Modelling of a Steam Generator Using Distributed Parameter Equations

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Abstract: Because of the complexity of the thermodynamic phenomena which govern the dynamic behavior of thermal power plants, the modelling problem is still with a great interest. This paper deals with the modelling of a steam generator, which complexity results in the existence of several problems, such as shrink-and-swell phenomena which introduce a non minimum phase.

In this work, the only required hypotheses are the thermodynamical equilibrium and the incompressibility of the supply fluid. In general, in the literature relative to this field [1], the modelling hypotheses are more constraining and the dynamics are described by means of ordinary differential equations (ODEs). Here, more realistic partial differential equations (PDEs) are considered.

After a presentation of the notations, the first part of the paper establishes the thermodynamical equation with the theoretical resolution of the energy equation applyied to the heat propagation phenomenon [2]. This first result (and related simulations) makes appear a propagation delay, that confirms the classical understanding of such plants.

Then, the second part provides an original contribution by taking into account the dynamic behaviors (this means, time varying inputs) with a homogeneous diphasic mixture. This leads to a models with partial differential equations (PDE), with a mathematical input/output form that is adapted to control issues:

$$\mathrm{d}x/\mathrm{d}t = f(x, u, k, t, z)$$
$$y = g(x, u, k, t, z)$$

Where f is an infinite dimensional function (depending on the spatial position z in the boiler), x the state (pressure, liquid and steam velocities), u the input (control: delivery pump flow, electrical heating, disturbances: charge variations) and k are environnemental variables (external temperature,…).

An other interesting point is that this a nonlinear distributed parameter model allows estimating the «**steam quality**» (steam-liquid mass ratio) in relation with the position z in the boiler. This last point constitutes a practical progress since such a variable cannot be directly measured on the plant.

Lastly, a third part deals with the numerical implementation of the related PDEs.

Key-Words: steam generator, partial differential equations, delay systems, mass/momentum/energy equations, diphasic mixture, steam quality, numerical implementation,.
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1 Introduction

The problem of modelling (and therefore of control) of thermodynamic systems still stays open, even if this type of process is present in a many industries with risk (nuclear, chemical, etc...). There is a real need of some control models that can be at the same time operable and precise [3].

The present work concerns the modelling of a steam generator, implemented at the L.A.I.L. *(Laboratoire d'Automatique et d'Informatique de Lille)* [1], which replicates, in a reduced scale, the working of a thermal power station. This technological process is mainly made up of a delivery pump, a water-steam boiler and a servovalve which represents the charge variations of an electric alternator. It presents several delay effects due to the heat transfer in the boiler and to the transit of the steam in the pipe.

To the best authors knowledge, the literature devoted to such processes mainly involves "localized parameters" systems [3], [4], [5], [6] and [7]. However, the information such approach can

provide is limited to the spatial average of the evolution of the variables in the steam generator. For this reason, it is interesting to look for an alternative approach leading to informations about the variables every where in the boiler: this means, partial differential equations. In this aim, a method was suggested [8], relying on the PDEs that govern the thermodynamic exchanges between two separated phases (liquid / steam). Unfortunately, such laws of exchange between phases are not well known. To overcome such difficulty, a method [9] available for the diphasic flows was applied, leading to statistical equations [8]. In this case, the production and the flow of the steam bubbles is considered as a random phenomenon, such model is not adapted to control.
The first part of this work concerns a theoretical study of the resolution of the energy equation applied to the heat propagation phenomenon. It reveals the delay effects linked to the heat propagation (Volterra equations [2]), as the simulations will confirm.
In the second part, a practical application to the steam generator leads to the homogeneous diphasic mixture model. This approach appears to be more effective and represents an original contribution of the paper.
Finally, the numerical implementation of this last model is discussed: the PDEs of the hydrodynamics (mass, energy and moment conservation) are transformed, by time and space discretization, into finite-difference equations, and numerically integrated without being linearized. Such approach allows determining the response of the system with high amplitude boundary conditions. Of course, it is also possible, starting from this nonlinear model, to derive a linear one valid around a working point.

2 Theoretical delay in the heat transfer equations

2.1 Heat transfer equation

In the usual applications, the energy equation written in integral form and with vector notation, for some volumic domain $V_m(t)$ (Fig.1), is:

$$\int_{V_m(t)} rc_p \frac{dT}{dt} dV_m = \int_{V_m(t)} rc_p (\frac{\partial T}{\partial t} + v\nabla T) dV_m$$
$$= \int_{V_m(t)} PdV_m + \int_{S_m(t)} -(q^{cd} + q^r) dS_m \quad (1)$$

with the following notations,
r , fluid density,
C_p , fluid calorific capacity,
u , local fluid velocity,
$S_m(t)$, boundary surface of the studied domain,
P , power of heat source,
q^{cd} , conduction heat flux,
q^r , radiance heat flux.

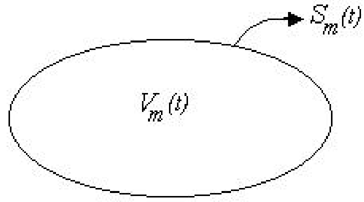


Fig.1 : Studied volumic domain

In any medium without heat source, radiance nor material transfer, the heat equation written in a local form is:

$$m \frac{\partial^2 T}{\partial z^2} - \frac{\partial T}{\partial t} = 0, \quad z \in [a, b] \quad t \geq 0, \quad (2)$$

where $m = l\ (r\ C_p)^{-1}$ denotes the heat diffusivity and l represents the heat transfer conduction coefficient.

2.2 Boundary and initial conditions

The initial temperature of the medium is in general depending on the space variable z :

$$T(z,0) = F(z) \quad (3)$$

Considering that the medium is delimited by the boundaries $z=a$ and $z=b$, the two imposed temperatures on these boundaries are:

$$\begin{cases} T(a,t) = f_1(t) \\ T(b,t) = f_2(t) \end{cases} \quad (4)$$

Remark : Equations (4) are Dirichlet boundary conditions, but heat equation (2) can also be considered to satisfy Neumann boundary conditions.

$$-l \left.\frac{\partial T}{\partial z}\right|_a = j,$$
$$-l \left.\frac{\partial T}{\partial z}\right|_b = h_c (T - T_a).$$

For instance, such Neuman conditions shall be considered for the simulation.

2.2 General solution

The general solution of heat equation, is expressed as a sum of three integrals :

$$T(z,t) = \int_a^b \frac{f(x)}{\sqrt{mt}} exp\left[-\frac{(z-x)^2}{4mt} \right] dx$$

$$+\frac{1}{2\sqrt{m}}\int_0^t \frac{z-b}{(t-t)^{\frac{3}{2}}} exp\left[-\frac{(z-b)^2}{4m(t-t)}\right] y_2(t)dt$$

$$+\frac{1}{2\sqrt{m}}\int_0^t \frac{z-a}{(t-t)^{\frac{3}{2}}} \exp\left[-\frac{(z-a)^2}{4m(t-t)}\right] y_1(t)dt$$

(5)

Functions f, y_1 and y_2, which define integration variables of equation (2), must be such that (5) satisfies the initial condition (3) and the boundary conditions (4). Since second and third integral in equation (5) tend to zero as *t* tends to zero [2], then:

$$\lim_{t \to 0} T(z,t) = 2\sqrt{p} f(z),$$

$$f(z) = \frac{1}{2\sqrt{p}} F(z). \qquad (6)$$

Then boundary conditions (4) hold if and only if the unknown functions y_1 and y_2 satisfy the following Volterra equation system (7)-(8):

$$f_1(t) - \frac{a-b}{2\sqrt{m}}\int_0^t \frac{y_2(t)}{(t-t)^{\frac{3}{2}}} \exp\left[-\frac{(a-b)^2}{4m(t-t)}\right]dt =$$

$$\sqrt{p} y_1(t) + \frac{1}{4\sqrt{pmt}}\int_a^b F(x)\exp\left[-\frac{(a-x)^2}{4mt}\right]dx,$$

(7)

$$-f_2(t) + \frac{b-a}{2\sqrt{m}}\int_0^t \frac{y_1(t)}{(t-t)^{\frac{3}{2}}} \exp\left[-\frac{(b-a)^2}{4m(t-t)}\right]dt =$$

$$\sqrt{p} y_2(t) - \frac{1}{4\sqrt{pmt}}\int_a^b F(x)\exp\left[-\frac{(b-x)^2}{4mt}\right]dx.$$

(8)

The kernels of these two equations are defined and continuous for all $t > t$ and they tend to zero when t tends to t.

According to the investigations on the Volterra integral systems [2], system (7)-(8) has a unique solution $(y_1(t), y_2(t))$ defined for all $t > 0$. The functions $y_1(t)$ and $y_2(t)$ could be expressed as sums of absolutely and uniformly convergent series. Equations (7) and (8) include some aftereffect, since functions $y_1(.)$ and $y_2(.)$ are expressed at instants belonging to $[0,t[$. By introducing these functions in (5), we obtain the final solution $T(z,t)$ satisfying the imposed conditions (3)-(4).

2.3 Simulation

As an example, the heat equation (2) has been simulated for a 1 meter column of water with $1m^2$ section, submitted at the bottom to a homogeneous heating flux (1kw), and set free to the ambient air at the top. A loss of heat by convection with the atmosphere (Neumann boundary Conditions) was then foreseen. Fig. 2 and 3 represent the evolution of the temperature at the bottom and top of the water (this means, the temperature at the point in contact with the heat flux and the temperature at the point in contact with the ambient air.

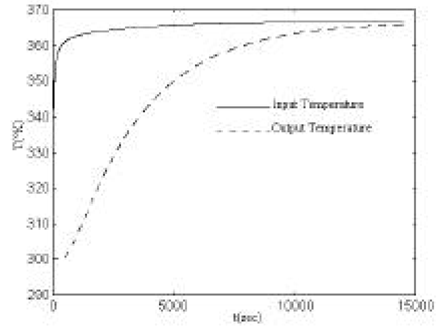


Fig.2: Temperature evolution

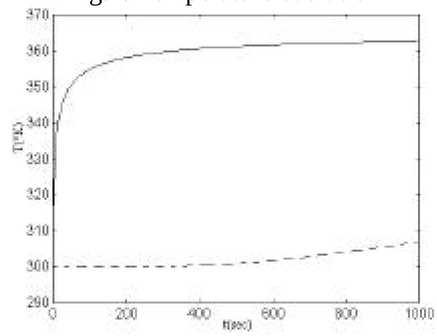


Fig.3: Temperature evolution (zoom)

Fig.3 shows the time delay bettween the input an the output temperature (about 380sec).

3 Modelling of the steam generator by PDEs

3.1 Description of the platform

The above Fig. 4 depicts a mini thermal power station (60 KW) that the L.A.I.L. has built so to reproduce, in a reduced scale, the same thermodynamic phenomena as a real power station. A large number of sensors and actuators equip the platform for control and supervision purposes.

The installation includes a boiler which produces a pressurized water steam. The size of the installation does not permit the working of a turbine: this steam is therefore relaxed by two modulated servovalves. The relaxed steam is cooled by a condenser coupled with a heat exchanger. The condensates are routed toward the tank to which are connected pumps that supply the boiler.

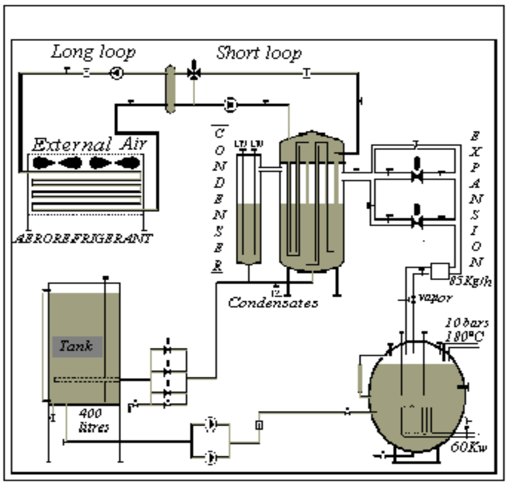


Fig.4: Test board of steam process

3.2 Modelling hypotheses

Two classical hypotheses will be adopted here:

H1. The water and the steam are in thermodynamic equilibrium, and the thermodynamic properties are calculated at the equilibrium state.

This hypothesis, generally admitted in the models of steam generators, is justified because the flow in the industrial steam generators is turbulent and produces a good homogenization of the emulsion (we shall consider here a homogeneous diphasic flow). This hypothesis H1 allows applying the global equations simultaneously to the two phases.

H2. The supplying liquid is incompressible (water at ambient temperature).

Note that conduction heat losses (from the boiler looses to the outside) are not neglected.

3.3 Homogeneous diphasic mixture

The global thermodynamic rules give the mass, momentum and energy equations. In diphasic mixture, they will be completed by the emptiness ratio correlation [10].

<u>Equation of the mass conservation:</u>

$$\frac{\partial \overline{r}}{\partial t} + \frac{\partial \overline{G}}{\partial z} = 0 \qquad (10)$$

$\overline{r}$, average density of mixture,

$\overline{G}$, density of mean rate flow,

$\overline{r} = a r_s + (1-a) r_w$

$\overline{G} = a r_s v + (1-a) r_w u$

r_s and r_w are the density of (respectively) steam and water in the boiler (given by the saturated water/steam tables) [7].

v and u are the velocity of the two phases, and a represents the emptiness ratio.

<u>Equation of the momentum conservation:</u>

$$\frac{\partial \overline{G}}{\partial t} + \frac{\partial \overline{j}}{\partial z} + \overline{r} g + F = -\frac{\partial p}{\partial z} \qquad (11)$$

where $\overline{j} = a r_s v^2 + (1-a) r_w u^2$, and g is the gravity. The variation of pressure by friction, F, calculated by the Fanning Equations [11], is given by:

$$F = f \frac{\overline{G}^2}{2 \overline{r} D_h}$$

where f is the pressure loss coefficient and D_h is the hydraulic diameter.

<u>Equation of the energy conservation:</u>

$$\overline{r} \frac{\partial \overline{h}}{\partial t} + \frac{\partial \overline{y}}{\partial t} + \overline{G} \frac{\partial \overline{h}}{\partial z} = E + \frac{\partial p}{\partial t} \qquad (12)$$

$\overline{h} = X h_s + (1-X) h_w$ is the average specific enthalpy, $E = l \frac{\partial^2 \overline{h}}{\partial z^2}$ represents the diffusion term, where l is the conduction heat coefficient, h_s and h_w are the specific enthalpy of (respectively) steam and water in the boiler (given by the satureted water\steam tables) [7] and X is the steam quality:

$$X = \frac{a r_s v}{a r_s v + (1-a) r_w u} \qquad (13)$$

$$\overline{y} = (h_v - h_l)\left[a(1-X) r_v - (1-a) X r_l\right]$$

3.3.1 Emptiness ratio correlation

In local equilibrium, we consider that the emptiness ration is null, since we suppose the thermodynamic equilibrium between phases. In reality (see [11]), steam bubbles are formed in the overheated liquid layer at the neighborhood of the heat flux (φ) position and condensed in the under saturated remaining liquid.

In mass-boiling zone, we use the Armand correlation modified by Massena [12], which was found on theoretical and experimental bases:

$$a = \frac{0.833 + 0.167 X}{X + g(1-X)\frac{r_v}{r_l}} X \qquad (14)$$

Where γ representes the sliding coefficient.

The three quantities a, X, and g are bounded by the following relation, since the two phases are in thermodynamic equilibrium:

$$g = \frac{X}{1-X} \frac{1-a}{a} \frac{r_l}{r_v} \qquad (15)$$

The sliding coefficient g keeps moderate values (between 1 and 3) [8].

3.3.2 The pressure loss coefficient

In simple phase, the pressure loss coefficient is calculated by the E.C.Koo correlation (cited by W.H. Mac Adams [11]) valid for a Reynolds number between 3000 and 3.10^6:

$$f = 0.0056 + 0.5 R_e^{-0.32} \tag{16}$$

However, in homogeneous diphasic flow, we have to use the Armand correlation [13] (here for a pressure $p > 8$ bars and a steam quality $X < 0.9$),

$$f = a d(a), \tag{17}$$

$$d(a) = \begin{cases} \dfrac{1}{(1-a)^{1.42}} \text{ if } 0.39 < 1-a < 1 \\ \dfrac{0.478}{(1-a)^{1.42}} \text{ if } 0.1 < 1-a \le 0.39 \\ \dfrac{1.73}{(1-a)^{1.42}} \text{ if } 0 < 1-a \le 0.1 \end{cases}$$

$$a = 0.0056 + 0.5 R_e^{-0.32}$$

3.4 Partial differential equations

In solving the system (13)-(14) in X and a, we notice that these two variables depend on u, v, r_w and r_s. Examining the water and steam saturation tables, we remark that these last two variables (r_w and r_s) and the specific enthalpy h_w and h_s, are function of the only pressure p. This remark allows us to finally write system (10), (11), (12) as function of the three variables *u*, *v* and *p* which turn to be, in our case, the state variables. This allow obtaining, after resolution of this system, all the other thermodynamic variables in the boiler from a state equation as:

$$\begin{aligned} \dot{x} &= f(x,i,k,t,z) \\ y &= g(x,i,k,t,z) \end{aligned} \tag{18}$$

where,

$$x = [u, v, p]^T$$

$i = [\dot{m}_{al}, \dot{m}_v, j]^T$ is the input vector including:

$\dot{m}_{al}$, feedwater flow (control),

$\dot{m}_v$, outlet steam flow (perturbation),

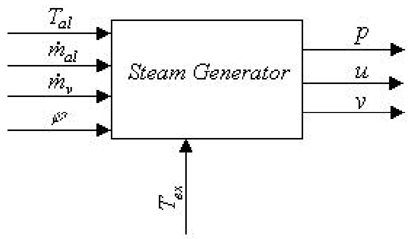
j, electrical heating flux (control),

$k = [T_{ex}, T_{al}]^T$ is an environment vector with:

T_{ex}, ambient temperature,

T_{al}, feedwater temperature.

The diagram corresponding to such relations is represented by the following figure:



Expressing all variables in function of *u*, *v* and *p*, the system (10),(11),(12) becomes:

$$\begin{bmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{bmatrix} \begin{bmatrix} \frac{\partial p}{\partial t} \\ \frac{\partial u}{\partial t} \\ \frac{\partial v}{\partial t} \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}$$

$$\begin{aligned} A_i &= A_i(p,u,v) \\ B_i &= B_i(p,u,v) \\ C_i &= C_i(p,u,v) \\ f_i &= f_i\left(p,u,v,\frac{\partial p}{\partial z},\frac{\partial u}{\partial z},\frac{\partial v}{\partial z},(\frac{\partial p}{\partial z})^2,\frac{\partial^2 p}{\partial z^2} \right) \\ i &= 1,2,3 \end{aligned} \tag{19}$$

Inverting the matrix in (19) leads to the final system to be discretized:

$$\begin{aligned} \frac{\partial p}{\partial t} &= a_1 \frac{\partial p}{\partial z} + b_1 \frac{\partial u}{\partial z} + c_1 \frac{\partial v}{\partial z} + d_1 \left[\frac{\partial p}{\partial z}\right]^2 + e_1 \frac{\partial^2 p}{\partial z^2} \\ \frac{\partial u}{\partial t} &= a_2 \frac{\partial p}{\partial z} + b_2 \frac{\partial u}{\partial z} + c_2 \frac{\partial v}{\partial z} + d_2 \left[\frac{\partial p}{\partial z}\right]^2 + e_2 \frac{\partial^2 p}{\partial z^2} \\ \frac{\partial v}{\partial t} &= a_3 \frac{\partial p}{\partial z} + b_3 \frac{\partial u}{\partial z} + c_3 \frac{\partial v}{\partial z} + d_3 \left[\frac{\partial p}{\partial z}\right]^2 + e_3 \frac{\partial^2 p}{\partial z^2} \end{aligned}$$

a_i, b_i, c_i, d_i, e_i are function of p,u,v

$i = 1,2,3$

(20)

Using water/steam tables [7], steam quality can be expressed as function of state *x*, after solving system (20):

$$X(x,t) = 5.988 \frac{u - .833v}{v - uf(p)}$$

where

$$f(p) = \frac{-.178710^{-2} p^3 + .459910^{-1} p^2 - .4091p + 1.486}{.766410^{-5} p + .105210^{-2}}$$

Remark: in [1], the expected state equation of the nonlinear system had a different type. The state $x = [T_G, T_m, m_G]^T$ was composed with:

T_G, emulsion temperature,

T_m, body boiler temperature

m_G, liquid mass in the boiler.

3.5 Numerical implementation

The principle of simulation is based on the discretization of system (20) into a finite difference equations.

$$\frac{\partial f(z,t)}{\partial t} = \frac{f_i^{k+1} - f_i^k}{\Delta t}$$

$$\frac{\partial f(z,t)}{\partial z} = \frac{f_{i+1}^k - f_i^k}{\Delta z}$$

$$\frac{\partial^2 f(z,t)}{\partial z^2} = \frac{f_{i+1}^k - 2 f_i^k + f_{i-1}^k}{(\Delta z)^2}$$

where Δt and Δz are respectively, the time and the space steps. Unfortunately, the theoretical approach of the choice of Δt and Δz necessitates a stability criterion which, in our case (non linear equations) is absent. However, in practice, we used an iterative scheme so to adapt, at each step of discretization, the space and time steps enabling the stability.

4 Conclusion

An original contribution was provided by taking into account the dynamic behaviors with a homogeneous diphasic mixture leading to a model with partial differential equations, with an input/output form that is adapted to control.

An other interesting point is that this nonlinear PDE model allows estimating the "**steam quality**" relating to the position in the boiler: this constitutes a practical progress, since no such sensor is available. Furthermore, this variables allows to modelize the shrink and swell effect [4], wich introduces a non minimum phase in the vapor generator.

Besides, the initial motivation for this study was to obtain a possible bentchmark for the control of delay systems [14]. The theoretical resolution of the heat equation has shown the existence of an aftereffect phenomenon in the heat propagation, which was confirmed by the simulation of the equation (2) (Fig. 2-3). This result encouraged us to make a more complete study of the heat propagation equation in the steam generator. A second cause of delay effect lays in the boiler output pipe (transportation time lag from the boiler to the modulated servovalves, see Fig. 4). This lengh will be increased on the plant.

References:

[1] B. Ould Bouamama and G. Dauphin-Tanguy, *Bond Graph model of a Steam Generator Process and its Environment,* In the 10th European, Simulation Multiconference ESM'96, SCS, Budapest Hungary June 2-6 1996, pp.238-242.

[2] W. Pogorzelski, *Integral Equations and their Applications*, Vol. I, Oxford New York Pergamon 1966.

[3] B. Ould Bouamama, R. Litwak and M. Staroswiecki, *Detection and Isolation of Process Faults in an Industrial Boiler,* 3rd IFAC Workshop, on-line-Fault detection and supervision in the chemical process industries,4-5 june 1998, IFP, Solaize (Lyon), vol. 2 France.

[4] S. Rimaux, B. Ould Bouamama and G. Dauphin-Tanguy, *Bond Graph Model of the Shrink and Swell phenomenon in a PWR Nuclear Power Plant,* IEEE-SMC, IMACS Multiconference CESA'96, Lille- France, july 9-12 1996, pp. 288-292.

[5] S. Rimaux, B. Ould Bouamama et G. Dauphin – Tanguy, *Modélisation Simplifiée des Centrales*, rapport scientifique N° 1, 2 3 et 4, contrat N° P33/2M3502/EP732, ECL-EDF, LAIL, URA CNRS 1440, december 1995, Mai 1996, June 1996 and march 1997.

[6] Tylee L. 1983, *A Bond Graph Description U-Tube Steam Generator Dynamics,* The Franklin Institute, vol. 315, N°. 3, pp. 165-178,

[7] A. W. Ordys, A. W. Pike, M. A. Johnson, R. M. Katebi and M. J. Grimble, *Modelling and Simulation of Power Generation Plants*, Springer-Verlag, 1994.

[8] J. M. Delhay, *Contribution à l'étude des Ecoulements Diphasisques Eau-Air et Eau-Vapeur*, PhD Thesis, 1970, Grenoble.

[9] G. Standart, *The Mass, Momentum and Energy Equations for Heterogeneous Flow Systems*, Science Ingeneering, Vol. 19, 1964, pp. 227-236.

[10] B. Dansac, *Modèle Mathématique d'un Générateur de Vapeur pour Centrale Nucléaire*, PhD Thesis, 1978, Univ. Paris XI.

[11] W. H. Mac-Adams, *Transmission de chaleur*, Dunod, 1964.

[12] Axelson and Rouhani, *Correlation of Void Volume Fraction in the Subcooled and Quality Boiling Regions*, International Journal of Heat and Mass Transfer, vol 13, n°2, pp 383-393, 1970.

[13] A. A. Armand, *The Resistant during the Mouvement of a Two-Phase System in Horizontal Pipes*, AERE 1959.

[14] J.P. Richard, *Some Trends and Tools for the Study of Time delay systems, Plenary lecture,* 2nd Conf. IMACS-IEEE CESA'98, Computational Engineering in Systems Applications, Tunisia, April 1998, Proc. Vol. P, pp 27-43.