Interpolation of B1-spline curve

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Abstract: - The recently introduced B1-spline curve has been drawing a great deal of attention, because B1-spline has special advantages in geometric modeling and in manufacturing processes using NC machines. This paper introduces a method of B1-spline interpolation. It generates almost the same shape as B-spline interpolation, but is particularly appropriate for designing corner points. Corner point often exists in design, but can cause a continuity problem or wiggling in the interpolation curve. B1-spline interpolation has the advantage of solving this problem. This paper shows examples proving that B1-spline interpolation has the merits compared with B-spline interpolation.

Key-Words: - B-spline, Knot vector, Basis function, B1-spline, parameterization, interpolation

1 Introduction

B-spline has become the de facto standard tool for the representation, design, and data exchange of geometric information in geometric modeling. Data fitting is an interesting challenge in engineering. There are two types of B-spline fitting: interpolation and approximation. Interpolation generates a B-spline curve or surface that passes through the data points. Parameterization, the method used to match between point and parameter, is an approach that has recently been attracting interest[1-7].

Recently, B1-spline, which has several special advantages and the same quality as B-spline, has been introduced [8]. B1-spline requires the number of knots to be the same as the number of control points. This property of B1-spline has two special merits, selecting the knot vector automatically even in nonuniform cases and generating spline with relative constant speed. These merits are very great advantages in geometric modeling and in manufacturing processes with NC machine[9-12].

This paper introduces the method of B1-spline interpolation and shows examples that confirm the special merits that B1-spline has in interpolation.

2 B1-Spline

A B1-spline curve is defined by
\[ C(u) = \sum_{i=0}^{n} P_i B_{i,p}(u) \] (1)
where \( P_i \) are the control points, and \( B_{i,p}(u) \) are B1-spline basis functions of \( p \) degree.

The knot vector and basis functions are defined in \( i=(0,\ldots,n) \) as follows, where \( s = \text{int}((p+1)/2) \)

\[ U=\{u_i\} = \{u_0, \ldots, u_0, u_{p+1-s}, \ldots, u_n, \ldots, u_n\} \] (2)

\( u_0 \) and \( u_n \) appear \( s \) times at the end.

\[ B_{i,p}(u) = \frac{u-a_2}{a_4-a_2} B_{i,p-1}(u) + \frac{a_3-u}{a_3-a_1} B_{i+1,p-1}(u) \]
\[ \cdots \] (3)

where \( B_{i,0}(u) = \begin{cases} 1 & \text{if } u_i \leq u \leq u_{i+1} \\ 0 & \text{otherwise} \end{cases} \)

\[ a_1 = \begin{cases} u_0 & \text{if } (i+1-s < 0) \\ u_{i+1-s} & \text{otherwise} \end{cases} \]

\[ a_2 = \begin{cases} u_0 & \text{if } (i-s < 0) \\ u_{i-s} & \text{otherwise} \end{cases} \]

\[ a_3 = \begin{cases} u_n & \text{if } (i+p+1-s > n) \\ u_{i+p+1-s} & \text{otherwise} \end{cases} \]

\[ a_4 = \begin{cases} u_n & \text{if } (i+p-s > n) \\ u_{i+p-s} & \text{otherwise} \end{cases} \]
3 Interpolation
Let consider a \( p \)-th degree B1-spline curve global interpolation[13] to a sequence of given points \( Q_i \) \( (i=0,...,n) \), where \( n \geq p \) and no two consecutive points are the same. B1-spline curve interpolation can be stated as the problem of constructing a B1-spline curve passing through \( Q_i \) \( (i=0,...,n) \). When parameter values \( \bar{u}_i \) of \( Q_i \), and knot vector \( U \) of a B1-spline curve \( C(u) \) are given, the problem leads to solving a set of linear equations:

\[
Q_i = C(\bar{u}_i) = \sum_{j=0}^{n} P_j B_{i,p}(\bar{u}_i) \quad (i=0,...,n)
\]

(4)

where \( P_j \) \( (i=0,...,n) \) are unknown control points and the system matrix is a \((n+1) \times (n+1)\) square matrix of scalars.

\[
\begin{bmatrix}
B_{0,p}(\bar{u}_0) & \cdots & B_{n,p}(\bar{u}_0) \\
B_{0,p}(\bar{u}_1) & \cdots & B_{n,p}(\bar{u}_1) \\
\vdots & \ddots & \vdots \\
B_{0,p}(\bar{u}_n) & \cdots & B_{n,p}(\bar{u}_n)
\end{bmatrix}
\begin{bmatrix}
P_0 \\
P_1 \\
\vdots \\
P_n
\end{bmatrix}
=\begin{bmatrix}
Q_0 \\
Q_1 \\
\vdots \\
Q_n
\end{bmatrix}
\]

(5)

The values \( \bar{u}_i \) range from 0 to 1. The choice of the parameter values \( \bar{u}_i \) and the knot vector \( U \) affects the shape and the parameterization of the curve \( C(u) \).

Although there are many ways to determine the parameter value \( \bar{u}_i \), the following three parameterization methods are well cited[13-16].

1. uniform method

\[
\bar{u}_0 = 0, \quad \bar{u}_n = 1 \\
\bar{u}_k = \frac{k}{n} \quad \text{for} \quad k = 1,\ldots,n-1
\]

(6)

2. chord length method

\[
\bar{u}_0 = 0, \quad \bar{u}_n = 1 \\
\bar{u}_j = \bar{u}_{j-1} + \frac{|Q_j - Q_{j-1}|}{\sum_{j=0}^{n}|Q_j - Q_{j-1}|} \\
(7)
\]

3. centripetal method

\[
\bar{u}_0 = 0, \quad \bar{u}_n = 1 \\
\bar{u}_j = \bar{u}_{j-1} + \frac{|Q_j - Q_{j-1}|^{1/2}}{\sum_{j=0}^{n}|Q_j - Q_{j-1}|^{1/2}} \quad (8)
\]

4 Experimental Results
Two examples are explained in detail, and general examples are also shown. Three parameterization methods, uniform, chord length and centripetal, are tested. But the uniform and chord length methods often result in difficulties with a cusp or a loop in the resulting curve. Therefore the centripetal method is used for all examples.

The “+” symbol in the examples indicates the location of control points. The blue line curve means the B-spline and the red line curve means the B1-spline.

Averaging method is used to define the knot vector \( U \) in B-spline [13-15].

\[
U = \{0,0,...,0,u_{p+1},...,u_n,1,1,...1\}
\]

where \( u_{j+p} = \frac{1}{p} \sum_{i=j}^{j+p-1} \bar{u}_i \) for \( j = 1,\ldots,n-p \)

(9)

The parameter values calculated from equation (8) are used as knot vector in B1-spline.

Cubic is selected as degree. Thus, \( p=3 \) and \( s=\text{int}((p+1)/2)=2 \).

4.1 Example 1

4.1.1 Data points

13 Data Points are input as follows:
\[Q_{0-12} = \{(200., 200., 0.), (201., 200., 0.),
(399., 200., 0.), (400., 200., 0.),
(400., 299.5, 0.), (400., 300., 0.),
(200.6, 300., 0.), (200., 300., 0.),
(200., 399.3, 0.), (200., 400., 0.),
(200.7, 400., 0.), (399., 400., 0.),
(400., 400., 0.)\}\]

For each corner point and end point, we input two points intentionally with a short distance between them. This helps us show the merits of B1-spline compared with B-spline.

4.1.2 Knot vector
For verification we use 2 knot vectors: \(u\) for B-spline and \(u1\) for B1-spline.

\[u = \{0.0000, 0.0000, 0.0000, 0.0000,
0.156711, 0.278824, 0.335780,
0.456707, 0.532778, 0.653985,
0.710425, 0.767167, 0.843982,
1.0000, 1.0000, 1.0000, 1.0000\}\]

\[u1 = \{0.0000, 0.0000, 0.220440,
0.235066, 0.380065, 0.391308, 0.597848,
0.609177, 0.754929, 0.767167, 0.779404,
1.0000, 1.0000\}\]

\(u\) is calculated from equation (8),(9) and \(u1\) from equation (8).

4.1.3 Control points
Control points calculated are as follows. We use \(P\) for B-spline and \(P1\) for B1-spline.

\[P = \{(200., 200., 0.), (191.7, 198.0, 0.),
(414.4, 235.1, 0.), (398.5, 173.5, 0.),
(390.8, 324.5, 0.), (424.5, 280.2, 0.),
(206.2, 271.8, 0.), (203.4, 394.3, 0.),
(206.9, 390.2, 0.), (408.7, 400.7, 0.),
(400., 400., 0.)\}\]

\[P1 = \{(200., 200., 0.), (199.2, 199.7, 0.),
(398.7, 204.1, 0.), (400.7, 196.6, 0.),
(397.9, 299.5, 0.), (403.4, 300.5, 0.),
(200.5, 303.0, 0.), (199.7, 297.4, 0.),
(200.1, 399.3, 0.), (200.0, 400.0, 0.),
(201.1, 399.8, 0.), (407.9, 400.0d, 0.),
(400., 400., 0.)\}\]

Fig.1 B-spline and B1-spline interpolation curve with “\(\rightarrow\)” shape
As shown in Fig.1, B1-spline keeps the pleasing shape but B-spline wiggles. The closer of the two data points on the corner wiggles more in the B-spline curve.

4.2 Example 2

4.2.1 Data points
6 Data Points are input as follows:
\[Q_{0-5} = \{(200., 200., 0.), (200., 200.2, 0.),
(200., 400., 0.), (200.5, 400., 0.),
(399.5, 400., 0.), (400., 400., 0.)\}\]

4.2.2 Knot vector

\[u = \{0.0000, 0.0000, 0.0000, 0.0000,
0.335721, 0.656273, 1.0000,
1.0000, 1.0000, 1.0000\}\]

\[u1 = \{0.0000, 0.0000, 0.484409,
0.507899, 1.0000, 1.0000\}\]

4.2.3 Control points

\[P = \{(200., 200., 0.), (199.5, 195.9, 0.),
(220.3, 439.6, 0.), (166.9, 373.0, 0.),
(406.1, 401.0, 0.), (400., 400., 0.)\}\]

\[P1 = \{(200., 200., 0.), (200.0, 196.1, 0.),
(199.5, 403.5, 0.), (200.6, 396.9, 0.),
(405.8, 400.1, 0.), (400., 400., 0.)\}\]
As example 2, we show one corner point with a "ㄴ" shape. We can obviously see the difference between B1-spline curve and B-spline curve.

4.3 General Examples

Several types of examples are shown. As shown, B1-spline generally has almost the same shape as B-spline. But in terms of the corner point, B1-spline generates a more pleasing curve.

5 Conclusion

Recently, B1-spline has been proposed as an alternative to B-spline. This paper introduces the interpolation of B1-spline, and shows its differences from B-spline in the sense of the resulting curve. B1-spline interpolation generates almost the same shape as B-spline interpolation, but is particularly appropriate for designing corner points. Corner point often exists in design, but can cause a continuity problem or wiggling in the interpolation curve. B1-spline interpolation has the advantage of solving this problem. Considering the importance of corner points in design work, this is highly advantageous for designers in geometric modeling and in manufacturing processes with NC machine.

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