Wavelet Analysis in the Study of the Nuclear Magnetic Resonance Spectroscopy of Head Trauma

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Abstract: - A continuous wavelet transform using the Morlet wavelet is applied to spectroscopic data obtained from individuals who have suffered various degrees of head trauma and compared against control subjects. Results are presented which demonstrate the advantages of employing wavelet analysis in the study of quasi-stationary nuclear magnetic resonance free-induction decay in comparison to using traditional Fourier transform techniques alone.

Key Words: - head trauma, spectroscopy, NMR, wavelets, continuous wavelet transform, Morlet wavelet

1 Introduction

Trauma to the head is an unfortunately common occurrence that is highly variable in degree, physical form, and above all, outcome. Diagnosing the manner in which trauma to the head occurs is often easy; determining intrinsic changes in the brain and predicting outcome, is not. Today, one of the best methods for diagnosing the effects of injury to the head is clinical nuclear magnetic resonance spectroscopy (MRS), a technique for extracting chemical information from regions inside the brain through perturbations of high-strength magnetic fields. The chemical concentrations measured by MRS are used to determine quantitatively the severity of trauma and subsequent outcome. For example, MRS is extremely useful in determining if a patient will awake from a coma after a traumatic injury.

In an effort to improve our understanding of head trauma we have applied wavelet analysis techniques to the spectroscopic data of patients who have suffered various degrees of head injury. MRS signals, in contrast to other subjects of wavelet analysis (e.g. signals of cataclysmic geophysical phenomena), are more dependent on frequency information and are smoother in time. Consequently, our analysis places a heavier emphasis on frequency localization versus time localization.

2 Background

2.1 Basics of MRS

Nuclear magnetic resonance spectroscopy (MRS) is a technique for extracting chemical information from a region localized in space. It provides more information and greater accuracy about biological systems than magnetic resonance imaging (MRI), its close cousin. One of the great strengths of MRS is its safety: it is non-invasive, requiring no surgery or biopsy, and operates at radio frequency in comparison to the far higher frequencies of X-ray and CT.

In MRS, molecules are placed in the presence of a powerful homogenous magnetic field, inducing a large difference in the populations of nuclear spins. Radio frequency pulses synchronize the precession of these spins and signal is recorded as the spins dephase. The resonant frequency $\grave{u}_k$ of a nucleus of type k can be described:

$$w_k = \frac{g}{2p} B_0 \left(1 - s_k\right) \qquad (1)$$

where $\tilde{a}$ is the magnetogyric ratio (constant for a particular isotope, ^{1}H in our studies), B_0 the magnetic field strength, and $\acute{o}_k$ the shielding constant for k [1]. Since metabolites with differing chemical structures have different shielding constants, they also have different resonant frequencies. As the population of a particular metabolite increases, so does the signal at the metabolite's resonant frequency. Thus, signal at certain frequencies is correlated with the chemical concentration of certain metabolites.

If equation (1) explained the entire story behind the behavior of different nuclei, the MRS signal would be a stationary wave, and consequently, there would be no need for further analysis: the Fourier transform would suffice. However, several factors such as *spin-lattice relaxation*, *spin-spin relaxation*, and magnetic field inhomogeneity cause an exponential decay of signal [2]. This composite signal, called the free induction decay (FID), contains not only the resonant frequencies of a wide range of chemicals, but also decay rates endemic to these frequencies.

2.2 The Fourier Transform in MRS

Traditionally, a Fourier transform is applied to an FID in order to extract frequency content. Given a discrete sequence $\boldsymbol{x} = \{ x_0 .. x_{N-1} \}$, the components of the discrete Fourier transform $\mathbf{F}(\boldsymbol{x}) = \{ \hat{x}_0 .. \hat{x}_{N-1} \}$ are defined:

$$\hat{x}_n = \frac{1}{N}\sum_{k=0}^{N-1} x_k e^{-\frac{2pikn}{N}} \qquad (2)$$

By examining amplitudes at various frequencies, a detailed profile of the chemical composition of a sample can be determined. MRS using Fourier techniques is highly effective in characterizing a wide range of pathologies [3]. Unfortunately, the global nature of the Fourier transformation hides local time information, and makes determination of time-related changes impossible. Furthermore, closely resonant frequencies add to one another in Fourier space when field homogeneity is not ideal. This can complicate diagnosis by eliminating the ability to distinguish one chemical from another. Consider the Fourier transform of two signals composed of three frequencies of equal amplitude and decay rate in Figure 1. The only difference between the two signals is the frequency separation between components.

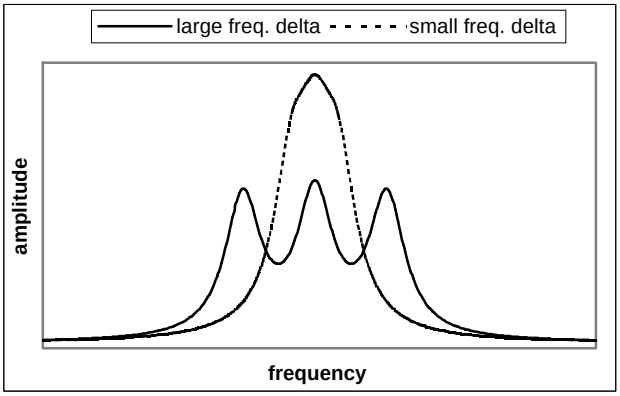

Fig.1: Peak spillover problem in the Fourier Transform

As the frequency difference between peaks decreases, the peaks merge. This causes an overestimation in the concentration of the central peak. In addition, as the frequency differences become even smaller, it becomes impossible to distinguish the three components of the signal, and estimations of decay rate based on peak broadness fail.

Another problem inherent to the Fourier transformation of MRS signals is the problem of correct phasing. In actuality, acquisition of an FID begins a few moments after the radio frequency pulse, as the transmitter is also the receiver. This small delay introduces a phase shift in the acquired signal. Figure 2 illustrates the effect.

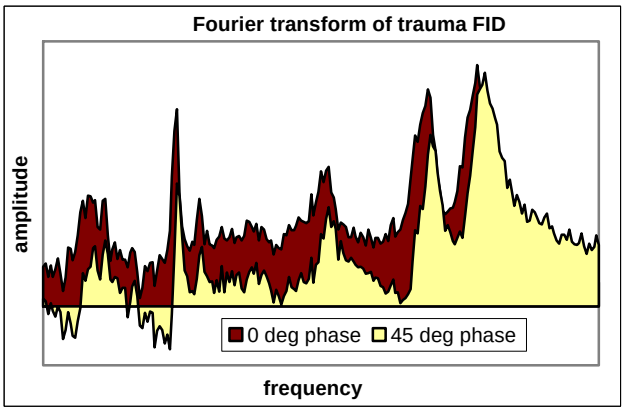

Fig.2: Fourier transform phasing effects

Phasing factors can drastically alter the appearance of data, giving false impressions of chemical concentrations. Simulated data is easy to phase correctly; real data, with noise and differences specific to individuals, is difficult. In fact, though phasing has been a problem since the beginning of clinical MRS, phase is still corrected manually, and the human eye regularly beats algorithmic solutions.

Windowed Fourier analysis helps to resolve the issue of obtaining time information and frequency content simultaneously, but suffers from constraints imposed by windowing (e.g. non-uniform resolution of features widely varying in size and determination of the "best" window size for analysis) [4]. In addition, phasing problems are multiplied as the window slides along the signal.

3 Wavelets

A better alternative to simultaneously obtain both time and frequency information is wavelet analysis. In wavelet analysis, a signal is convolved with a set of functions well localized in both time and frequency domains. These wavelet functions are scaled and translated versions of each other. In other words, wavelet analysis employs windows of varying sizes and positions to analyze a signal. This allows fine resolution of both time and frequency without restrictions on the scale of detectable phenomena.

Wavelet analysis falls under two categories: discrete and continuous. The discrete wavelet transform (DWT) uses window scales and translations that vary in a discrete, pre-set, manner and employs wavelet basis functions that are orthonormal. The continuous wavelet transform (CWT), in comparison, uses window scales of any size or position and does not require orthoganality in its basis functions. The DWT allows perfect reconstruction of the original signal, is orders of magnitude faster, and is simpler to implement than the CWT. These properties make the DWT ideal for compression, denoising, and decomposing extremely large data sets [5].

The more flexible nature of the CWT, however, allows information to be analyzed in a much finer and controllable manner. For this reason, we focus on the CWT exclusively.

3.1 Continuous Wavelet Transform (CWT)

Given a discrete sequence $\boldsymbol{x} = \{ x_0 .. x_{N-1} \}$, and a wavelet function Ø with complex conjugate Ø*, the wavelet decomposition of sequence $\boldsymbol{x}$ and scale s is $\mathbf{W}(\boldsymbol{x}, s) = \{W_0(s) .. W_{N-1}(s)\}$, where $W_n(s)$ is defined:

$$W_n(s) = \frac{1}{\sqrt{s}} \sum_{m=0}^{N-1} x_m \text{Ø}^* \left(\frac{m-n}{s} \right) \qquad (3)$$

In essence, the CWT is a convolution of the wavelet function Ø and sequence $\boldsymbol{x}$. By applying the convolution property of the Fourier transform, and utilizing a fast Fourier transform algorithm, it is possible to perform this convolution extremely efficiently [6]. Note that the square root before the summation is a normalization factor taken up by Ø in other conventions.

3.2 The Morlet Wavelet

Different wavelet basis are better for different tasks. For the case of resolving strong frequency components, the Morlet wavelet is a good choice. The Morlet wavelet is defined, for time t, as:

$$\Psi(t) = p^{-\frac{1}{4}} (e^{-i w_0 t - e^{-\frac{1}{2} i w_0^2}}) e^{-\frac{1}{2} t^2} \qquad (4)$$

Where $p^{-\frac{1}{4}}$ is a normalization factor and $\grave{u}_0$ is a parameter controlling oscillation frequency. For $\grave{u}_0 > 5$, the exponential term $\exp(-i\grave{u}_0^2/2)$ is less than 4×10^{-6} and so (4) can be approximated:

$$\Psi(t) = p^{-\frac{1}{4}} (e^{-i w_0 t}) e^{-\frac{1}{2} t^2} \qquad (5)$$

The Morlet wavelet, then, is an oscillating wave enveloped by a gaussian. To examine the properties of the Morlet wavelet at difference scales, we take the Fourier transform of (5) keeping in mind the form of (3), i.e. $\mathbf{F}(\text{Ø}(t/s))$, to get $\hat{\Psi}(sw)$:

$$\hat{\Psi}(sw) = p^{-\frac{1}{4}} e^{-\frac{1}{2}(sw - w_0)^2} \qquad (6)$$

From (3,5,6) we can see that the Morlet wavelet has the center t and spread s in time, and the center $\grave{u}_0 / s$ and spread $1/s$ in frequency (for a more detailed treatment, see [4]). When examining decompositions of data with the Morlet wavelet it is important to keep in mind these changes in function spread since these factors play significant roles in both peak height and range of effect.

The relationship between scale s and frequency f can be calculated for the Morlet wavelet [7], and is given by:

$$s = \frac{w_0 + \sqrt{2 + w_0^2}}{4pf} \qquad (7)$$

Since, in MRS, we have prior knowledge of the frequencies at which important metabolites resonate, we can choose a range $R = [f_0, f_1]$ of frequencies such that by applying (7) we obtain corresponding scales. Note that for fixed R, the choice of a higher $\grave{u}_0$ implies correspondingly larger scales, and therefore greater precision in frequency, but less precision in time. Often, in the study of geophysical phenomena, an $\grave{u}_0$ between 5 and 6 is used to meet the approximation result in (5) and still retain as much time domain precision as possible, but the study of MRS FIDs requires much higher frequency to resolve peaks, and thereby necessitates an $\grave{u}_0$ of 20 and higher. The larger scales require careful consideration, a good treatment of which can be found in [8].

4 Application of CWT to Head Trauma

Physical injury to the head can cause severe disruption of neurological function, coma and even death. While the more obvious physical changes (e.g. fractures, bleeding, or contusion) can be identified by CT scans or by water-based MRI, the best description of events may be in terms of disordered brain biochemistry [9]. In our study, ^{1}H (proton) MRS was performed using a 1.5 Tesla clinical scanner on a number of patients who had suffered traumatic brain injury. The FIDs from these cases was then wavelet transformed using (3) and (5) and compared against controls. The following questions were addressed:

1. Are the frequency and/or temporal wavelet transforms able to distinguish head injured from normal MRS studies?
2. Does visual examination of the wavelet transform enable distinction between mild and severe head injury?
3. Is there "predictive value" in the wavelet transform of head-injured patients?
4. Does the wavelet transform offer any advantages over the Fourier transform?

A case study of a control subject (Figure 3) versus patients suffering moderate (Figure 4) and severe (Figure 5) head injury should help answer some of these questions.

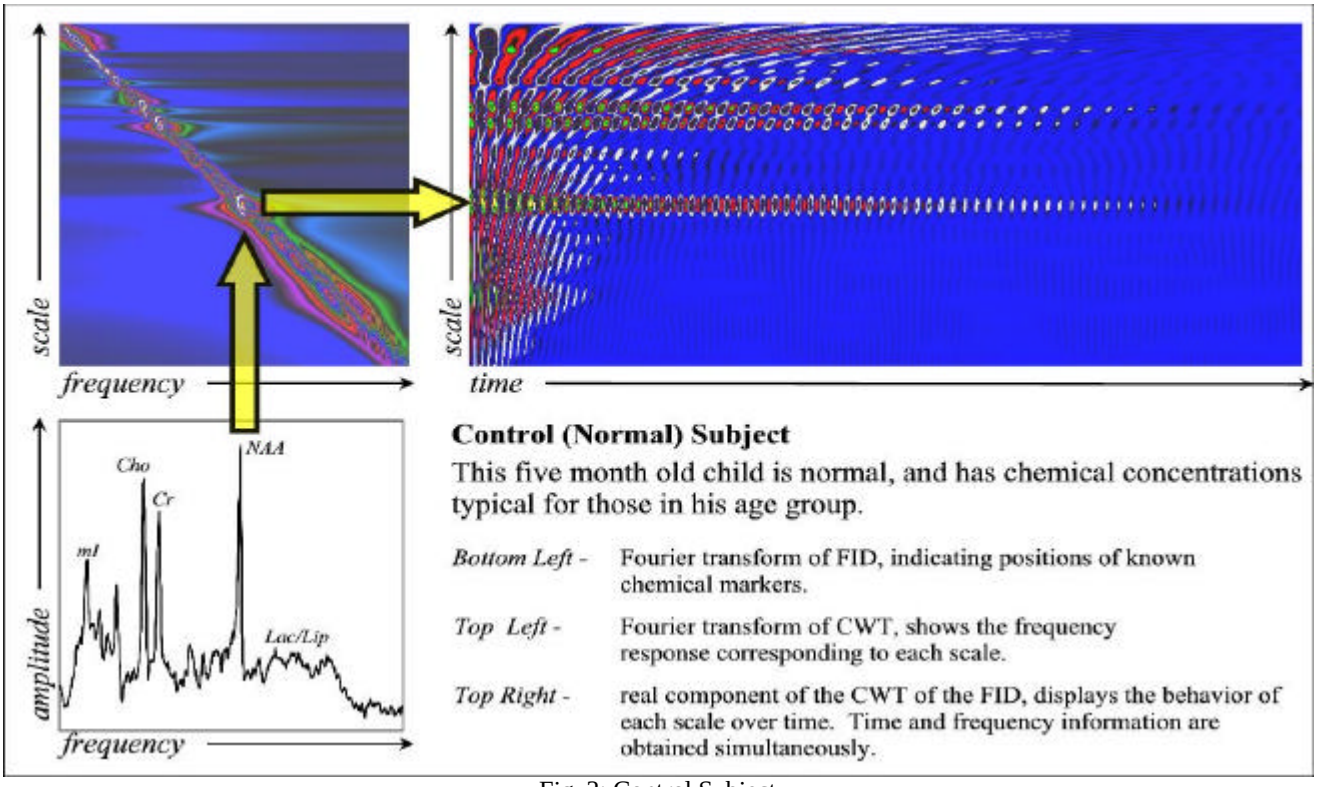

Fig. 3: Control Subject

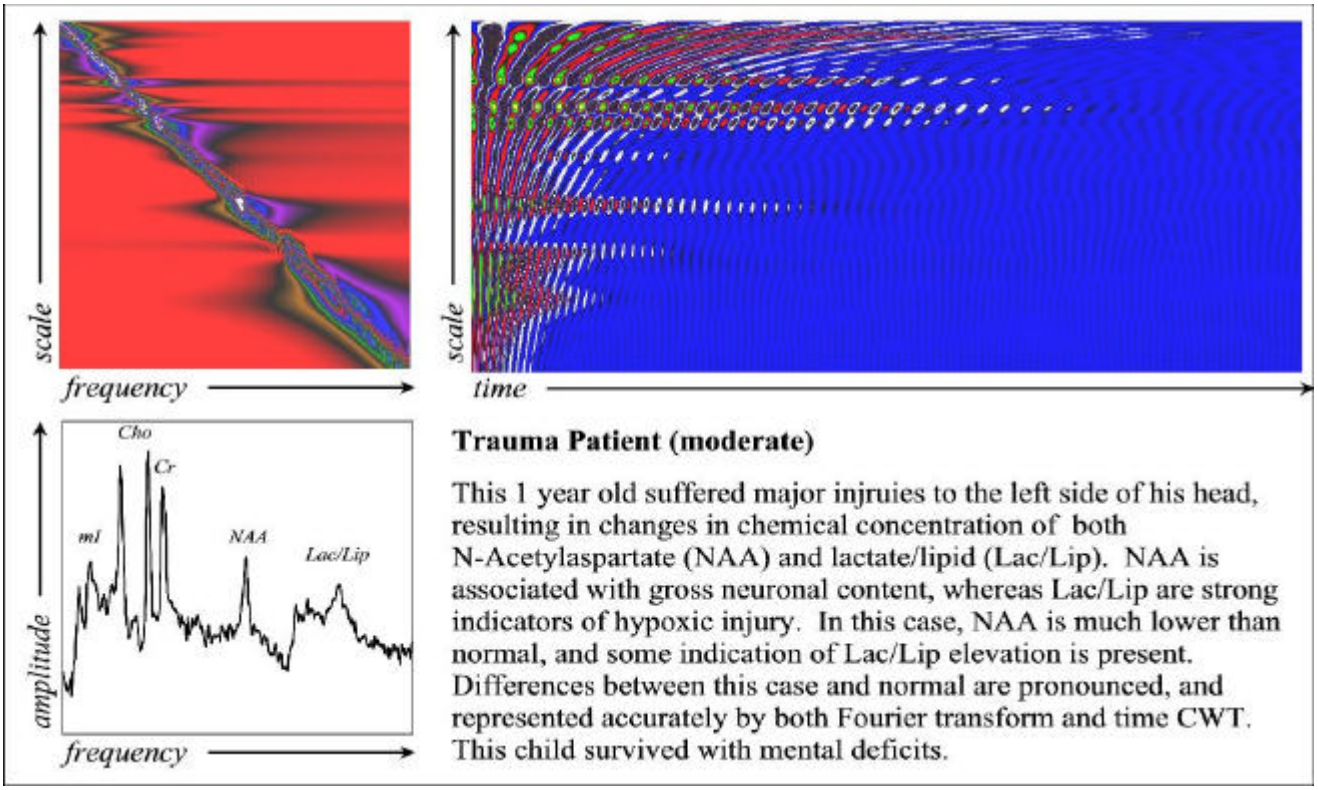

Fig. 4: Head Trauma Case (Moderate)

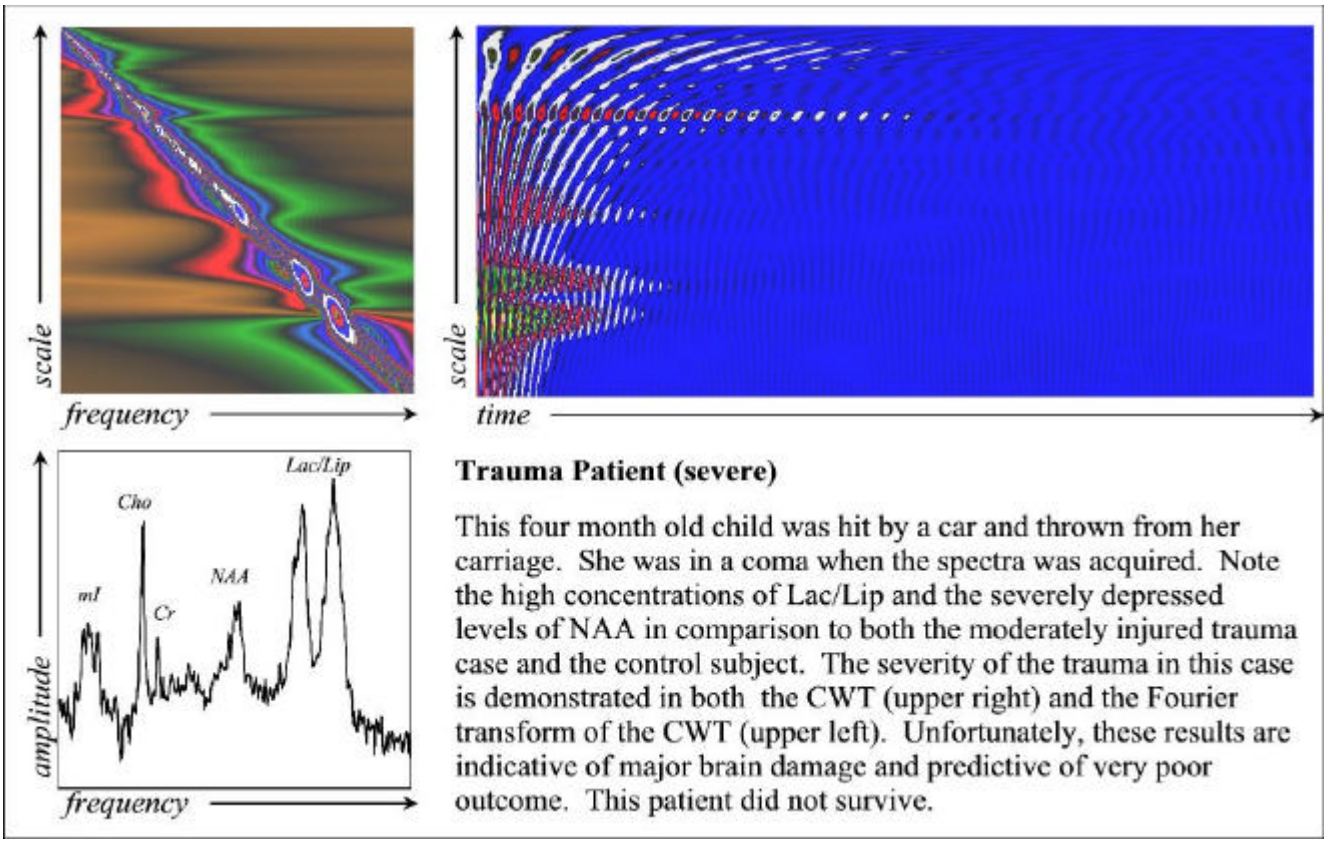

Fig. 5: Head Trauma Case (Severe)

Note that the wavelet transform in time strongly coincides with results from Fourier transform. In addition, had we not chosen to plot scales directly corresponding to frequencies by (7), the Fourier transform of the wavelet transforms (upper left plates in Figure 3-5) would be useful as a roadmap to translate scales into corresponding frequencies. Analysis of a number of cases demonstrated the ability of the wavelet transform to:

1. Clearly distinguish between head injury and normal subjects.
2. Gauge the severity of head trauma through corresponding chemical markers.
3. Collaborate results attained from Fourier transform MRS that are predictive of outcome [9].

To see the advantages of the wavelet transform versus Fourier transform, we re-examine the peak spillover problem (Figure 1) discussed in section 2.2 and plot the *scalogram*. The *scalogram*, or *wavelet power spectrum*, is a plot of power at varying times and scales, where power $P_n(s)$ at time n and scale s is defined:

$$P_n(s) = \left|W_n(s)\right|^2 \qquad (8)$$

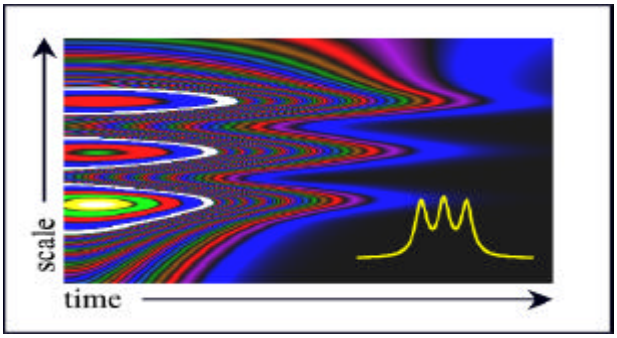

Fig. 6: Scalogram of large delta frequency

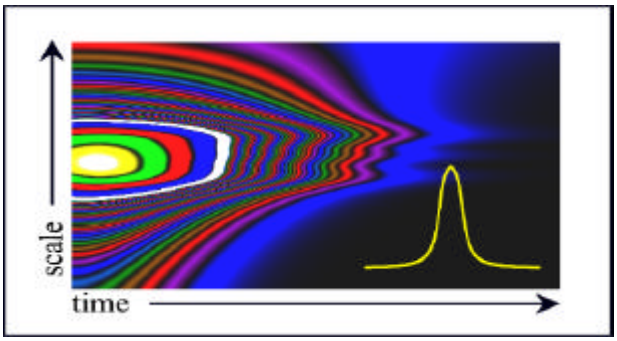

Fig. 7: Scalogram of small delta frequency

Note in Figure 7 that the central peak retains its behavior in time despite the presence of neighboring peaks that make resolution in frequency difficult. Also note that as time progresses the differences between peaks becomes more pronounced, and the presence of multiple peaks becomes obvious.

In addition, phasing problems, which cause massive changes in the Fourier spectrum, result in only a small period shifting in time (Figure 8, 9). Moreover, this shift in time does not change the intrinsic exponential decay rate found at each scale. In other words, as the oscillating frequency shifts in time, it correspondingly shifts in amplitude. Thus, continuous wavelet transform analysis employing Morlet wavelets is more resistant to the problems presented in section 2.2. However, as noted in 3.2, and shown by the differences between Figures 6 and 7, different scales have varying spreads. These differences are intrinsic to the wavelet transform of a finite discrete series and not to the nature of the data itself (i.e. spread differences do not represent contamination of data by other data). This implies that even though irregularities exist at various scales, they are consistent and will not affect the reproducibility of clinical work.

5 Conclusions

In short, the wavelet transform allows the use of time information to augment analytical ability. Even when the signal is quasi-stationary, as in the case of MRS FID signals, use of time domain information can make analysis more robust and precise. In the case of examining head trauma, wavelet analysis not only concurs with the current bulk of medical knowledge, but also allows access to time information that is hidden by using Fourier transform methods alone. Though more research is necessary to reveal the limitations and capabilities of wavelet analysis in studying MRS signals, preliminary results are very promising and may point to new possibilities in improving the state of the art in medical technology.

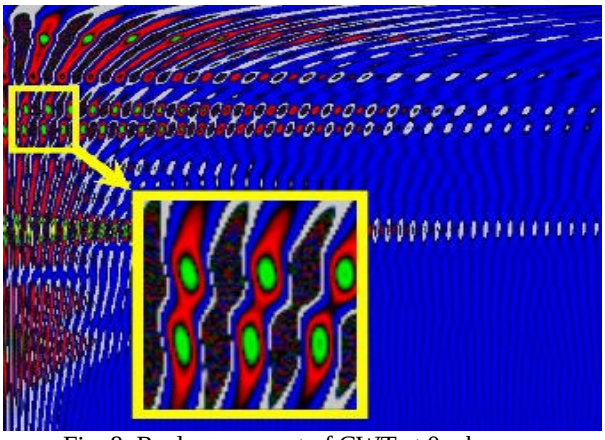

Fig. 8: Real component of CWT at 0 phase

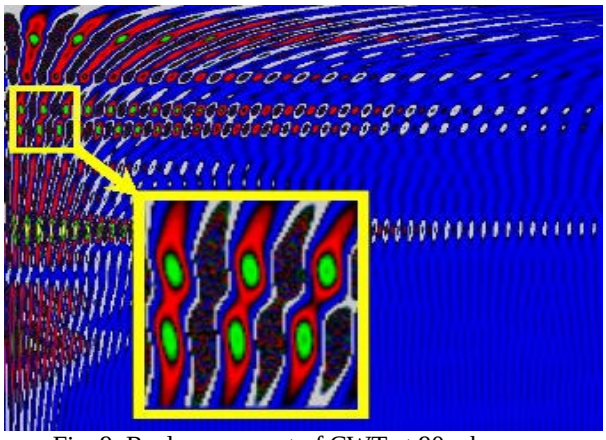

Fig. 9: Real component of CWT at 90 phase

References

[1] Gadian, DG, *Nuclear Magnetic Resonance and its Application to Living Systems*, Oxford University Press, 1982.

[2] Salibi N and Brown M, *Clinical MR Spectroscopy*, Wiley-Liss Inc., 1998.

[3] Ross BD, Blüml S, "Neurospectroscopy", *Neuroimaging*, McGraw Hill, 1999, pp. 727-773.

[4] Kumar P and Foufoula-Georgiou E, *Wavelets in Geophysics*, American Press, 1994.

[5] Hubbard B, *The World According to Wavelets*, AK Peters LTD, 1998.

[6] Torrence C and Compo G, "A Practical Guide to Wavelet Analysis", *Bulletin of the American Meteorological Society*, Vol. 79, 1998, pp 61-78.

[7] Meyers SD, Kelly BG and O'Brien JJ, "An Introduction to Wavelet Analysis in Oceanography and Meteorology: With Application to the Dispersion of Yanai Waves", *Monthly Weather Review*, Vol. 121, 1993, pp. 2858-2865.

[8] Jordan D, Miksad RW, and Powers EJ, "Implementation of the Continuous Wavelet Transform for Digital Time Series Analysis", *Review of Scientific Instruments*, Vol. 68(3), 1997, pp. 1484-1494.

[9] Ross BD, Ernst T, Kreis R, Haseler LJ, Bayer S, Danielsen E, Blüml S, Shonk T, Mandigo JC, Caton W, Clark C, Jensen SW, Lehman NL, Arcinue E, Pudenz R, Shelden CH, "^{1}H MRS in Acute Traumatic Brain Injury", *JMRI*, Vol.8, 1998, pp. 829-840.