MIXED H_2/H_∞ DESIGN OF A MULTIRATE OUTPUT CONTROLLER

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Abstract- Multirate output controllers (MROCs) sample the plant outputs at a faster rate than they change the plant inputs. Consequently, they offer greater flexibility for controller design than conventional single-rate controllers. MROCs designed under mixed H_2/H_∞ performance have offered a new dimension in the design process. In this paper, we introduce the concept of lifting technique, fast discretization and multirate output sampling and calculate the mixed H_2/H_∞ norm of the sampled-data control systems induced via multirate output sampling.

Keywords: mixed H_2/H_∞ control, MROC, lifting technology, FIC controller

1. Introduction

Sampled-data control systems consist of a continuous-time plant to be controlled, a discrete-time controller controlling it, and ideal A/D and D/A converters. Figure 1 shows the idealized model of the standard sampled-data control system.

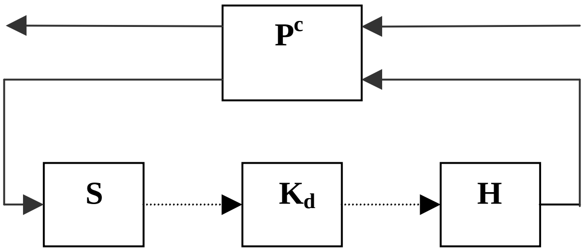

Figure 1: The standard sampled-data control system

In this paper, MROCs are engaged as the controller K_d in Figure 1. The hybrid closed-loop control system with a MROC is shown in Figure 2, where continuous-time signals are indicated in solid lines, discrete-time signals shown in dotted lines and continuous-time signals denoted with superscript "c". The block P^c is a continuous-time LTI plant. The inputs to the plant, P^c are the exogenous input, $w^c(t)$, containing commands, disturbances and sensor noise, and the control input, $u^c(t)$. The outputs of P^c are the controlled output, $z_0^c(t)$, $z_1^c(t)$, and the measured output, $y^c(t)$. MROCs detect the *ith* plant output, $y^c(t)$ at N_i uniformly spaced times and changes the plant input once during one frame period, T_0. For simplicity, in this paper, we assume that all the plant output channels are sampled at the same rate, m/T_0 and the rate of $1/T_0$ is applied to all channel holders, i.e. a MROC of uniform output sampling rate is used here. Readers can refer to [1], [2] for details of MROCs.

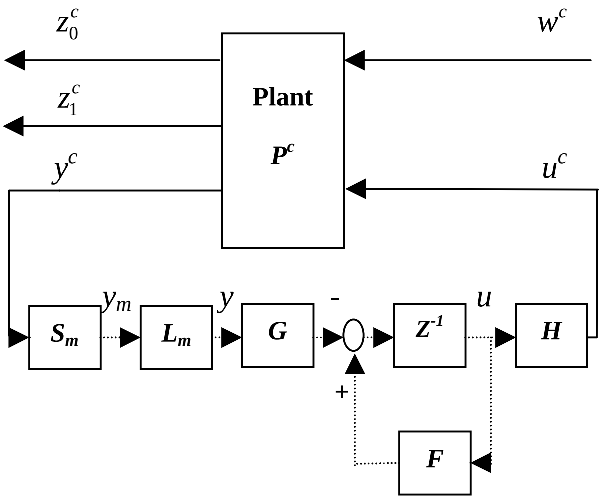

Figure 2: Hybrid closed-loop control system with a MROC

Design of control systems almost inevitably involves tradeoffs among competing objectives. It is often the case that the controller is required to meet several different performance and robustness goals, and all of these cannot be met simultaneously. In this paper, we solve the problem of finding MROC parameters F and G to minimize the H_2 norm from w^c to z_0^c in one channel while keeping the conflicting H_∞ performance on another channel below a postulated level. This is the question introduced by Bernstein and Haddad [3]. Instead of minimizing $\|T_{z_0w}\|_2$, we considered the minimization of an "upper bound" for $\|T_{z_0w}\|_2$, subject to the constraint $\|T_{z_1w}\|_\infty < \gamma$. In this paper, we focused on this kind of mixed H_2/H_∞ problem. This problem not only provides a tractable approach to the problem of minimizing nominal performance subject to a robust stability constraint, but also it can be interpreted as an optimal performance problem.

We solve this problem in three steps. First, we transfer a sampled-data system employing a MROC to a norm-equivalent discrete-time system. Then, we transfer it to a full-information controller (FIC) system. Third, we will show that the mixed H_2/H_∞ problem with full-information feedback can be reduced to a convex optimization problem over a convex bounded set of real matrices.

2. Lifting and Fast Discretization of a MROC System

The system shown in Figure 2 is a hybrid, periodic time-varying system. So, we cannot solve it directly using techniques for tackling LTI systems. However, using the technique of fast discretization [4], we compute a discrete-time model for P^c and obtain a discrete-time LTI system, accounting for intersample behavior. Furthermore, a technology called discrete lifting is used to lift slow discrete-time signals to suitably fast discrete-time signals.

Given $\tau > 0$ and an integer $l \geq 1$, we define three basic elements, namely the periodic sampler, the zeroth-order hold and the lifting operator. The periodic sampler, S_l samples the continuous-time signal y^c at a rate of l/τ. The zeroth-order hold, H_l operates on the discrete-time signal u_l at the rate of l/τ. The lifting operator, L_l is norm preserving, see [5] for details.

We denote $e^c = \begin{bmatrix} z_0^c \\ z_1^c \end{bmatrix}$, and $e = \begin{bmatrix} z_0 \\ z_1 \end{bmatrix}$, and choose an integer n and an integer q, which divides m exactly. We then discretize P^c by introducing the fictitious operators, S_m and H_l to both sides of the generalized plant P^c, which is partitioned as $P^c = \begin{bmatrix} P_{11}^c & P_{12}^c \\ P_{21}^c & P_{22}^c \end{bmatrix}$. In order to get a time-invariant plant, we also introduce the operators L_m and L_l^{-1} as in Figure 3. Finally we get the lifted plant in Figure 3, which is given as:

$$\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} L_m S_m & 0 \\ 0 & L_m S_m \end{bmatrix} P^c \begin{bmatrix} H_l L_l^{-1} & 0 \\ 0 & H \end{bmatrix} = \begin{bmatrix} L_m S_m P_{11}^c H_l L_l^{-1} & L_m S_m P_{12}^c H \\ L_m S_m P_{21}^c H_l L_l^{-1} & L_m S_m P_{22}^c H \end{bmatrix} \quad (1)$$

The lifted system shown in Figure 3 and the system depicted in Figure 2 are equivalent in norm so that if and only if the norm of the transformation matrix T_{ew} of the first system is less than γ, the norm of $T_{e^c w^c}$ of the second system is less than γ. We can transform the original system from one form to another form using these norm-preserving methods.

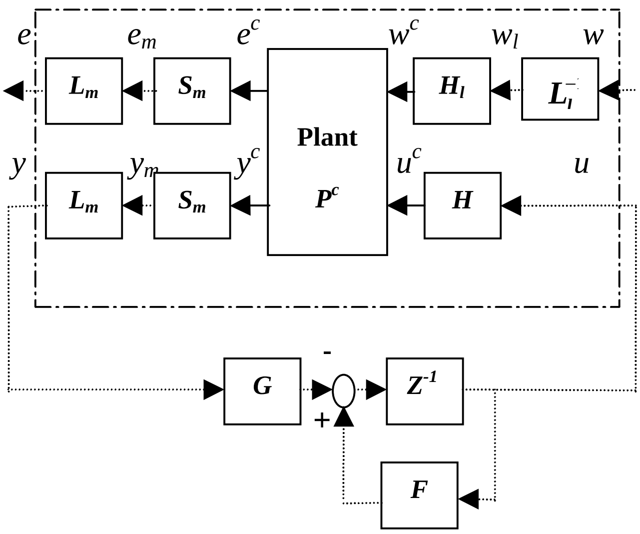

Figure 3: Closed-loop control system with fast-discretized and lifted plant

The discrete-time lifted plant P inside the dashed box in Figure 3 is time-invariant.

Suppose that the state equation for P^c is given by:

$$\begin{bmatrix} \dot{x}^c(t) \\ e^c(t) \\ y^c(t) \end{bmatrix} = \begin{bmatrix} A^c & B_1^c & B_2^c \\ C_1^c & D_{11}^c & D_{12}^c \\ C_2^c & D_{21}^c & D_{22}^c \end{bmatrix} \begin{bmatrix} x^c(t) \\ w^c(t) \\ u^c(t) \end{bmatrix} \quad (2)$$

Discretizing using a zeroth-order hold at the rate of m/τ, we can show that the discretization of P^c is:

$$\left[\begin{array}{c|cc} A_m & B_{m1} & B_{m2} \\ \hline C_1^c & D_{11}^c & D_{12}^c \\ C_2^c & D_{21}^c & D_{22}^c \end{array}\right] \quad (3)$$

where

$$A_m = e^{A^c \frac{\tau}{m}}, \quad B_{mj} = \int_0^{\frac{\tau}{m}} e^{A^c \sigma} d\sigma \, B_j^c, \quad j = 1,\ 2 .$$

Given an integer $m>1$, an integer $q \geq 1$ that divides m exactly, and the realization for $S_m G H_m$, the state-space realization for the lifted plant P in Figure 3 is given by

$$\left[\begin{array}{c|cc} A & B_1(q) & B_2 \\ \hline C_1 & D_{11}(q) & D_{12} \\ C_2 & D_{21}(q) & D_{22} \end{array}\right] \quad (4)$$

where

$$\left[\begin{array}{c|c} A & B_2 \\ \hline C_i & D_{i2} \end{array}\right] = \left[\begin{array}{c|c} A_m^m & (A_m^{m-1} + A_m^{m-2} + \cdots + I) B_{m2} \\ \hline \begin{bmatrix} C_i^c \\ C_i^c A_m \\ \vdots \\ C_i^c A_m^{m-1} \end{bmatrix} & \begin{bmatrix} D_{i2}^c \\ C_i^c B_{m2} + D_{i2}^c \\ \vdots \\ C_i^c (A_m^{m-2} + \cdots + I) B_{m2} + D_{i2}^c \end{bmatrix} \end{array}\right]$$

$$\left[\begin{array}{c} B_1(q) \\ \hline D_{i1}(q) \end{array}\right] = \left[\begin{array}{cccc} A_m^{m-1} B_{m1} & A_m^{m-2} B_{m1} & \cdots & B_{m1} \\ \hline D_{i1}^c & 0 & \cdots & 0 \\ C_i^c B_{m1} & D_{i1}^c & \cdots & 0 \\ C_i^c A_m B_{m1} & C_i^c B_{m1} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ C_i^c A_m^{m-2} B_{m1} & C_i^c A_m^{m-3} B_{m1} & \cdots & D_{i1}^c \end{array}\right] \times \ diag(B_1, \cdots, B_{\frac{m}{q}})$$

For i=1, 2, blocks B_k, $k=1,\dots.m/q$ are built by stacking q identity matrices as

$$B_k = \begin{bmatrix} I_{nw} \\ \vdots \\ I_{nw} \end{bmatrix} \quad (5)$$

The proof of this result can be found in [6].

3. Full Information Controller

Consider Figure 4, which depicts a full information controller (FIC) of the MROC system. Note that Figure 4 is obtained by redrawing Figure 3.

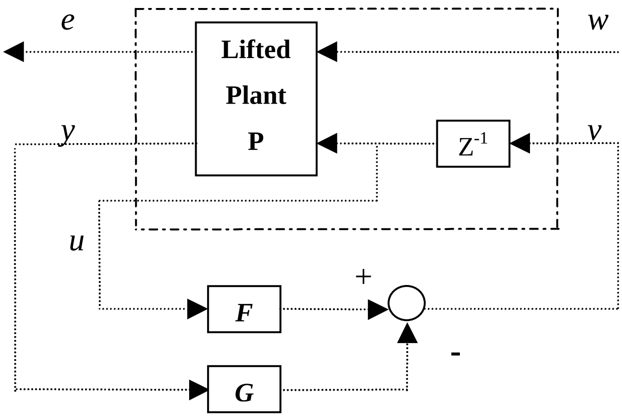

Figure 4 : Closed-loop Configuration with a Full Information Controller

Let us define the state and measured output vectors as follows:

$$\tilde{x}(k) = \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \in R^{n+n_u},$$
$$\tilde{y}(k) = \begin{bmatrix} y(k) \\ u(k) \end{bmatrix} \in R^{mn_y+n_u} \quad (6)$$

where $x \in R^n$ is the state of the realization for P in Eq. (4) and v denotes the input to the delay block. We have then from Eqs. (4) and (6) that the following equation exists:

$$\begin{bmatrix} \tilde{x}(k+1) \\ e(k) \\ \tilde{y}(k) \end{bmatrix} = \begin{bmatrix} \tilde{A} & \tilde{B}_1 & \tilde{B}_2 \\ \tilde{C}_1 & \tilde{D}_{11} & 0 \\ \tilde{C}_2 & \tilde{D}_{21} & 0 \end{bmatrix} \begin{bmatrix} \tilde{x}(k) \\ w(k) \\ v(k) \end{bmatrix} \quad (7)$$

where

$$\begin{bmatrix} \tilde{A} & \tilde{B}_1 & \tilde{B}_2 \\ \tilde{C}_1 & \tilde{D}_{11} & 0 \\ \tilde{C}_2 & \tilde{D}_{21} & 0 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} A & B_2 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} B_1 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ I_{n_u} \end{bmatrix} \\ \begin{bmatrix} C_1 & D_{12} \end{bmatrix} & D_{11} & 0 \\ \begin{bmatrix} C_2 & D_{22} \\ 0 & I_{n_u} \end{bmatrix} & \begin{bmatrix} D_{21} \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{bmatrix} \quad (8)$$

From Figure 4, v is given by

$$v(k) = \begin{bmatrix} -G & F \end{bmatrix} \tilde{y}(k) \quad (9)$$

Replacing $\tilde{y}(k)$ by $\tilde{y}(k)$ from Eq. (7), we get the following equation:

$$v(k) = \begin{bmatrix} -G & F \end{bmatrix} \times \begin{bmatrix} \begin{bmatrix} C_2 & D_{22} \\ 0 & I_{n_u} \end{bmatrix} & \begin{bmatrix} D_{21} \\ 0 \end{bmatrix} \end{bmatrix} \begin{bmatrix} \tilde{x}(k) \\ w(k) \end{bmatrix} \quad (10)$$

Now, we can let the linear static FIC law be defined as

$$v(k) = \begin{bmatrix} K_{\tilde{x}} & K_w \end{bmatrix} \begin{bmatrix} \tilde{x}(k) \\ w(k) \end{bmatrix} \quad (11)$$

where $K_w \in R^{n_u \times \ln_w}$ and $K_{\tilde{x}} \in R^{n_u \times (n+n_u)}$. Then, we can see that the control law in Eqs. (10) and (11) are equal if and only if

$$\begin{bmatrix} -G & F \end{bmatrix} \begin{bmatrix} \begin{bmatrix} C_2 & D_{22} \\ 0 & I_{n_u} \end{bmatrix} & \begin{bmatrix} D_{21} \\ 0 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} K_{\tilde{x}} & K_w \end{bmatrix} \quad (12)$$

It follows from the above equation that if we can get the FIC gains $K_{\tilde{x}}$ and K_w, we can easily get the parameters of F and G from the above equation without difficulty. However, in order to ascertain that Eq. (12) holds, we need the following condition.

Let us first partition $K_{\tilde{x}}$ as $K_{\tilde{x}} = [K_x \quad K_u]$, where $K_x \in R^{n_u \times n}$ and $K_u \in R^{n_u \times n_u}$. We now derive the MROC gains F and G from the given FIC gains $K_{\tilde{x}}$ and K_w. From standard results in linear algebra, we can solve the parameters of F and G satisfying Eq. (12) if and only if

$$rank\begin{bmatrix} C_2 & D_{22} & D_{21} \\ 0 & I_{n_u} & 0 \\ K_x & K_u & K_w \end{bmatrix} = rank\begin{bmatrix} C_2 & D_{22} & D_{21} \\ 0 & I_{n_u} & 0 \end{bmatrix} \quad (13)$$

From Eq. (13), we can say that if and only if the following condition holds

$$\begin{bmatrix} C_2 & D_{22} & D_{21} \\ 0 & I_{n_u} & 0 \end{bmatrix} \text{ is of full column rank} \quad (14)$$

we can find F and G solving Eq. (12) for any pair $K_{\tilde{x}}$ and K_w.

Readers can refer to [6] for a detailed discussion of the condtion that the FIC model can be made equivalent to the MROC model. We assume that all the condtions satisfying these criteria when adopting the model of the plant in the following sections are true.

4. The Mixed H_2/H_∞ Control Problem

Let us begin by considering a finite-dimensional linear time-invariant (FDLTI) discrete-time system P as shown in Figure 5.

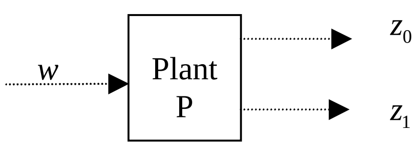

Figure 5: Definition of the Mixed H_2/H_∞ Performance Measure

Suppose that P is internally stable with the discrete-time state-space model:

$$L := \begin{cases} x_{k+1} = Ax_k + Bw_k \\ z_{0k} = H_0 x_k + J_0 w_k \\ z_{1k} = H_1 x_k + J_1 w_k \end{cases} \quad (15)$$

The following lemma is adopted directly from Theorem 2.1 of [7].

Lemma 1:
Consider the stable system P defined in Eq. (15) and let T_{zw} denote the transfer matrix from w to z. Suppose that $\|T_{z_1 w}\|_\infty < 1$, then the mixed H_2/H_∞ performance measure $J(T_{zw})$ for the LTI system should be:

$$\begin{aligned} &J(T_{zw}) \\ &= \inf\left\{tr(H_0 Y H_0' + J_0 J_0') : Y = Y' > 0\right\} \quad (16) \\ &\text{such that } M(Y) > 0 \text{ and } R(Y) < 0 \end{aligned}$$

with

$$M(Y) := \gamma^2 I - J_1 J_1' - H_1 Y H_1' > 0, \text{ and}$$

$$\begin{aligned} R(Y) &:= AYA' - Y + (AYH_1' + BJ_1') \\ &\times M^{-1}(H_1 YA' + J_1 B') + BB' < 0 \end{aligned} \quad (17)$$

Let us then consider the FDLTI discrete-time feedback system depicted in Figure 6 with the state-space expression as shown in Eq. (18). In the sequel, we will consider the mixed H_2/H_∞ synthesis problem for the full-information plant.

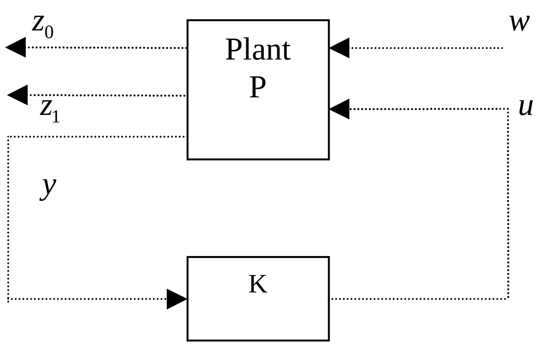

Figure 6: The H_2/H_∞ Synthesis Framework

$$\begin{aligned} x_{k+1} &= Ax_k + B_1 w_k + B_2 u_k \\ z_{0k} &= C_0 x_k + D_{01} w_k \\ z_{1k} &= C_1 x_k + D_{11} w_k \quad (18) \\ y &= \begin{bmatrix} x \\ w \end{bmatrix} \end{aligned}$$

Consider Figure 6, let J denotes the "mixed H_2 / H_∞ performance measure" and let

$\Psi :=$ inf { J: K internally stabilizing and $\|T_{z_1 w}\|_\infty < \gamma$ }

denote the optimal mixed H_2 / H_∞ performance measure. Then, we can consider the following problem:

" Compute Ψ and given $\alpha > \Psi$, find an internally stabilizing controller K such that $\|T_{z_1 w}\|_\infty < \gamma$, and the mixed H_2 / H_∞ performance measure satisfies $J < \alpha$".

Let

$$\|T_{zw}\| = \begin{bmatrix} T_{z_0 w} \\ T_{z_1 w} \end{bmatrix} \qquad (19)$$

denotes the closed-loop transfer matrix, where $T_{z_0 w}$ and $T_{z_1 w}$ are the closed-loop transfer matrices from w to z_0 and w to z_1, respectively.

Consider the feedback system shown in Figure 6. Given a plant P and an internally stabilizing controller K, the mixed H_2 / H_∞ cost $J(T_{zw})$ of the closed-loop system is a function of the transfer matrix T_{zw} only. We then define the sub-optimal mixed H_2 / H_∞ controller synthesis problem considered in this paper to be the following:

To calculate the optimal mixed H_2 / H_∞ performance measure

$$\Psi := \inf \{J(T_{zw})\} \qquad (20)$$

while the controller K is admissible and given any $\alpha > \Psi$, find a controller K such that $J(T_{zw}) < \alpha$.

It is easy to see that as long as such a realization is internally stable, $J(T_{zw})$ is only a function of the transfer matrix T_{zw}, and does not depend on the choice of realization. The mixed H_2 / H_∞ performance measure $J(T_{zw})$ is also a function of the parameter γ. Without loss of generality, we set $\gamma = 1$ for the remainder of this paper.

It will be shown later that in the full-information case, the mixed H_2 / H_∞ optimal performance Ψ and a static gain controller that satisfies $J < \alpha$ (for any $\alpha > \Psi$) can be obtained by solving a finite-dimensional convex programming problem over a bounded set of real matrices. Here, a solution to this convex programming problem is a global solution to the mixed H_2 / H_∞ synthesis problem.

5. A Convex Approach to Full Information Feedback Problem

Now, we reduce the full information feedback problem to a memoryless feedback problem. The following theorem is directly adopted from Theorem 4.1 of [7]. This theorem applies to the case of a full information (FIC) model with the state-space model given by Eq. (18).

Theorem 1:
Consider the full-information plant defined by Eq. (18), then we get

$$\Lambda_\infty(K) \neq \varnothing \iff \Lambda_{\infty,m}(K) \neq \varnothing \qquad (21)$$

where $\Lambda_\infty(K)$ means that there exists an admissible FIC controller $K = \begin{bmatrix} K_{\tilde{x}} & K_w \end{bmatrix}$, while $\Lambda_{\infty,m}(K)$ means that there exists an admissible memoryless controller K. In this case

$$\Psi(K) = \Psi_m(K) \qquad (22)$$

Furthermore, given any $\alpha > \Psi(K)$, there exists a static FIC $K \in \Lambda_{\infty,m}(K)$ such that $J(G,K) < \alpha$.

With reference to the full-information plant defined in Eq. (18), let $n_x = \dim(x)$,

$n_u = \dim(u)$ and $n_w = \dim(w)$. Furthermore, let $\sum$ denote the set of all real $n_x \times n_x$ symmetric matrices, and define

$$\Omega := \left\{(W, Y, k_w) \in R^{n_u \times n_x} \times \sum \times R^{n_u \times n_w}, Y > 0\right\} \quad (23)$$

Note that Ω is a strictly convex open subset of $R^{n_u \times n_x} \times \sum \times R^{n_u \times n_w}$. Given $(W, Y, K_w) \in \Omega$, let us then define

$$f(W, Y, K_w) := tr(C_0 Y C_0{}' + D_{01} D_{01}{}') \quad (24)$$

Given any $(W, Y, K_w) \in \Omega$, define the following:

$$L(W, Y, K_w) := \begin{bmatrix} AY + B_2 W \\ C_1 Y \end{bmatrix} Y^{-1} \begin{bmatrix} AY + B_2 W \\ C_1 Y \end{bmatrix}' + \begin{bmatrix} B_1 + B_2 K_w \\ D_{11} \end{bmatrix} \begin{bmatrix} B_1 + B_2 K_w \\ D_{11} \end{bmatrix}' - \begin{bmatrix} Y & 0 \\ 0 & I \end{bmatrix} \quad (25)$$

Consider the set of real matrices:

$$\Phi(K) := \left\{(W, Y, K_w) \in \Omega : L(W, Y, K_w) < 0\right\} \quad (26)$$

and the constrained optimization problem

$$\Psi(K) := \inf\left\{ f(W, Y, K_w) : (W, Y, K_w) \in \Phi(K)\right\} \quad (27)$$

Now, we present the main result of the mixed H_2 / H_∞ problem under the full-information model.

Theorem 2:
Consider the system defined in Eq. (18) with transfer matrix G_{fi}. Let $\Lambda_{\infty,m}(K)$ be the set of static full information controllers. Then

$$\Lambda_{\infty,m}(K) \neq \varnothing \Leftrightarrow \Phi(K) \neq \varnothing \quad (28)$$

where $\Phi(K)$ is given by Eq. (26). In this case,

$$v_m(K) = \Psi(K) \quad (29)$$

where $v_m(K)$ is a memoryless controller and $\Psi(K)$ is a controller defined in Eq. (27) respectively. Furthermore, given any $\alpha > V_m(K)$, there exists a triple $(W, Y, K_w) \in \Phi(K)$ such that the static FIC

$$K := \begin{bmatrix} K_{\tilde{x}} & K_w \end{bmatrix} = [WY^{-1} \ K_w] \quad (30)$$

satisfies

$$K \in \Lambda_{\infty,m}(K) \text{ and } J(G_{fi}, K) < \alpha \quad (31)$$

This result is a direct and straightforward generalization of Theorem 4.2 of [8].

Let us rewrite Eq. (25) as

$$\begin{aligned} L(W,Y,K_w) &= \begin{bmatrix} A & B_2 \\ C_1 & 0 \end{bmatrix} \begin{bmatrix} Y \\ W \end{bmatrix} Y^{-1} \begin{bmatrix} Y' & W' \end{bmatrix} \begin{bmatrix} A' & C_1' \\ B_2' & 0 \end{bmatrix} \\ &+ \begin{bmatrix} B_1 & B_2 \\ D_{11} & 0 \end{bmatrix} \begin{bmatrix} I \\ K_w \end{bmatrix} \begin{bmatrix} I & K_w' \end{bmatrix} \begin{bmatrix} B_1' & D_{11}' \\ B_2' & 0 \end{bmatrix} \\ &- \begin{bmatrix} Y & 0 \\ 0 & I \end{bmatrix} \\ &= \tilde{F}\tilde{W}Y^{-1}\tilde{W}'\tilde{F}' + \tilde{G}\tilde{K}\tilde{K}'\tilde{G}' - \begin{bmatrix} Y & 0 \\ 0 & I \end{bmatrix} \end{aligned} \quad (32)$$

where

$$\tilde{F} := \begin{bmatrix} A & B_2 \\ C_1 & 0 \end{bmatrix}, \quad \tilde{G} := \begin{bmatrix} B_1 & B_2 \\ D_{11} & 0 \end{bmatrix}$$

$$\tilde{W} := \begin{bmatrix} Y' & W' \end{bmatrix}' \quad \tilde{K} := \begin{bmatrix} I & K_w' \end{bmatrix}'$$

From the Proposition E.7.f in [9], the mappings $(\tilde{W}, Y) \to \tilde{F}\tilde{W}Y^{-1}\tilde{W}'\tilde{F}'$, and $\tilde{K} \to \tilde{G}\tilde{K}\tilde{K}'\tilde{G}'$ are convex in their domains. Since the maps $(W, Y) \to \begin{bmatrix} Y' & W' \end{bmatrix}$ and $K_w \to \begin{bmatrix} I & K_w' \end{bmatrix}$ are affine linear, the convexity of L follows. Finally, the convexity of $\Phi(K)$ follows from the convexity of L.

As we have seen that the optimization problem defined by Eq. (27) is a convex problem, we solve this kind of problem using the popular LMI method.

Finally, let us summarize the procedures to solve the mixed H_2 / H_∞ control problem using a FIC model.

- Solve the constrained convex optimization problem of

$$\Phi(K) := \inf\left\{ tr(C_0 Y C_0^{'} + D_{01} D_{01}^{'}) : L(W, Y, K_w) < 0 \right\}$$

- Get the required FIC from Eq. (30). Then, according to Eq. (12), based on which a MROC model is made equivalent to a FIC model, we get the final parameters F and G of the MROC system.

6. Design Example

In order to fully examine the effectiveness of our design, we choose the original plant P^c to be as follows:

$$\dot{x}^c(t) = \begin{bmatrix} 3 & 6 \\ 2 & -5 \end{bmatrix} x^c(t) + \begin{bmatrix} 0 \\ 0.6 \end{bmatrix} w^c(t) + \begin{bmatrix} 1 \\ 1.2 \end{bmatrix} u^c(t)$$

$$e^c(t) = \begin{bmatrix} z_0^c \\ z_1^c \end{bmatrix} = \begin{bmatrix} 0.2 & 0.1 \\ 1 & -0.2 \end{bmatrix} x^c(t)$$

$$y^c(t) = \begin{bmatrix} 5 & 1 \end{bmatrix} x^c(t)$$

We choose the lifting rate l=1 and the frame period T_0=0.2s with m=4. The state-space representation of the FIC model after it is lifted is the following:

$$\left[\begin{array}{ccc|c|c}
2.1059 & 1.1764 & 0.4300 & 0.0690 & 0 \\
0.3921 & 0.5374 & 0.2046 & 0.0831 & 0 \\
0 & 0 & 0 & 0 & 1 \\
\hline
0.2 & 0.1 & 0 & 0 & 0 \\
0.2451 & 0.1370 & 0.0182 & 0.0035 & 0 \\
0.3017 & 0.1793 & 0.0412 & 0.0083 & 0 \\
0.3724 & 0.2291 & 0.0702 & 0.0144 & 0 \\
1 & -0.2 & 0 & 0 & 0 \\
1.1579 & 0.1303 & 0.0518 & -0.001 & 0 \\
1.3757 & 0.4375 & 0.1320 & 0.0076 & 0 \\
1.6615 & 0.7438 & 0.2430 & 0.0253 & 0 \\
\hline
5 & 1 & 0 & 0 & 0 \\
5.9822 & 2.2358 & 0.3706 & 0.0486 & 0 \\
7.2573 & 3.4984 & 0.8719 & 0.1344 & 0 \\
8.8799 & 4.8669 & 1.5240 & 0.2595 & 0 \\
0 & 0 & 1 & 0 & 0
\end{array}\right]$$

Keeping the H_∞ performance at one in one channel, while achieving the "upper bound" of the H_2 performance on the other channel at 2.6608, the controller parameters are given below:

$$F = 9.2032$$
$$G = \begin{bmatrix} -0.2187 & 3.2290 & 2.1987 & 5.3286 \end{bmatrix}$$

7. Conclusions

In this paper, we solve the problem of designing a multirate output controller under mixed H_2/H_∞ performance. We use fast-discretization and discrete-lifting technologies to first convert the periodic time-varying multirate system to a norm-equivalent discrete system. Then, we transfer the discrete system to a FIC system. Using the convex approach to full information feedback problem, we solve the mixed H_2/H_∞ problem by keeping the H_∞ performance at one in one output channel, while achieving a minimum "upper bound" of H_2 performance on the other.

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