A Novel Approach to Curvature Estimation.

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Abstract: - The ability to describe and accurately measure shape in digital images is a useful tool in a number of applications. The performance of methods such as scene reconstruction, object segmentation and object recognition and classification can be facilitated by low level shape descriptors. However, curved image boundaries are seriously degraded by the imaging process itself and accurate curvature measurements are difficult to recover. In this paper, a novel approach to discrete curvature estimation is presented that makes use of the varying responses of spatial filters to curved edge stimuli. A limited set of basis filters are used to detect edge features and their apparent orientations. A second set of elliptical filters are then applied to provide an estimate of local curvature. The resulting curvature measurement is invariant to the contrast, relative position and orientation of the feature and is suitable for use with a curve/contour model.

Key-Words: - Curvature, Feature detection, Spatial Filter, Steerable, Elliptical

1 Introduction

Edge or discontinuity detection and estimation plays an important role in a number of technological areas such as image analysis, signal processing, pattern recognition and image understanding. It is of particular importance in image processing where low level tasks such as edge and line detection are used to locate structure or regions of interest within an image. However, traditional edge detection methods are designed with the detection of locally 'straight edges' in mind and cannot discriminate between these and a feature which may be locally curved. This is particularly highlighted by edge detectors which are based upon spatial filters, such as the Canny[1]. In general a filter can give an identical response to curved and straight features depending upon the amount of curvature, contrast differences and the feature position relative to the filter centre.

1.1 Curvature estimation.

A number of methods for curvature estimation have been developed in the past including orientation based techniques, linefitting, tangent based techniques and path based techniques,[2][3]. The latter has received the most attention and has given rise to a number of path based algorithms such as snakes or active contours. However, all these techniques act upon the edge boundary once its location in the image has been determined. This is achieved either by preprocessing with an edge detector or, in the case of snakes, allowing the contour to move itself into position[3]. Further processing is then conducted on the set of points within the image contour to estimate curvature which, can involve re-sampling of points to non image grid locations and smoothing of the curvature data. These procedures increase the complexity of the technique and can cause instability in the algorithm[2]. In particular deciding on the correct amount of smoothing and its effect on error within the data presents a significant problem to current techniques.

The approach presented here, attempts to link between the pre-process edge detection stage and later curvature estimation stage of the algorithms. In particular we are interested in deriving a measure for curvature based upon the response of spatial filters with different magnitude envelopes[4]. Use is made of a steerable filter basis set, which allows continuous estimation of orientation and measurement of feature position to sub-pixel accuracy,[5][6]. The differences in reported positions of features at a particular image site can be used to estimate amount of curvature at the apex of the detected curve. Using this approach a discrete set of curvature measurements can be recovered from the contour by the use of 2D spatial/frequency information only. This information could then be applied to a curve model to allow for post refinement and description of the entire contour via a suitable model.

2 Orientation Analysis

The first step in the curvature estimation process is to detect the apparent orientation of an edge feature within the image. We use a computational approach based upon the responses of spatial basis filter sets oriented evenly over the range, $0 \leq q \leq \pi$, to sampling orientation over, n, discrete angles. As an efficient method for orientation analysis is required, the filters are designed to be used within a steerable framework. This allows the response of a filter at an arbitrary orientation to be synthesized accurately from a limited basis set[5], by providing a known interpolation function over orientation. In order to resolve certain ambiguities in the responses of such filters, a combination of symmetric and anti-symmetric filters are used which provide a quadrature pair filter[6].

2.1 Steerable filters

The quadrature pair filters are implemented separately as two steerable filter banks. The first, symmetrical filter is given by:

$$G(x,y) = A\left(B_g\left(\frac{x}{s}\right)^2 + C_g\right)e^{-\frac{1}{2s^2}\left(x^2+y^2\right)} \qquad (1)$$

Which is a close approximation to a Gabor cosine function. The filter is a second order odd parity polynomial, coefficients B_g, C_g, times a circular gaussian window with standard deviation, s. From the theory of steerable filters[10], (1) requires 3 basis filters for steering and these are positioned at angles, $q_j = 0, \pi/3, 2\pi/3$ for j = 1,2,3 respectively.

The response of a filter at an arbitrary orientation can then be generated from a linear combination of the responses of those basis filters and interpolation functions, k^{qj},[5]:

$$G^q = \sum_{j=1}^{n} G^{q_j} k^{q_j} \qquad (2)$$

The anti-symmetric filter is constructed in the same manner from a third order odd parity polynomial times a circular Gaussian window and requires four basis filters for steering.

$$H(x,y) = A\left(B_h\left(\frac{x}{s}\right)^3 + C_h\left(\frac{x}{s}\right)\right)e^{-\frac{1}{2s^2}\left(x^2+y^2\right)} \qquad (3)$$

The filter given by (3) is a close approximation to a Gabor sine function where B_h and C_h are coefficients of the polynomial. The complete basis set of four filters are positioned at, $q_j = 0, \pi/4, 2\pi/4\ 3\pi/4$.

2.2 Scale considerations

Basis filters at the specific set of angles, q_j, provides a means to set the angular frequency of each of our filters. However, the frequency response of a particular filter in terms of it's radial frequency allows the scale of the filter, in the spatial domain, to be varied. In this case, a high radial frequency is used for the filters, of 0.375, and bandwidth covering approximately one octave in frequency. The resulting filter can be implemented using 4 x 4 taps, which is sufficient for the purposes of this paper.

2.3 Elliptical Window

The theory described above has been used to design filters with circular magnitude envelopes, which are a class of steerable filters. However, in this application a basis set of filters with non-circular envelopes is used to perform data extraction alongside the circular filters. This is achieved by modification of the Gaussian window in equations (1) and (3) to:

$$W(x,y) = e^{-\frac{1}{2s^2}\left(x^2+\frac{y^2}{m}\right)} \qquad (4)$$

Where the parameter, m, represents the axis ratio of the elliptical window. By designing filters with, $m > 1$, the orientation selectivity of the filters are reduced and the sensitivity of the filter is increased to orientations near its central frequency.

Although, not formally steerable, the responses of elliptical filters can be steered sufficiently well [4], as the change in filter response due to, $m^{1}1$, does not affect the distribution of maxima in the energy function of the quadrature pair. Hence, the steering theorem can be used to determine dominant orientations detected by both elliptical and circular filters.

2.4 Detecting dominant orientation

In order to determine to the dominant orientation detected by a filter at a particular image site, I(x,y), the angle which maximises the energy function of the filter must be found. Where energy is defined as:

$$E(q) = G(q)^2 + H(q)^2 \qquad (5)$$

where G and H are the responses of the even and odd filters steered to an orientation, q. Use of a well known technique[7][6][4] is adopted to determine the orientation of a dominant feature.

Given a set of basis filter responses at I(x,y), a total of seven possible orientations are generated (based on second and third order filters). The orientation which maximises the filter energy function, equation (5), is selected as the dominant orientation (see [4] for a more detailed explanation). Once the dominant orientation has been determined the filters can be steered, using equation (2), to give the response of a filter at that orientation.

3 Estimating Curvature

Existing techniques for curvature estimation have been shown to be prone to error under quantative analysis[2]. The fact that existing techniques rely upon resampling and smoothing of the image boundary may be a contributing factor. However, the effects of quantisation on a curved feature are severe and a great deal of information is lost in the imaging process. The approach presented in this paper, attempts to overcome this by basing curvature measurements on the local spatial frequency characteristics of the boundary, via the use of orientation selective filters, which are disscussed in the previous section. Use is made of the apparent ambiguity between straight and curved edge/line features as detected by spatial filtering.

3.1 Ambiguity in edge detection

Feature detectors such as those based on orientation selective filters must assume that a detected edge profile forms a straight edge in the image. This can cause a number of problems when attempting to recover sub-pixel information from their responses in the presence of a curved edge, as in the technique of Folsom and Pinter[6]. Their technique allows recovery of position relative to the filter centre of a straight edge to sub-pixel accuracy from phase based measurements. However, if the edge is curved locally this position estimate insufficient to describe the edge's true position. The reported edge position deviates from the actual boundary position as the amount of curvature increases. It is this effect which allows a filter with an elliptical magnitude profile to be applied together with its circular counterpart to form a differential measure of curvature.

3.2 Estimation of curvature maxima

For the purposes of this investigation the application of this approach is limited to the case of curve extrema. That is, curve points corresponding to a maxima in the curvature function. It is assumed that a filter can be positioned so that the curved edge falls equally across its receptive field. In this case the orientation detected, as the filters are steered, is parallel to the tangent to the curve at the maximum. Once orientation has been determined using the circular filters, an elliptical filter is steered to that same orientation. As the filters are identical, except for extension of the receptive field in the direction parallel to the filters orientation, the slope of the phase surface of each filter is also identical. The phase of each quadrature pair is calculated and the difference, f_p, is taken as.

$$f_p = \arg(G_c(q_1), H_c(q_1)) - \arg(G_e(q_1), H_e(q_1)) \quad (6)$$

where G_c,H_c and G_e,H_e are the responses of circular and elliptical filters respectively. Figure 2, shows the phase difference of the filters in response to a second order curve maxima.

4 Results

4.1 Orientation Analysis

It is important to quantify the effects of extending the Gaussian envelope of the filter to form elliptical filters. This extension narrows the angular frequency tuning of the filters and hence reduces the band of orientation in the spatial domain to which the filter will react significantly. The effects of this modification also have a direct consequence on the characteristic of the quadrature filters energy function. It is argued that the maximum of the energy function is preserved at the orientation of the stimuli and so conforms to the relation given by equation (2). This being the case, the response of elliptical basis filters can be steered in the same way as circular filters and can be used independently for orientation analysis, as described in section 2.4. Figure 1(a), shows an image taken from an MRI data set to which the orientation estimation algorithm is applied. The filters used are circular, elliptical $m=3$ and $m=5$ in figures 1(b),(c) and (d) respectively. A threshold is applied to the resulting image and are reconstructed at four times the resolution using straight line segments whose orientation and position are determined by the algorithms output. From the resulting re-constructions shown in Figure 1, the elliptical filter basis sets appear to steer as well as the circular set shown in figure 1(a). However, as the axis ratio of the elliptical window, m, increases, areas which contain edges of low contrast tend to give differing results. This may be attributed to the estimation algorithm itself. The algorithm for dominant feature detection can be effected by the presence of multiple oriented edges or local texture, if edge contrast is low. Which can tend to cause the function to extract an orientation close to that of the basis function orientations. The number of edge detections also increases with, m, illustrating the increased sensitivity to tuned edges possessed by the elliptical filters.

5.2 Measuring Curvature

In order to determine the relationship between the responses of circular and elliptical filter banks after convolution with a curved edge, a set of test image features were constructed. The features chosen were those that represent a second order curve around its maximum (or minimum) point. At this maximum on the parabola the curvature of the edge profile is also at a maximum and the radius of curvature, R, is a minimum. Using this point as reference, it is intended to determine the relationship between this value and the deviation in the relative distance measurement from the filter centre to the apparent feature position. As this distance measurement is derived from the linear phase property possessed by each filter, the radius of curvature of the second order test curve was related to the phase difference between the responses of the two filter types. The resulting plots are shown in figure 2, for various values of m. For sake of clarity, R, is plotted on a logarithmic scale against phase difference as defined in equation (6). The phase difference increases with R, towards the point R = -3.82 at which the phase difference peaks and drop off to zero as the filters response drops off to zero. This peak signifies the transition between a detectable edge parallel to the filters orientation and a line feature normal to the filters orientation. For R greater than –3.82 the filters magnitude response drops. This suggests that the phase difference measure can be adopted to estimate curvature which corresponds to R below the peak in phase difference, as it is in this region where magnitude is at near peak value and likely to be identified as a feature by the detection algorithm.

5 Conclusion

Orientation selective filtering provides an efficient vehicle for extraction of angular frequency information from digital images. The computational efficiency of filter based analysis techniques is further improved by such sub groups as wavelet analysis. However, when more flexibility over filter design is required, non-orthogonal analysis schemes may be more suited to a particular application. Steerable filters allow an increase in the efficiency of the analysis process by providing a derivable interpolation function over orientation. Steerable filters which are constructed using circular window functions are well known and have been applied successfully in a variety of applications. However, little research has been carried out on such filters constructed using non-circular receptive fields. Generally, non-circular windows are avoided as they cannot be implemented by separable one-dimensional filters that provide a reduction in computational load. However, the use of filters with different receptive fields can provide additional domains of analysis. The increased angular selectivity of the elliptical filters used can provide a superior estimate of the orientation of a strong edge or line feature than their circular counterparts in terms of the second order filter sets used. However, where no strong feature or multiple oriented features exist the algorithm used for dominant feature detection can give anomalous results when the axis ratio, m, of the elliptical receptive field becomes large. Within the scope of curvature estimation combinations of circular and elliptical filters have been shown to react in a discriminatory manner between curved and straight edge stimuli. In particular, the case of curvature maxima of a family of second order curves has been studied. It has been determined that the phase difference of steered filters with circular and elliptical receptive fields can provide a suitable measure of these curved features for detection and estimation purposes.

References:

[1] J.F. Canny, *"A Computational Approach to Edge Detection"*, IEEE Trans. Pattern Analysis and Machine Intelligence, Vol.6, 1986, pp. 679-698.
[2] M. Worring, A.W.M. Smeulders, *"Digital Curvature Estimation"*, CVGIP: Image Understanding, Vol.58, No.3, 1993, pp.366-382.
[3] D.J. Williams, M. Shah, *"A fast algorithm for active contours and curvature estimation"*, CVGIP: Image Understanding, Vol.55, No.1, 1992, pp.14-26.
[4] S.J.Purves, S.S.Dlay, *"Near Gabor Filters for Feature Detection and Curvature Estimation"*, Proc. SPIE Conf. On Human Vision and Electronic Imaging, San Jose, CA, January 2000, *In Press*.
[5] W.T Freeman, E.H. Adelson, *"The Design and Use of Steerable filters"*, IEEE Trans. on Pattern Analysis and Machine Intelligence, Vol.13, No.9, 1991, pp.891-906.
[6] T.C. Folsom, R.B. Pinter, *"Primtive Features by Steering, Quadrature, and Scale"*, IEEE Trans. Pattern Analysis and Machine Intelligence, Vol.20, No.11,1998, pp.1161-1173.
[7] H. Knutson, G.H.Granlund, *"Texture Analysis Using Two Dimensional Quadrature Filters"*, IEEE Comp. Soc. Workshop on Computer Architecture Pattern Analysis & Image Database Management, 1983, pp206-213.
[8] H. Asada, M. Brady, *"The curvature primal sketch"*, IEEE Trans. on Pattern Analysis and Machine Intelligence, Vol.8, No.1, 1986, pp.2-14.
[9] Rafael C. Gonzalez, Richard E. Woods, *"Digital image Processing"*, Addison-Wesley, 1993.

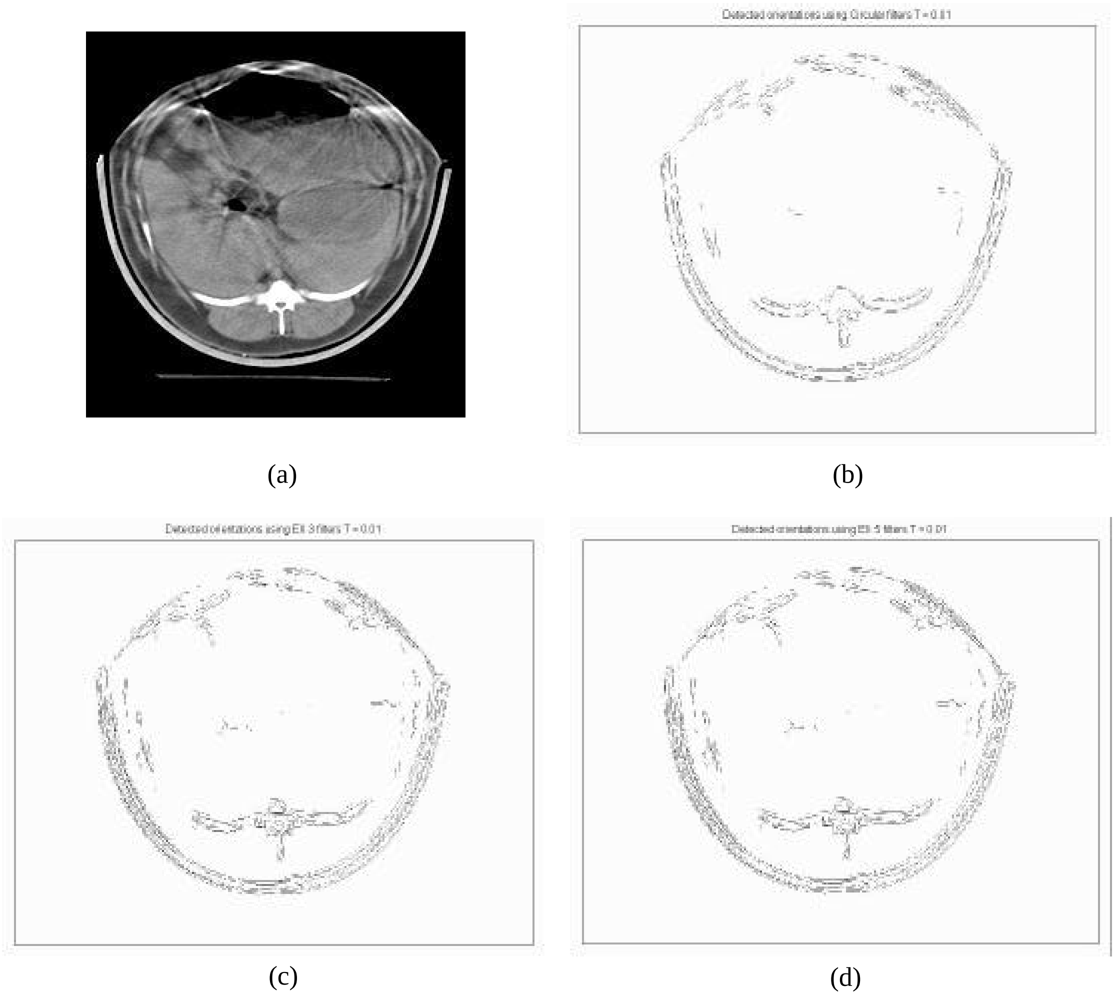

Figure 1. Reconstructed results from orientation analysis – (a) Test image (b) Response circular filters (c) Response elliptical filters, $m = 3$ (d) Response elliptical filters, $m = 5$.

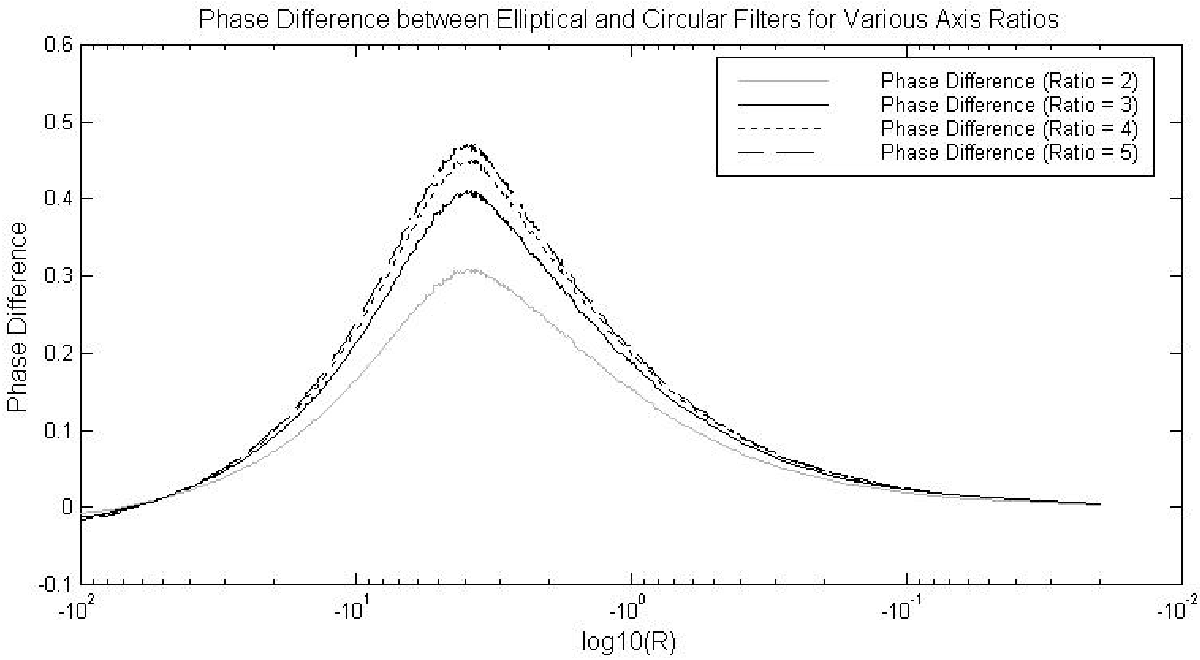

Figure 2. Plot of the phase difference between circular and elliptical filter responses of various axis ratios, *m*, against the minimum radius of curvature.