Mathematical Modeling of The Pest Control

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Abstract: - In this paper the mathematical modeling problem of the pest control was formulated. The computer programs were developed in the form of an easy-to-handle software based on six mathematical models. The methods of the parameter estimation were considered. The potential benefits of the mathematical modeling were illustrated by applying the software to the pest control in soybeans. The computer simulations allowed the comparison of results between different methods of pest control in soybeans (chemical, biological, and natural) and was chosen the most appropriate one for the considered ecosystem.

Key-Words: - Mathematical modeling, Pest control, Prey-predator model, Computer simulation, Soybean caterpillar. *IMACS/IEEE CSCC'99 Proceedings*, Pages:3061-3067

1 Introduction

One of the important elements of the agricultural technology is pest management and control. Most methods of the pest control, in agriculture, are based on chemical insecticides. These insecticides are toxic chemicals with a negative effect on beneficial species in the ecosystem. As time goes by, the insecticides become ineffective against the insects that we want to control, because the insects become genetically resistant to them. On the other hand, biological control makes use of predators and diseases to reduce the population of the pest species. Biological pest control is a method in which pest population is restrained by its natural enemies. These feed on the pest insect and thus reduce its biotic potential. The biological control of undesired insects is a secure and efficient way in reducing pest damages in crop quality and quantity without the inconveniences consequent of chemical insecticides. Integrated pest management combines the features of biological, chemical and other methods of insect control and thus minimizes environmental degradation. In order for integrated control to be successful, the dynamics of pest population have to be understood.

Although the mathematical models of the ecological systems have been analyzed mathematically since the classical papers of Lotka, Volterra and others which appeared in the period between 1920 and 1930 [1], [2] the mathematical modeling didn't become an indispensable element of the agricultural technology. In order to organize the mathematical modeling of an analyzed ecosystem, the following steps must be taken: mathematical formulation of the problem and development or adaptation of mathematical models; elaboration of algorithms and computer programs; collection of statistical data; identification of mathematical models; computer simulations; and technical recommendations. This can only be achieved by a close cooperation between experimentalists, the model and program builders among others.

It has to be admitted that nowaday the main difficulty in performing a realistic and convincing ecosystem simulation is not the implementation of the model on a computer but rather the incomplete information on basic model parameters [3]. Often, it is difficult even to assess the correct order of magnitude of important parameters.

The main objective of this investigation was to develop a simple tool which allows to formulate the pest control problems mathematically, solve them, and analyze the solution.

2 Mathematical models

There are several mathematical models that describe the interaction process between insects and its natural enemies of the certain ecosystem. This happens because the behavior of populations of each ecosystem depends on many factors that don't enter explicitly in the mathematical model. The detailed models admit a larger number of factors and contain a large number of parameters. Apart from sophisticated numerical techniques, the parameter estimation problem obviously demands a sufficiently large body of data, which in most cases is not available. Too many variables have to be measured simultaneously during long time. The short period that a farmer has to take the decision on the pest control, limits the use of the detailed models. Therefore, a group of simple prey - predator models was considered in this paper. In general form a model of this group can be written as the system of two differential nonlinear equations:

$$\frac{dx}{dt} = x\,F(x,y) \qquad \frac{dy}{dt} = y\,G(x,y) \tag{1}$$

In the first version of the software the following models were included:

1. The Lotka - Volterra classic model [1], [2], in this case the functions F and G have the form:

$$F(x,y) = a - \alpha\, y; \qquad G(x,y) = -b + \beta\, x \tag{2}$$

2. The Lotka - Volterra model with competition [4]:

$$F(x,y) = a - \gamma_1\, x - \alpha\, y; \qquad G(x,y) = -b + \beta\, x - \gamma_2\, y \tag{3}$$

3. The Leslie - Gower model [4]:

$$F(x,y) = a - \gamma\, x - \alpha\, y; \qquad G(x,y) = \beta - \frac{b\,y}{x} \tag{4}$$

4. The Pianka model [5]:

$$F(x,y) = a - \gamma\, x - \alpha\, y; \qquad G(x,y) = \beta\, x - \frac{b\,y}{x} \tag{5}$$

5. The Holling - Tanner model [4]:

$$F(x,y) = a - \gamma\, x - \frac{\alpha\, y}{D + x}; \qquad G(x,y) = \beta - \frac{b\,y}{x} \tag{6}$$

6. The “realistic” predator - prey model according Murray [6]:

$$F(x,y) = a - \gamma\, x - \frac{\alpha\, x\, y}{D^2 + x^2}; \qquad G(x,y) = \beta - \frac{b\,y}{x} \tag{7}$$

Some remarks are in order. In the books [4] - [6] the models (4) - (7) were called “more realistic” that the Lotka - Volterra classic model. There are not more or less realistic models. There is a more or less adequate model to an object or real process. Therefore, in the mathematical modeling of ecosystems several models can be tested in order to choose the most appropriate.

3 Parameter Estimation

It is the purpose of parameter estimation techniques to derive reasonable estimates of the parameters from observations. First, performance criteria have to be formulated pertaining to the goodness of fit. The classical criterion is the “sum of squares”, i.e. the sum of squared deviations between model predictions and data [3. For the models considered, the sum of squares is given by:

$$I(p) = \sum_{j=1}^{M} \sum_{i=1}^{N} w_{ji} [\hat{y}_i(t_j) - y(t_j, p)]^2 \tag{8}$$

where y is model predictions vector; $\hat{y}$ is data vector, p is parameter vector, M is number of measurements, N is dimension of the model prediction vector. *I(p)* is a function of the unknown vector *p*. Therefore it is quite straightforward to define as estimates those values of components of vector *p*, which minimize *I(p)*. The parameter estimation problem is thus reduced to the numerical problem to minimize a nonlinear function of several variables. The least squares criterion (8) is only one of many possible criteria. Another function that is used in this paper is given by:

$$I(p) = \frac{1}{M} \sum_{j=1}^{M} \sum_{i=1}^{N} \left| \frac{\hat{y}_i(t_j) - y(t_j, p)}{\hat{y}_i(t_j)} \right| \tag{9}$$

4 General Information About The Software

Software MMPPS (Mathematical Modeling of Prey - Predator Systems) a tool which allows to formulate the pest control problems, solve them, and analyze the solution.

The main menu of the MMPPS software is horizontal. It includes the following options: *File, Model, Data, Results, Graphs, Exit.*

The main menu option *File* allows to open a vertical menu, with the options:

New, Open, Save, Save as, Delete.

The *New* option allows to input a new data file.

The *Open* option reads a saved file from disk and places it in MMPPS.

The *Save* option saves the contents of the data file to disk using the existing file name.

The *Save as* option allows to save the contents of the data file to disk under a new file name.

The *Delete* option allows to delete the contents of the existing data file.

The main menu option *Model* allows to open other vertical menu with the names of mathematical models:

Classic Lotka - Volterra Model, Lotka - Volterra Model with Competition, Leslie - Gower Model, Pianka Model, Holling - Tanner Model, Realistic Model that correspond to the models (2) - (7), respectively. There is a possibility of choosing a method of the parameter identification for each model.

The main menu option *Data* allows to exhibit the contents of an opened data file.

The main menu option *Results* allows to open a vertical menu, with the options:

Video, Print.

The *Video* option allows to exhibit the results of the resolved problem to the screen.

The *Print* option allows to send the results of the resolved problem to the printer.

The main menu option *Graphs* allows to open a vertical menu with the options of the result graphs:

Variation of the Preys Population Density,

Variation of the Predator Population Density,

Variation of the Preys - Predator Population Density,

Ecological cycle of the Preys - Predator System.

The main menu option *Exit* is used to quit MMPPS.

5 Numeric simulations

We hope to illustrate the potential benefits of the mathematical modeling by applying MMPPS to the pest control in soybeans. As an example, consider the data from [7] concerning the incidence the soybean caterpillar (*Anticarsia gematalis*) and its predators (*Nabis spp, Geocoris, Aracnideo, etc*), see Table 1.

Table 1. Incidence the soybean caterpillar (*Anticarsia gematalis*) and its predators

Time t [days]	0	6	22	29	34
Density of soybean caterpillar $\hat{x}$ [number/m^2]	35	16	114	81	32
Density of predator $\hat{y}$ [number/m^2]	15	9	8	22	24

The parameter estimation was accomplished for models (2) - (7). The classic Lotka - Volterra model was the model, whose parameters supplied the smaller value to the functional (9). The results of parameter estimation are shown in the Table 2.

Table 2: Parameter estimates for the classic
Lotka – Volterra model

Parameter	a	α	b	β	x_0	y_0
Estimate	0,216	0,02088	0,1728	0,00288	32	16

Three simulations of the pest control were considered. The first simulation was made using the Lotka - Volterra classic model with parameters of Table 2. This simulation corresponds to the natural development of the modeled system. The uninterrupted curves in Fig.1 and Fig.2 show the resultant fit. The comparison of the natural development curve with the data points in the interval of 34 days exhibits the good quality of the parameter estimation. It is evident from the graphs that the ecosystem in this case has periodic oscillations occurred in a 34 day period and the maximum numbers of prey and predators densities reached 119 and 19, respectively.

In the second simulation the application of the chemical pest control was modeled on the twentieth fourth day when the maximum number of the caterpillar appeared. In this case the initial value problem for the Lotka - Volterra classic model was solved using the coefficients from Table 2 and initial conditions:

$$x(24) = 0,5 \qquad y(24) = 0,05 \qquad (10)$$

The initial conditions (10) were obtained supposing that after the chemical control survived less than 0,5 % of the prey and predator populations. The interrupted curves in Fig.1 and Fig.2 show that the applied chemical control enhanced the oscillation period to 68 days and the maximum number of caterpillars grew to 614 while the number of predators reached 87.

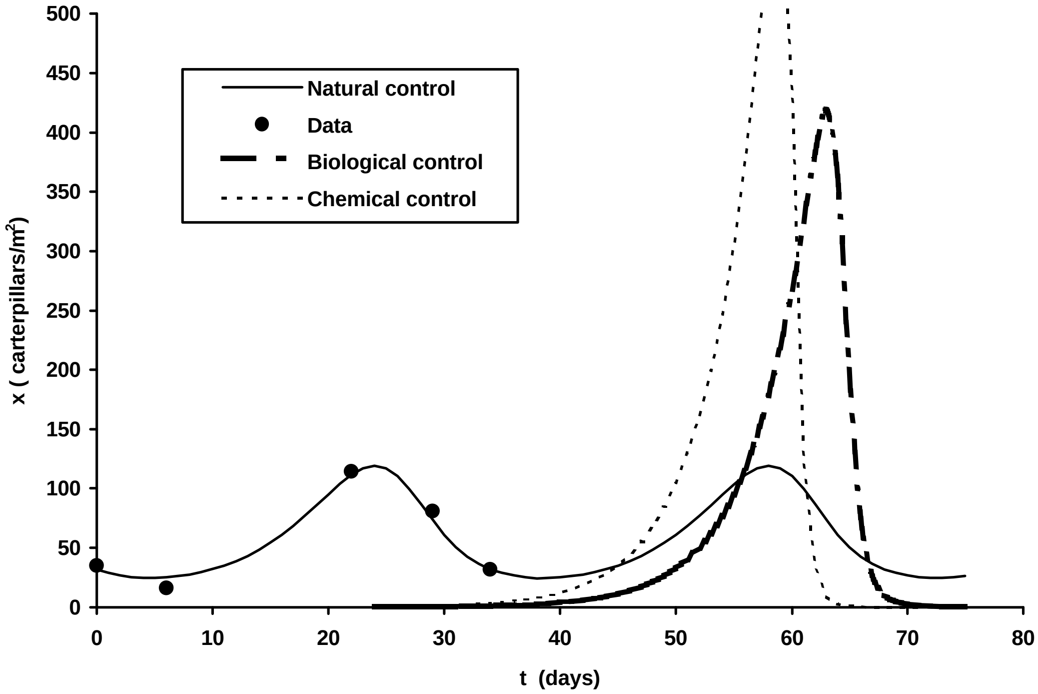

Fig. 1. Density of *Anticarsia gematalis* for several types of the pest control

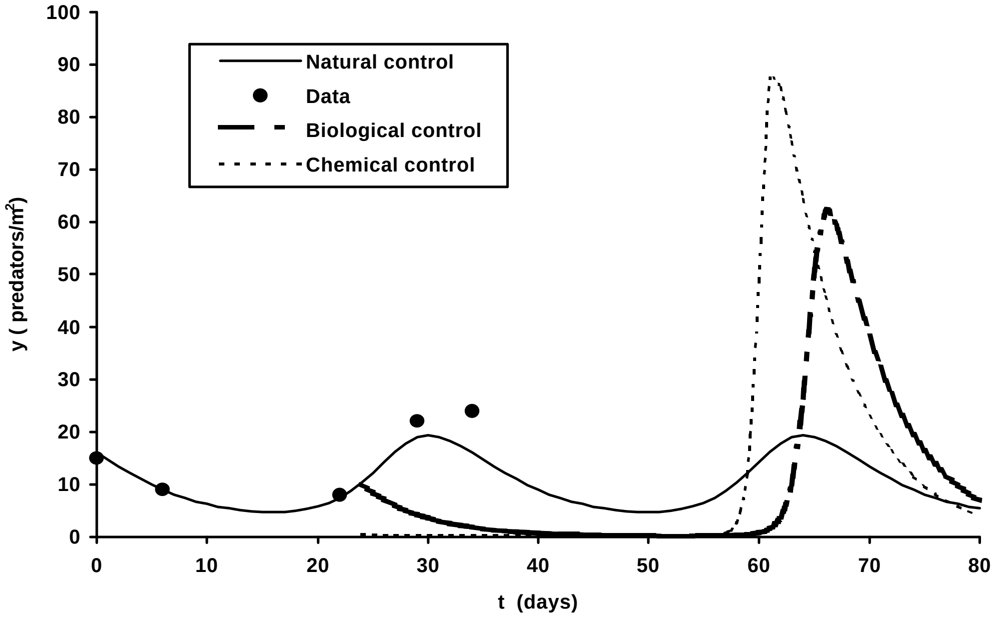

Fig. 2. Predators density for several types of the pest control

The explanation of this phenomenon becomes clear if we use the phase diagram of the ecosystem (ecological cycle) showed in Fig. 3. At the moment of the chemical control application, the ecosystem was in the point (119, 10) of the uninterrupted curve that characterizes the ecological cycle of the ecosystem natural development. Due to the application of the chemical control, the system passed into another ecological cycle curve (interrupted curve) that count the point (0.5, 0.05). The simulation shows that the critical point of 20 caterpillar (according to recommendations of Brazilian Research Agricultural Enterprise (EMBRAPA) the critical density of caterpillar is 20 big caterpillars or 40 small caterpillars for 1 m^2 of the soybean) the system surpasses at the 43.

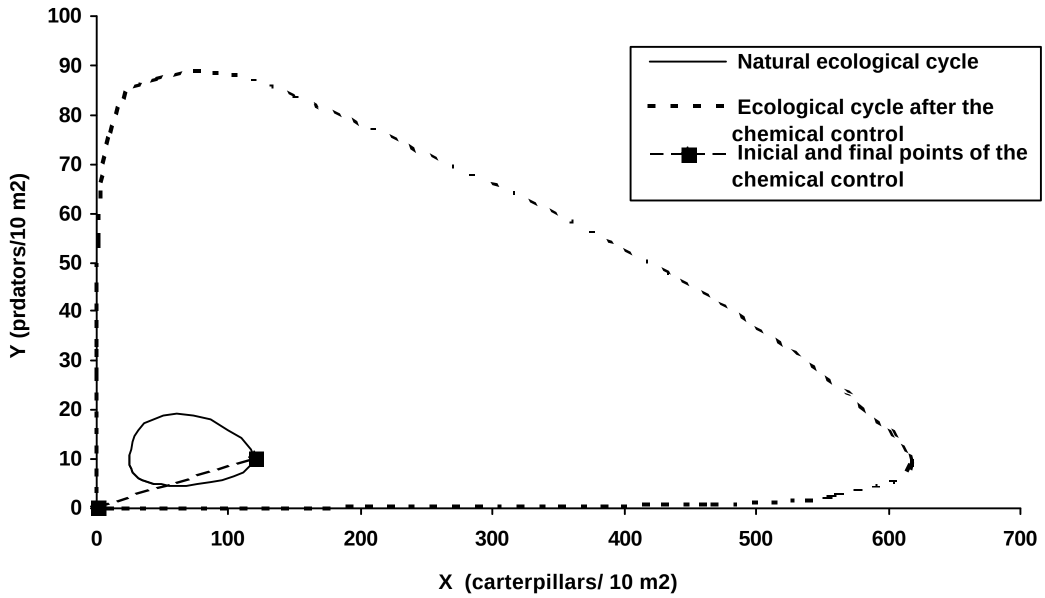

Fig. 3. Ecological cycle graphs of the ecosystem before and after the chemical control

In the third simulation the application of the biological pest control was modeled in same day (24) and the initial value problem for the Lotka - Volterra classic model was solved using the coefficients from Table 2 and initial conditions:

$$x(24) = 0{,}5 \qquad y(24) = 10 \qquad (11)$$

The initial conditions (11) were obtained supposing that after the biological control survived less than 0,5 % of the prey population and all predators.

The biological control curve (see Fig.1 and Fig.2) shows that in this case the ecosystem has periodic oscillations occurred at a 54 day period and the maximum numbers of prey and predators densities reached 418 and 62, respectively. The simulation showed that the ecosystem surpassed the critical point of 20 caterpillars in the 48th day.

The computer simulations showed that the better result of the biological control application is on the initial day. The results of the simulation of the biological control application on this day show in Fig.4.

6 Conclusion

The mathematical models and parameter estimation techniques were presented for mathematical modeling of the pest control. The computer programs were developed in the form of an easy-to-handle software based on mathematical models. This software permits the elaboration and control of statistical data; the parameter estimation of the ecosystems models; an overview of the systems development, becoming possible a projection of the population growth of the species that are being considered. The numerical simulations by this software allowed the comparison of results between different methods of pest control in soybeans (chemical, biological, and natural) and was chosen the most appropriate one for the considered ecosystem.

There is the possibility to include more models and parameter estimation methods in this software.

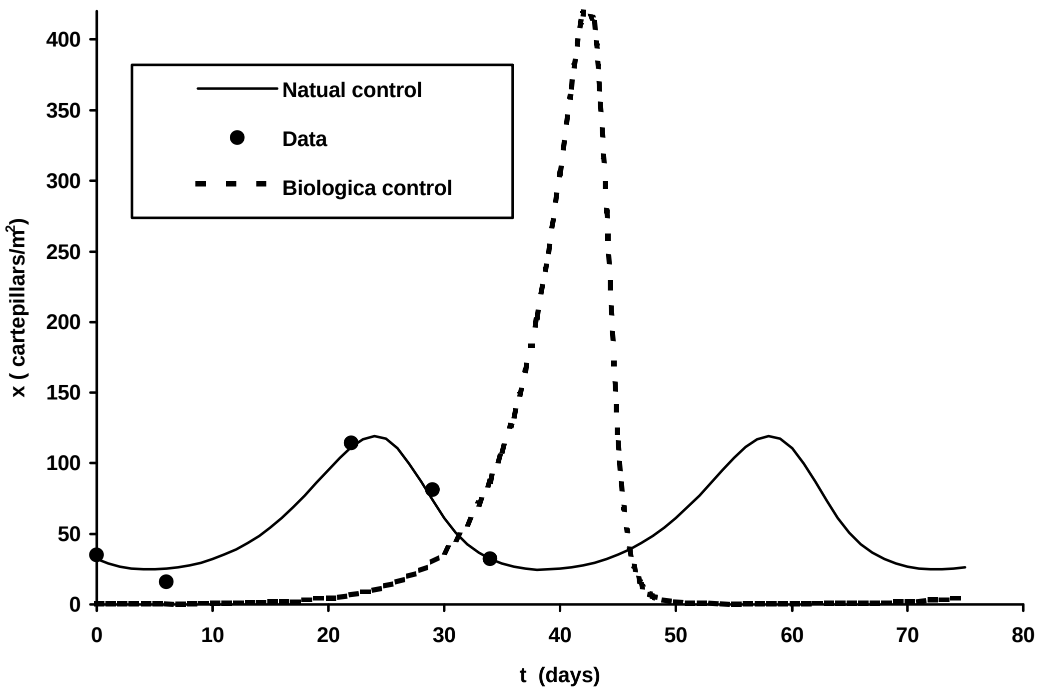

Fig. 4. Density of *A. gematalis* for the biological pest control applied on the initial day

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