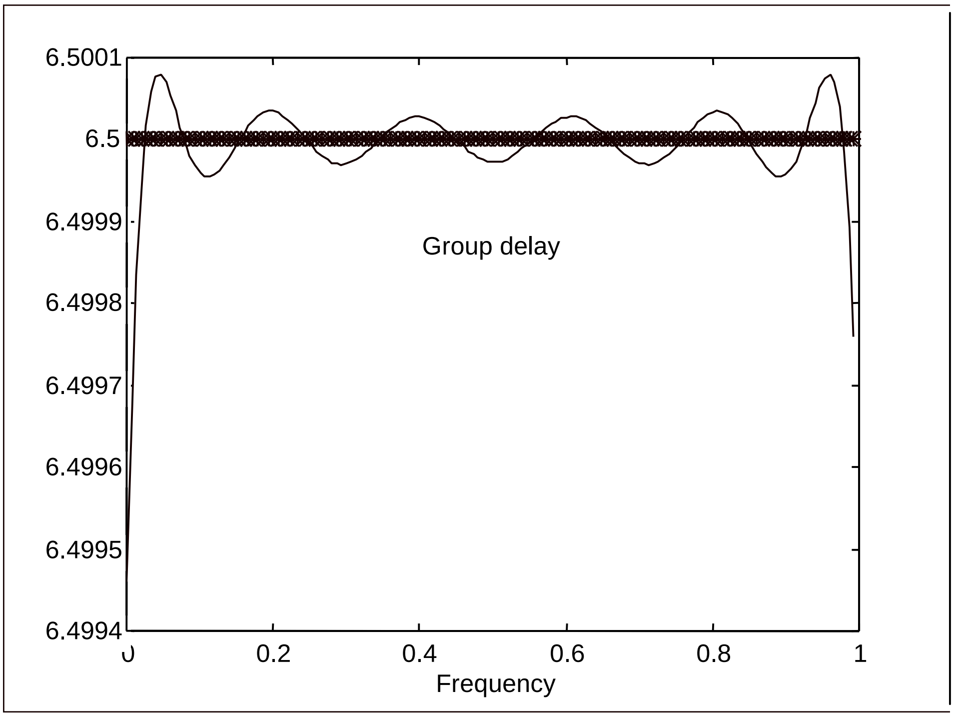

Y(n)
$Z^{-p/q}$
w_n
-
e(n)
FIR
Model
system
$\hat{Y}(n)$
Minimize sum
of squared
error

Fig .2



1
1
1
1
1
1
1
1
1
1
Magnitude
0
0.2
0.4
0.6
0.8
1
Frequency

(a)



6.5001
6.5
6.4999
6.4998
6.4997
6.4996
6.4995
6.4994
Group delay
0
0.2
0.4
0.6
0.8
1
Frequency

(b)