Genetic Methodology for Linear Output Feedback Control Low Design

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Abstract

The multivariable linear output feedback technique is recast as a constrained nonlinear optimization problem. An evolutionary, multiple-objective genetic algorithm is applied to encapsulate and globally optimally reconcile stability, robustness, performance enhancement, reliability, actuator limitations, numerical and computational pitfalls, and tracking and regulation, faced to structured or unstructured system uncertainties. The potentials and effectiveness of the proposed method are substantiated by simulation results.

Keywords: Eigenstructure assignment, multivariable linear output feedback, constrained nonlinear optimization, GAs.

I. Introduction

Consider any minimal linear-time-invariant multivariable system described by

$$\dot{x} = Ax + Bu \qquad (1)$$

$$y = Cx \qquad (2)$$

where $A \in \mathbf{R}^{n\times n}, B \in \mathbf{R}^{n\times m}, C \in \mathbf{R}^{l\times n}$, and B and C without loss of generality are of full rank. Applying the static output feedback control law

$$u = Gy \qquad (3)$$

the closed loop system state equations would be

$$\dot{x} = (A + BGC)x. \qquad (4)$$

The closed loop system is desired to have a prescribed eigenstructure. In this paper, for notational simplicity it is assumed that the desired closed loop eigenvalues are distinct. All the developments can easily be extended to the case of repeated eigenvalues.

The output feedback design problem has always been a pole of attraction for many researchers due to various unanimous reasons. The entire eigenstructure assignment (EEA) and multivariable pole placement (MVPP) through linear output feedback have been intensively studied over the last three decades [1]-[60]. In [1] it was shown that if the system is completely controllable and observable, then l poles of the closed loop system can be almost arbitrarily assigned via static output feedback, where l is the number of independent system outputs. In [2] and [3] the results were extended and it was shown that under the same conditions as that of [1], $max(m,l)$ poles of the closed loop system are almost arbitrarily assignable by constant output feedback, where m is the number of independent system inputs. Later in [4] and [5] it was shown that subject to the condition $m + l > n$, where n is the number of states, all of the n poles can unconditionally be assigned if a slight modification in their location is tolerable. In [6] the results of [5] were generalized but a limitation is that the desired poles of the closed loop system are distinct and different from those of the open loop system. By reformulating the output feedback problem as an eigenvalue/eigenvector one, new sufficient conditions to assign $min(n, m + l - 1)$ eigenvalues using gain output feedback were derived in [7] and it was substantiated that, in general, in addition to assigning $m + l - 1$ eigenvalues, $l - 1$ eigenvectors can be partially assigned with m entries in each vector arbitrarily chosen. In [8], [9], [10], and [11] restricted to some decoupling or full rank Plucker matrix conditions, it was shown that $ml \geq n$ is a lesser restrictive sufficient condition for arbitrary pole assignment via linear output feedback, but it was in fact in the seminal work of [12] that $ml > n$ was shown a generic (i.e. for almost all systems) sufficient condition for that of linear multivariable systems (In case of equality real solutions may not exist.). Some necessary and sufficient conditions for accomplishing MVPP and/or EEA using gain output feedback were presented in [10], [11], [13], [14], [15], [16], [17], [18], [19], [20], [21], and [22] where none of them is as tangible as that of [21] and [22], none so generic as that of [17], and the necessary and sufficient conditions for pole assignability with complex gain output feedback established in [11] later shown to be only necessary for the case of real gain output feedback. In [10] this condition was strengthened using the Plucker matrix. For the results reported in [8], [9], [10], [11], [15], [23], and [24] some recursive or analytical algorithms were proposed for determination of the linear output feedback matrix, which however sound numerically cumbersome to apply. Some parametric solutions [17], [20], [24], [25], [26], [27], [28], [29], [30], [31], and [32], two coupled full-order nonsquare Sylvester matrix equation approaches [16], [18], [20], and [29], and bilinear transformation strategies [18] and [33] can also be found in the literature, where by the latter the links between geometric and algebraic results were provided and the equality of the existing necessary and sufficient conditions was shown. Recursive and analytical approaches of [15] and [34] are refutable under some unknown conditions, and the integrated approaches of [35] and [36] highly dependent on the insight of the designer. Nonlinear optimization techniques were also utilized in [37] and [38]. The problem of stabilization via static output feedback was addressed in [22], [39], and [40]. Tracking systems design was also tackled in [22], [32], [41], [42], and [43], and a convex analysis provided in [19] for the nonconvex problem of optimal H_2 control of the aforementioned problem under some special conditions. The notion of (C, B, A)-invariance was utilized in [18], [29], and [44], the problem of numerical algorithms tackled in [45], and the problem of output feedback in the most general framework considered in [46], where it neatly utilized geometric framework involving lattices, but is however difficult to be translated to computational techniques. The EEA for the specific class of rank-one systems was addressed in [47], and that for time-varying linear systems in [48]. Some promising, yet embryonic, results were also provided in [49], [50], and [51]. Grassman space (i.e. exterior algebra) was invoked in [10], [12], and [52], and the control canonical form in [53]. The Hessenberg form was utilized to solve the two single-output problems and a germane algorithm was provided in [54]. Optimal and/or robust performance through EEA were addressed in [22], [33], [37], [38], [55], and [49]. Decoupling and decentralized eigenstructure assignment were considered in [25] and [49]. The EEA has been invoked pervasively, e.g. [41]-[43], and yet is gaining more attraction.

The above works, though prolific, all suffer from some of the sequel defects: unguaranteed workability, convergence, and global optimality, nongenericness, numerical problems, computational heaviness, etc., and are not comprehensive, in other words, none of them considers the following criteria simultaneously: stability, robustness, optimality, performance enhancement, reliability, feasibility (actuator limitation), numerical and computational problems, and tracking and regulation.

In this paper, first the nonlinearity intrinsically involved in the output feedback design problem is elucidated, recasting the problem as a nonlinear optimization one and introducing the notion of achievability. Then, based on the EEA concept, the aforementioned objectives are formulated to be simultaneously satisfied by designing a constant output feedback. The above problem will be solved by a novel evolutionary, multiple-objective genetic algorithm, triggering and paving the way for practical applications. The convergence and global optimality of the solution is guaranteed. Finally, simulation results are presented to attest to the potentials of the proposed method.

II. Output Feedback Control Law Design

Let K be a state feedback gain matrix required to place the eigenvalues of the closed loop plant described by equations (1), (2), and (3). Then

$$K = GC. \tag{5}$$

Consequently, from one standpoint, the problem of output feedback is to find G from (5), which is possible iff K is in the range space of C, or

$$rank \begin{pmatrix} K \\ C \end{pmatrix} = m. \tag{6}$$

In other words, ml unknown entries of G can be found from mn equations (5) iff they are consistant. In such a case G will be given by

$$G = KC^T(CC^T)^{-1} \tag{7}$$

where $\|.\|_2$ denotes the matrix two norm. Hence, regarding that for a MIMO system K is not unique, the problem can in fact be to minimize

$$\|K(I - C^T(CC^T)^{-1}C)\|_2. \tag{8}$$

Definition: When (5) can be solved exactly, s.t. (6) holds, the given eigenvalue spectra is said to be *achievable* via linear output feedback.

The *achievability* is always but not a necessity. In fact, the desired closed-loop poles suffice to be assigned in some admissible regions. Therefore, the problem will be tackled through optimization techniques by finding G directly such that the design criteria be optimally reconciled. In the next section, the design objectives of the above optimization problem will be formulated.

III. Optimization Criteria

A. Stability Robustness

Structured Perturbations

Let the nominal system described by equations (1) through (4) be perturbed and discribed by

$$\dot{x} = (A + BGC + \Sigma_{i=1}^{k} \Delta p_i E_i)x \tag{9}$$

where E_i, $(i = 1, ..., k)$ are matrices determined by the structure of parameter perturbations Δp_i $(i = 1, ..., k)$. The nominal closed loop system matrix is assumed to be asymptotically stable. Then,

Theorem 1 [56] : The perturbed system (9) is stable for all Δp_i satisfying

$$|\Delta p_i| < \frac{2}{\|\Sigma_{i=1}^{k}(E_i^T P + PE_i)\|_2} \tag{10}$$

where P denotes the unique positive definite symmetric matrix solution of

$$A_c^T P + PA_c + 2I_n = 0. \tag{11}$$

Unstructured Perturbations

Let the closed loop perturbed system with parameter uncertainties be represented by

$$\dot{x} = (A + BGC + E)x \tag{12}$$

where E is the 2-norm bounded perturbation matrix and the nominal closed loop system matrix is assumed to be asymptotically stable. Then,

Theorem 2 [57] : The perturbed system (12) is stable for all E satisfying

$$\sigma_{max}(E) \leq \frac{1}{\sigma_{max}(P)} \tag{13}$$

where $\sigma_{max}(.)$ is the maximum singular value, and P is the unique symmetric positive definite matrix solution of Lyapunov equation (11).

B. Performance Robustness

A system is said to have some properties of performance robustness if the closed loop eigenvalues are guaranteed to stay in a specified performance region. In case of Ω region, defined below, performance robustness of the system is equivalent to the stability robustness of an augmented system.

Ω *region* [37] : represents the whole part of the left-half complex s-plane which is left to both lines

$$y = \pm cotan(\delta)(x + \sigma) \tag{14}$$

where x and y represent $\Re(s)$ and $\Im(s)$ in the complex s-plane, respectively, $0 < \delta < \pi/2$, and $\sigma > 0$.

Augmented System

The system described by [37]

$$\dot{x} = A_{c\delta}x \tag{15}$$

is called the augmented system, where

$$A_{c\delta} = \Theta(\delta) \otimes (A + BGC + \sigma I_n) \tag{16}$$

$$\Theta(\delta) = \begin{pmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{pmatrix} \tag{17}$$

with $\sigma > 0$, $0 < \delta < \frac{\pi}{2}$, and where $\otimes$ is the Kronecker product of matrices. The main property of the augmented system is that [58] the necessary and sufficient condition to have the eigenvalues of the real matrix $A + BGC$ in the region Ω is that

$$\Re[\lambda_i(A_{c\delta})] \leq 0. \tag{18}$$

This property will be used to extend the stability robustness measure to include performance robustness. The idea is to find the stability robustness measure for the augmented system which is tantamount to the performance robustness measure of the original system.

To deal with the robustness problem two cases of perturbations will be considered in the sequel.

Structured Perturbations

The perturbed augmented system in this case can be written as

$$\dot{x} = (A_{c\delta} + \Sigma_{i=1}^{k} \Delta p_i E_{\delta i})x \tag{19}$$

where

$$E_{\delta i} = \Theta \otimes E_i. \tag{20}$$

The performance robustness result is resumed in the following theorem.

Theorem 3 [37] : The perturbed system (9) has its eigenvalues in the region Ω for all Δp_i satisfying

$$|\Delta p_i| < \frac{2}{\|\Sigma_{i=1}^{k}(E_{\delta i}^T P_\delta + P_\delta E_{\delta i})\|_2} \tag{21}$$

where P_δ is the symmetric positive definite matrix solution of the following Lyapunov equation

$$A_{c\delta}^T P_\delta + P_\delta A_{c\delta} + 2I_{2n} = 0 \tag{22}$$

Thus, a minimization objective is

$$J_1 = \|\Sigma_{i=1}^{k}(E_{\delta i}^T P_\delta + P_\delta E_{\delta i})\|_2. \tag{23}$$

Unstructured Perturbations

In this case, the augmented perturbed system is defined by

$$\dot{x} = (A_{c\delta} + E_\delta)x \tag{24}$$

with $E_\delta = \Theta \otimes E$. The performance robustness result is resumed in the sequel theorem.

Theorem 4 [37] : The perturbed system (12) has its eigenvalues in the region Ω if

$$\sigma_{max}(E) < \frac{1}{\sigma_{max}(P_\delta)}. \tag{25}$$

Consequently, another minimization objective is

$$J_2 = \|P_\delta\|_2. \tag{26}$$

C. Decoupling Performance

Let the effect of disturbance u^{dist} on the state equations (1) be considered through the additive term Du^{dist} , where $D \in \mathbf{R}^{n \times p}$, and v_i and w_i denote the eigenvectors and reciprocal eigenvectors of the closed loop system. As in [22] and [42] it can be shown that $w_i^T B = w_i^T \begin{pmatrix} b_1 & \ldots & b_m \end{pmatrix}$ is an inputs-to-mode decoupling vector, $w_i^T D = w_i^T \begin{pmatrix} d_1 & \ldots & d_p \end{pmatrix}$ a disturbances-to-mode decoupling vector, $c_k V = c_k \begin{pmatrix} v_1 & \ldots & v_n \end{pmatrix}$ a modes-to-output decoupling vector (Output decoupling is of paramount significance in partially uncontrollable systems), and $c_k v_i w_i^T B$ an inputs-to-output decoupling vector, where $1 \leq i \leq n$, $1 \leq j \leq p, m$, and $1 \leq k \leq l$.

Therefore, necessary and sufficient condition for decoupling of the jth input to the ith mode is that $w_i^T b_j$=0, that of the jth disturbance to the ith mode $w_i^T d_j$=0, sufficient condition for decoupling of the ith mode to the kth output is that $c_k v_i$=0, and necessary and sufficient condition for decoupling of the jth input to the kth output $c_k v_i w_i^T b_j$=0, for $i = 1, ..., n$.

A system described as above is said to have high decoupling performance if the above decoupling conditions can be satisfied.

However, in most practical situations it is neither possible nor expected to have complete decoupling. In fact, the criterion for each of them is respectively the minimization of the following cost functions

$$J_3 = (w_i^T b_j)^2 \tag{27}$$

$$J_4 = (w_i^T d_j)^2 \tag{28}$$

$$J_5 = (c_k v_i)^2 \tag{29}$$

$$J_6 = \Sigma_{i=1}^n (c_k v_i w_i^T b_j)^2. \tag{30}$$

D. Actuator Performance and Reliability

To comply with practical implementation requirements it is significant to check the feedback gains magnitudes when assigning an eigenstructure. Thus, a minimization objective is [55]

$$J_7 = \|G\|_2. \tag{31}$$

Reliability can be increased by suppressing some certain gains to zero [59]. This, reduces the controller complexity and alleviates its implementation, too . Hence, another minimization objective is

$$J_8 = G_{ij}^2. \tag{32}$$

E. Numerical and Computational Problems

By modifying the locations of the eigenvalues in the prescribed admissible region a well conditioned solution will be achieved. Therefore

$$J_9 = \kappa(V) = \|V\|_2 \|V^{-1}\|_2 \tag{33}$$

is included in the minimization. Note that the minimum achievable condition number $\kappa(V)$ is limited [33]. Hereby the sensitivities of the assigned eigenvalues to perturbations are minimized, too [60].

The optimization is performed over G which has less entries than K. Thus, the computational burden is constricted, and execution time (Sec. IV) reduced.

IV. Evolutionary Algorithm

As stated in Sec. II, [12], [22], [33], [34], [37], and [61] the problem at hand is a nonliear optimization problem. Consequently, EAs, in particular GAs, will be utilized. One of the key features of GAs is that they search from a population of points and not from a single point. In addition they invoke objective function (fitness values) instead of derivative or auxiliary knowledge. These features render GAs robust and thus attractive to practical engineering applications. Another outstanding property of GAs is their commensuration to parallel processing. If coding, for some reasons, is such that the majority of computational time is perished by function evaluation, the simple $master - slave$ scheme [62] can be utilized where the slaves simply perform the function evaluation. In response to the need that most engineering problems stipulate a simultaneous optimization of multiple, often competing criteria (MO problems), a class of GAs for MO problems, often called Pareto GAs [63], have also been designed.

Now, in light of the preceding developments, the problem of multivariable linear output feedback is formulated as the sequel constrained optimization problem, well suited to the MOGA algorithm, specifically developed for this purpose.

Problem: Minimize the performance index J

$$J = \rho_i J_i + \rho_3 J_3 + \rho_4 J_4 + \rho_5 J_5 + \rho_6 J_6 + \rho_7 J_7 + \rho_8 J_8 + \rho_9 J_9 \tag{34}$$

where i=1 or 2, with respect to elements of G, subject to

$$\lambda_{i_{min}} \leq \lambda_i \leq \lambda_{i_{max}} \tag{35}$$

for a real eigenvalue λ_i, and

$$\lambda_{i_{\Re}\ min} \leq \Re(\lambda_i) \leq \lambda_{i_{\Re}\ max} \tag{36}$$

$$\lambda_{i_{\Im}\ min} \leq \Im(\lambda_i) \leq \lambda_{i_{\Im}\ max} \tag{37}$$

for a complex eigenvalue $\lambda_i = \Re(\lambda_i) + j\Im(\lambda_i)$, in the solution space

$$G_{ij}^{LL} \leq G_{ij} \leq G_{ij}^{UL} \tag{38}$$

where G_{ij}^{LL} and G_{ij}^{UL} are the lower and upper limits of G_{ij}. Coefficients ρ_1 through ρ_9 are positive scalar weightings that must be determined properly to achieve overall satisfactory performance. A rich literature survey for handling the constraints is proffered in [64]. Selection of the method hinges on the specific application.

Remark : The algorithm is readily applicable to the two techniques offered in [22] for tracker design.

V. Illustrative Example

Consider the system described by equations (1), (2), and (3) where

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

along with the desired closed loop eigenvalue spectra

$$\Lambda = \{\lambda_{1_d} = -1, \lambda_{2_d} = -2, \lambda_{3_d} = -3, \lambda_{4_d} = -4\}.$$

This system has been shown to exhibit -1 as a singular point in its eigenvalue spectra, i.e. unreachable through finite gain output feedback, [22] and [34]. In this work, the desired closed loop eigenvalue spectra is considered $\lambda_{1_d} = -1, \lambda_{2_d} = \lambda_{3_d} = -2.65$, and $\lambda_{4_d} = -4$, while the admissible closed loop eigenvalue spectra region is set to

$$-1.3 \leq \lambda_1 \leq -0.1$$

$$-3.3 \leq \Re(\lambda_2) = \Re(\lambda_3) \leq -1.5$$

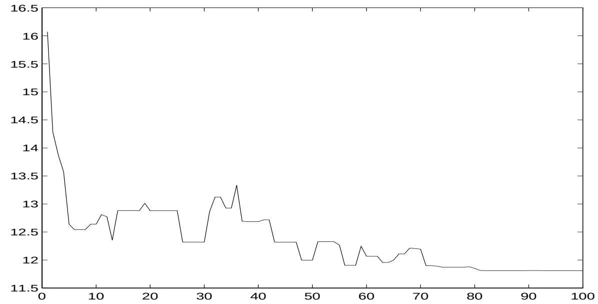

Fig. 1. Performance index minimization versus iteration

$$0 \leq \Im(\lambda_2) = -\Im(\lambda_3) \leq 1$$

$$-6 \leq \lambda_4 \leq -3.5.$$

The performance index to be minimized is

$$J = \Sigma_{i=1}^{4} (\lambda_i - \lambda_{i_d})^2 + 0.1 J_7 + 0.1 J_9.$$

in the solution space

$$-30 \leq G_{ij} \leq 30.$$

Floating point representation, *no encoding*, is applied as the encoding technique [65], so as to impose some shrinkage on the computational burden and expedite the algorithm convergence. Crossover operator is selected to be linear [66], in order to search through the whole solution space. Mutation operator is picked dynamic, so as to yield fine-tuning capabilities aimed at high precision [67]. The algorithm is run with crossover and mutation probabilities of 0.3 and 0.1, respectively. Performance index minimization is shown in Figure 1, and results listed in Table 1, where N represents the number of iterations. In this run, the minimum occurred at the 94-98 th iterations. The output feedback gain obtained is

$$G = \begin{pmatrix} -4.8112879016 & -3.7410642242 \\ -27.8975670092 & -16.4578934182 \end{pmatrix}.$$

As it is observed, the constraints are all met and the results are patently acceptable.

VI. **Conclusion**

An EEA-based evolutionary panacea, multiple-objective genetic algorithm, was adduced to globally optimally reconcile stability, robustness, performance enhancement, reliability, actuator limitations, numerical and computational problems, and tracking and regulation, faced to structured or unstructured system uncertainties via linear output feedback of general linear-time-invariant multivariable systems. Global optimality of the solution was guaranteed. Simulation resulte were provided.

Parameter	Best Iteration
N	94-98
J	11.8090357368
J_7	32.9499079064
J_9	37.9851598271
λ_1	-0.1722819132
λ_2	-1.6415801076 + j 0.6298328475
λ_3	-1.6415801076 - j 0.6298328475
λ_4	-5.0969099973

Table 1: Results of the algorithm run.

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