Statistics of k-μ Random Variable

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Abstract: -In this paper, the *k-μ* random variable will be considered and closed form expressions for probability density function (PDF), cumulative distribution function (CDF) and moments for this distribution will be derived. Also, product, ratio, maximum and minimum of two *k-μ* random variables will be studied and PDFs of these functions calculated in the closed forms. Then, obtained statistical functions will be shown graphically by using simulations in MATLAB. The influence of Rician factor *k* and *k-μ* multipath fading severity parameter *μ* on PDFs of considered distributions will be analyzed.

Key-Words: - Cumulative distribution function, *k-μ* distribution, probability density function, random variable

1 Introduction

The *k-μ* distribution can be used to describe signal envelope variation in multipath fading channels with two or more dominant components and two or more clusters in propagation environment. This distribution has two parameters, *k* and *μ*. *k* is Rician factor. It can be calculated as the ratio of dominant component power and scattering components powers. Second parameter, *μ*, is in relation with the number of clusters in propagation environment [1] [2].

Proposed *k-μ* distribution as a fading model is more realistic than other distributions such as Rayleigh, Rician, and Nakagami-*m* fading models [3]. The derivation of the *k-μ* distribution is completely based on inhomogeneous scattering environment unlike other fading models. Moreover, it has been reported that *k-μ* distribution has better fit to measured experimental data than other fading models such as Rician, Nakagami-*m*, and Rayleigh [4]. It is important to note that *k-μ* distribution is a general fading model which includes Rayleigh, the one-sided Gaussian, Rician (Nakagami-*n*), and Nakagami-*m* models as special cases. The relation between *k-μ* distribution and Rayleigh, Rician and Nakagami-*m* distributions has been shown in [5]. Namely, for $k = 0$, the *k-μ* distribution reduces to Nakagami-*m* distribution; Rician distribution can be derived from *k-μ* distribution by setting $\mu=1$ and Rayleigh distribution approximates *k-μ* distribution for $k=0$ and $\mu=1$.

The paper [4] presents two general fading distributions, the *k-μ* distribution and the *η-μ* distribution, for which fading models are proposed. These distributions are fully characterized in terms of measurable physical parameters. In particular, the *k-μ* distribution is better suited for line-of-sight applications, whereas the *η-μ* distribution gives better results for non-line-of-sight implementations.

A method to approximate the distribution of the sum of M independent, non-identically distributed Ricean random variables by the *k-μ* distribution is proposed in [6]. It is shown that the differences between exact and approximate distribution curves are almost imperceptible.

There are more papers in available technical literature considering statistical characteristics of *k-μ* random variable and analysing performance of wireless communication systems operating over *k-μ* multipath fading channels.

In paper [7], wireless communication system with L branches maximal ratio combining (MRC) diversity receiver operating over *k-μ* multipath fading channel is discussed. The probability density function and cumulative distribution function of MRC receiver output signal are calculated. These formulas are used for estimation of outage probability and bit error probability of proposed wireless communication system. The level crossing rate (LCR) of MRC receiver over *k-μ* multipath fading environment is analyzed in [8]. Further, the general exact expression for average fade duration (AFD) of dual selection diversity over independent and identically distributed (i.i.d) k-μ fading channel is derived in [9]. AFD is the average time duration to keep envelope below specific level when it crosses it in the negative direction. The AFD has many applications in the evaluation and design of wireless communication systems.

The infinite series expressions for the second-order statistical measures of a macro-diversity structure operating over Gamma shadowed k-μ fading channels are provided in [10]. The authors focused on MRC combining at each base station (micro-diversity), and selection combining (SC), based on output signal power values, between base stations (macro-diversity). Some numerical results of the system's level crossing rate and average fading duration are presented, in order to examine the influence of various parameters on observed sizes.

In [11], the wireless communication system with dual branch SC diversity receiver operating over k-μ multipath fading environment is considered. The closed form expressions for average level crossing rate of SC receiver output signal envelope and average fade duration of proposed system are evaluated. Numerical results are presented graphically to show the influence of Rican factor k and fading severity μ on average level crossing rate and average fade duration.

Statistics for ratios of Rayleigh, Rician, Nakagami-*m*, and Weibull distributed random variables is determined in [12]. The distribution of minimum of ratios of two random variables is derived in [13] and its uses in analysis of multi-hop systems presented in [13] [14].

The paper [15] acts with distribution ratio of random variable and product of two random variables for the cases where random variables are Rayleigh, Weibull, Nakagami-*m* and α-μ distributed. The closed-form expressions for the probability density function (PDF) and cumulative distribution function (CDF) of the ratio of random variable and product of two random variables for these cases are derived and an application of obtained results in performance questioning of multi-hop wireless communication systems in different transmission environments is detailed described.

LCR of the ratio of product of two *k-μ* random variables and *k-μ* , i.e. Nakagami-*m* random variable are observed in [16], i.e.[17]. This scenario presents an interference limited *k-μ* multipath fading line-of-sight environment. The ratio of product of two *k-μ* random variables and *k-μ* (Nakagami-*m*) random variable represent signal-to-interference envelope ratio.

In this paper, statistical characteristics of *k-μ* random variable are examined. The closed form expression for probability density function, cumulative distribution function, moments and average level crossing rate of *k-μ* random process are calculated. Also, probability density function of product, ratio and maximum of two *k-μ* random variables are evaluated. Statistics of product of two *k-μ* random variables can be used in performance analysis of wireless relay communication system with two sections working over *k-μ* multipath fading channel. Statistics of ratio of two *k-μ* random variables can be used in performance analysis of wireless relay communication system that runs over *k-μ* multipath fading channel in the presence of co-channel interference which suffers *k-μ* multipath fading. Probability density function of maximum of two *k-μ* random variables can be used in study of the system characteristics of wireless communication system with selection combining receiver in the presence of *k-μ* multipath fading.

2 The *k*-μ Random Variable

The *k-μ* random variable follows distribution [eq. (10), 3]:

$$p_x(x)=\frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{\mu k}\Omega^{\frac{\mu+1}{2}}}\cdot e^{-\frac{\mu(k+1)}{\Omega}x^2}\cdot I_{\mu-1}\left(2\mu\sqrt{\frac{k(k+1)}{\Omega}}x\right)=$$

$$=\frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{\mu k}\Omega^{\frac{\mu+1}{2}}}\cdot\sum_{i_1=0}^{\infty}\left(\mu\sqrt{\frac{k(k+1)}{\Omega}}\right)^{2i_1+\mu-1}\frac{1}{i_1!\Gamma(i_1+\mu)}\cdot$$

$$\cdot x^{2i_1+2\mu-1}\cdot e^{-\frac{\mu(k+1)}{\Omega}x^2},\quad x\geq 0 \tag{1}$$

where $k>0$ is the ratio of the total power of dominant components to that of scattered waves and:

$$\mu=\frac{E^2\{\gamma\}}{\mathrm{var}\{\gamma\}}\frac{1+2k}{(1+k)^2}.$$

Cumulative distribution function of *k-μ* random variable is:

$$F_x(x)=\int_0^x dt\,p_x(t)=$$

$$=\frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{\mu k}\Omega^{\frac{\mu+1}{2}}}\cdot\sum_{i_1=0}^{\infty}\left(\mu\sqrt{\frac{k(k+1)}{\Omega}}\right)^{2i_1+\mu-1}\frac{1}{i_1!\Gamma(i_1+\mu)}\cdot$$

$$\cdot\frac{1}{2}\left(\frac{\Omega}{\mu(k+1)}\right)^{i_1+\mu}\cdot\gamma\left(i_1+\mu,\frac{\mu(k+1)}{\Omega}x^2\right) \tag{2}$$

where $\gamma(n,x)$ is incomplete Gamma function:

$$\gamma(n,x)=\frac{1}{n}e^{-x}x^n\cdot\sum_{j=0}^{\infty}\frac{1}{(n+1)(j)}x^j\ .$$

Moment of n-th order of k-μ random variable is [1]:

$$m_n=\overline{x^n}=\int_0^\infty dx\, x^n p_x(x)=$$

$$=\frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{\mu k}\Omega^{\frac{\mu+1}{2}}}\cdot\sum_{i_1=0}^{\infty}\left(\mu\sqrt{\frac{k(k+1)}{\Omega}}\right)^{2i_1+\mu-1}\frac{1}{i_1!\Gamma(i_1+\mu)}\cdot$$

$$\cdot\frac{1}{2}\left(\frac{\Omega}{\mu(k+1)}\right)^{i_1+\mu+n/2}\cdot\Gamma\left(i_1+\mu+\frac{n}{2}\right). \qquad (3)$$

The k-μ random variable and its first derivative are independent. The first derivative of k-μ random variable follows Gaussian distribution. Therefore, the joint probability density function of k-μ random variable and its first derivative is:

$$p_{x\dot{x}}(x\dot{x})=p_x(x)\,p_{\dot{x}}(\dot{x})=$$

$$=\frac{2\mu(k+1)^{\frac{\mu+1}{2}}}{k^{\frac{\mu-1}{2}}e^{\mu k}\Omega^{\frac{\mu+1}{2}}}\cdot\sum_{i_1=0}^{\infty}\left(\mu\sqrt{\frac{k(k+1)}{\Omega}}\right)^{2i_1+\mu-1}\frac{1}{i_1!\Gamma(i_1+\mu)}\cdot$$

$$\cdot x^{2i_1+2\mu-1}\cdot e^{-\frac{\mu(k+1)}{\Omega}}\cdot\frac{1}{\sqrt{2\pi}\sigma_{\dot{x}}}\cdot e^{-\frac{\dot{x}^2}{2\sigma_{\dot{x}}^2}} \qquad (4)$$

where

$$\sigma_{\dot{x}}^2=\pi^2 f_m^2\frac{\Omega}{k(\mu+1)}\ ,$$

and f_m is maximal Doppler frequency.

The level crossing rate of k-μ random process is:

$$N_x=\int_0^\infty d\dot{x}\,\dot{x}p_{x\dot{x}}(x\dot{x})=$$

$$=f_m\frac{\mu^{1/2}(k+1)^{\frac{\mu}{2}}}{k^{\frac{\mu-1}{2}}e^{\mu k}\Omega^{\frac{\mu}{2}}}\cdot$$

$$\cdot\sum_{i_1=0}^{\infty}\left(\mu\sqrt{\frac{k(k+1)}{\Omega_1}}\right)^{2i_1+\mu-1}\frac{1}{i_1!\Gamma(i_1+\mu)}\cdot$$

$$\cdot x^{2i_1+2\mu-1}\cdot e^{-\frac{\mu(k+1)}{\Omega}}. \qquad (5)$$

3 Product, Ratio, Maximum and Minimum of Two *k*-μ Random Variables

The k-μ random variables x_1 and x_2 follow distribution [3]:

$$p_{x_j}(x_j)=\frac{2\mu_j(k_j+1)^{\frac{\mu_j+1}{2}}}{k_j^{\frac{\mu_j-1}{2}}e^{\mu_j k_j}\Omega_j^{\frac{\mu_j+1}{2}}}\cdot$$

$$\cdot\sum_{i_j=0}^{\infty}\left(\mu_j\sqrt{\frac{k_j(k_j+1)}{\Omega_j}}\right)^{2i_j+\mu_j-1}\frac{1}{i_j!\Gamma(i_j+\mu_j)}\cdot$$

$$\cdot x_j^{2i_j+2\mu_j-1}\cdot e^{-\frac{\mu_j(k_j+1)}{\Omega_j}x_j^2}. \qquad (6)$$

The product of two k-μ random variables x_1 and x_2 is:

$$x=x_1\cdot x_2,\quad x_1=\frac{x}{x_2}. \qquad (7)$$

The product of x_1 and x_2 has distribution:

$$p_x(x)=\int_0^\infty dx_2\frac{1}{x_2}p_{x_1}\left(\frac{x}{x_2}\right)p_{x_2}(x_2)=$$

$$=\frac{2\mu_1(k_1+1)^{\frac{\mu_1+1}{2}}}{k_1^{\frac{\mu_1-1}{2}}e^{\mu_1 k_1}\Omega_1}\cdot\frac{2\mu_2(k_2+1)^{\frac{\mu_2+1}{2}}}{k_2^{\frac{\mu_2-1}{2}}e^{\mu_2 k_2}\Omega_2}\cdot$$

$$\cdot\sum_{i_1=0}^{\infty}\left(\mu_1\sqrt{\frac{k_1(k_1+1)}{\Omega_1}}\right)^{2i_1+\mu_1-1}\frac{1}{i_1!\Gamma(i_1+\mu_1)}\cdot$$

$$\cdot\sum_{i_2=0}^{\infty}\left(\mu_2\sqrt{\frac{k_2(k_2+1)}{\Omega_2}}\right)^{2i_2+\mu_2-1}\frac{1}{i_2!\Gamma(i_2+\mu_2)}\cdot$$

$$\cdot x^{2i_1+2\mu_1-1}\cdot\left(\frac{\mu_1(k_1+1)\Omega_2x^2}{\Omega_1\mu_2(k_2+1)}\right)^{i_1+\mu_1+i_2+\mu_2}$$

$$\cdot K_{-2i_1-2\mu_1+2i_2+2\mu_2}\left(2\sqrt{\frac{\mu_1(k_1+1)\mu_2(k_2+1)}{\Omega_1\Omega_2}}\right). \qquad (8)$$

Probability density function of product of two k-μ random variables can be used in performance analysis of wireless relay communication system with two sections operating over k-μ multipath fading channel. Cumulative distribution function and moments of product of two k-μ random variables can be calculated by using this PDF.

Random variable, calculated as product of two k-μ random variables is denoted with (k-μ)*(k-μ). The (k-μ)*(k-μ) is general distribution, and Nakagami-m*Nakagami-m, Rician*Nakagami-m and Rician* Rician distributions can be derived from (k-μ)*(k-μ) distribution. By setting k_1=0 and k_2=0, the (k-μ)*(k-μ) distribution reduces to Nakagami-m*Nakagami-m distribution; Rician* Rician distribution is performed from (k-μ)*(k-μ) distribution for μ_1=1 and μ_2=1 and Nakagami-m*Rician distribution is carried out from (k-μ)*(k-μ) distribution by setting k_1=0 and μ_2=1.

The ratio of two k-μ random variables x_1 and x_2 is:

$$x = \frac{x_1}{x_2}, \quad x_1 = x \cdot x_2. \tag{9}$$

Probability density function of the ratio of two k-μ random variables is:

$$p_x(x) = \int_0^\infty dx_2 x_2 p_{x_1}(xx_2) p_{x_2}(x_2) =$$

$$= \frac{2\mu_1 (k_1+1)^{\frac{\mu_1+1}{2}}}{k_1^{\frac{\mu_1-1}{2}} e^{\mu_1 k_1} \Omega_1^{\frac{\mu_1+1}{2}}} \cdot \frac{2\mu_2 (k_2+1)^{\frac{\mu_2+1}{2}}}{k_2^{\frac{\mu_2-1}{2}} e^{\mu_2 k_2} \Omega_2^{\frac{\mu_2+1}{2}}} \cdot$$

$$\cdot \sum_{i_1=0}^{\infty} \left(\mu_1 \sqrt{\frac{k_1(k_1+1)}{\Omega_1}} \right)^{2i_1+\mu_1-1} \frac{1}{i_1! \Gamma(i_1+\mu_1)} \cdot$$

$$\cdot \sum_{i_2=0}^{\infty} \left(\mu_2 \sqrt{\frac{k_2(k_2+1)}{\Omega_2}} \right)^{2i_2+\mu_2-1} \frac{1}{i_2! \Gamma(i_2+\mu_2)} \cdot$$

$$\cdot x^{2i_1+2\mu_1-1} \cdot \frac{1}{2} \left(\frac{\Omega_1 \Omega_2}{\mu_1 (k_1+1) x^2 \Omega_2 + \mu_2 (k_2+1) \Omega_1} \right)^{i_1+\mu_1+i_2+\mu_2}$$

$$\cdot \Gamma(i_1+\mu_1+i_2+\mu_2) \tag{10}$$

Probability density function of the ratio of two k-μ random variables is calculated. In interference limited environment where interference power is significantly higher than noise power and noise effects on system performance can be ignored, the ratio of desired signal envelope and cochannel signal envelope is important performance measure. By using PDF of two k-μ random variables, outage probability and bit error probability of wireless communication system operating over k-μ multipath fading channel in the presence of co-channel interference subjected to k-μ multipath fading, can be calculated.

Random variable calculated as ratio of two k-μ random variables is denoted as (k-μ)/(k-μ). This random variable is general random variable. By placing k_1=0 and k_2=0, (k-μ)/(k-μ) random variable reduces to Nakagami-m/Nakagami-m random variable, for μ_1=1 and μ_2=1, Rician/Rician random variable is derived from (k-μ)/(k-μ) random variable; for k_1=0 Nakagami-m)/(k-μ) random variable is obtained from (k-μ)/(k-μ) random variable and for k_1=0, k_2=0, μ_1=1 and μ_2=1, Rayleigh/Rayleigh random variable approximates (k-μ)/(k-μ) random variable.

The maximum of two k-μ random variables x_1 and x_2 is:

$$x = \max(x_1, x_2).$$

Probability density function of variable x is:

$$p_x(x) = p_{x_1}(x) F_{x_2}(x) + p_{x_2}(x) F_{x_1}(x) =$$

$$= \frac{2\mu_1 (k_1+1)^{\frac{\mu_1+1}{2}}}{k_1^{\frac{\mu_1-1}{2}} e^{\mu_1 k_1} \Omega_1^{\frac{\mu_1+1}{2}}} \cdot$$

$$\cdot \sum_{i_1=0}^{\infty} \left(\mu_1 \sqrt{\frac{k_1(k_1+1)}{\Omega_1}} \right)^{2i_1+\mu_1-1} \frac{1}{i_1! \Gamma(i_1+\mu_1)} \cdot$$

$$\cdot x^{2i_1+2\mu_1-1} \cdot e^{-\frac{\mu_1(k_1+1)}{\Omega_1} x^2}$$

$$\cdot \frac{2\mu_2 (k_2+1)^{\frac{\mu_2+1}{2}}}{k_2^{\frac{\mu_2-1}{2}} e^{\mu_2 k_2} \Omega_2^{\frac{\mu_2+1}{2}}} \cdot$$

$$\cdot \sum_{i_2=0}^{\infty} \left(\mu_2 \sqrt{\frac{k_2(k_2+1)}{\Omega_2}} \right)^{2i_2+\mu_2-1} \frac{1}{i_2! \Gamma(i_2+\mu_2)} \cdot$$

$$\cdot \frac{1}{2} \left(\frac{\Omega_2}{\mu_2 (k_2+1)} \right)^{i_2+\mu_2} \cdot \gamma\left(i_2+\mu_2, \frac{\mu_2(k_2+1)}{\Omega_2} x^2 \right) +$$

$$+ \frac{2\mu_2 (k_2+1)^{\frac{\mu_2+1}{2}}}{k_2^{\frac{\mu_2-1}{2}} e^{\mu_2 k_2} \Omega_2^{\frac{\mu_2+1}{2}}} \cdot$$

$$\cdot \sum_{i_1=0}^{\infty} \left(\mu_2 \sqrt{\frac{k_2(k_2+1)}{\Omega_2}} \right)^{2i_1+2\mu_2-1} \frac{1}{i_1! \Gamma(i_1+\mu_2)} \cdot$$

$$\cdot x^{2i_1+2\mu_2-1} \cdot e^{-\frac{2i_1+2\mu_2-1}{\Omega_2}} \cdot$$

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$$\cdot\frac{2\mu_1\left(k_1+1\right)^{\frac{\mu_1+1}{2}}}{k_1^{\frac{\mu_1-1}{2}}e^{\mu_1 k_1}\Omega_1^{\frac{\mu_1+1}{2}}}\cdot$$

$$\cdot\sum_{i_2=0}^{\infty}\left(\mu_1\sqrt{\frac{k_1\left(k_1+1\right)}{\Omega_1}}\right)^{2i_2+\mu_1-1}\frac{1}{i_2!\Gamma\left(i_2+\mu_1\right)}\cdot$$

$$\cdot\frac{1}{2}\left(\frac{\Omega_1}{\mu_1\left(k_1+1\right)}\right)^{i_2+\mu_1}\cdot\gamma\left(i_2+\mu_1,\frac{\mu_1\left(k_1+1\right)}{\Omega_1}x^2\right) \quad (11)$$

The expression above is probability density function of maximum of two *k-μ* random variables. This expression can be used in performance analysis of wireless communication system with dual diversity SC receiver, working over *k-μ* multipath fading channels. Output signal from dual SC receiver can be calculated as the maximum of *k-μ* random variables at its inputs [18]. Cumulative distribution function of SC receiver output signal is equal to the product of cumulative distribution functions of *k-μ* signals at its inputs.

Random variable which can be calculated as maximum of two *k-μ* random variables is represented as max(*k-μ*, *k-μ*). This random variable is general random variable. By setting k_1=0 and k_2=0, the max(*k-μ*, *k-μ*) random variable reduces to max(Nakagami-*m*, Nakagami-*m*) random variable; for μ_1=1 and μ_2=1, the max(Rician, Rician) is obtained from max(*k-μ*, *k-μ*) random variable; for μ_1=1 and k_2=0, max(Rician, Nakagami-*m*) random variable is carried out from max(*k-μ*, *k-μ*) random variable and the max(Rayleigh, Rayleigh) random variable is performed from max(*k-μ*, *k-μ*) random variable for k_1=0, k_2=0, μ_1=1 and μ_2=1.

The minimum of two *k-μ* random variables x_1 and x_2 is:

$$x=\min\left(x_1,x_2\right).$$

Probability density function of variable *x* is:

$$p_x\left(x\right)=p_{x_1}\left(x\right)\left(1-F_{x_2}\left(x\right)\right)+p_{x_2}\left(x\right)\left(F_{x_1}\left(x\right)\right)=$$

$$=\frac{2\mu_1\left(k_1+1\right)^{\frac{\mu_1+1}{2}}}{k_1^{\frac{\mu_1-1}{2}}e^{\mu_1 k_1}\Omega_1^{\frac{\mu_1+1}{2}}}\cdot$$

$$\cdot\sum_{i_1=0}^{\infty}\left(\mu_1\sqrt{\frac{k_1\left(k_1+1\right)}{\Omega_1}}\right)^{2i_1+\mu_1-1}\frac{1}{i_1!\Gamma\left(i_1+\mu_1\right)}\cdot$$

$$\cdot x^{2i_1+2\mu_1-1}\cdot e^{-\frac{\mu_1\left(k_1+1\right)}{\Omega_1}x^2}\cdot$$

$$\cdot\left(1-\frac{2\mu_2\left(k_2+1\right)^{\frac{\mu_2+1}{2}}}{k_2^{\frac{\mu_2-1}{2}}e^{\mu_2 k_2}\Omega_2^{\frac{\mu_2+1}{2}}}\cdot\right.$$

$$\cdot\sum_{i_2=0}^{\infty}\left(\mu_2\sqrt{\frac{k_2\left(k_2+1\right)}{\Omega_2}}\right)^{2i_2+\mu_2-1}\frac{1}{i_2!\Gamma\left(i_2+\mu_2\right)}\cdot$$

$$\left.\cdot\frac{1}{2}\left(\frac{\Omega_2}{\mu_2\left(k_2+1\right)}\right)^{i_2+\mu_2}\cdot\gamma\left(i_2+\mu_2,\frac{\mu_2\left(k_2+1\right)}{\Omega_2}x^2\right)\right)+$$

$$+\frac{2\mu_2\left(k_2+1\right)^{\frac{\mu_2+1}{2}}}{k_2^{\frac{\mu_2-1}{2}}e^{\mu_2 k_2}\Omega_2^{\frac{\mu_2+1}{2}}}\cdot$$

$$\cdot\sum_{i_1=0}^{\infty}\left(\mu_2\sqrt{\frac{k_2\left(k_2+1\right)}{\Omega_2}}\right)^{2i_1+2\mu_2-1}\frac{1}{i_1!\Gamma\left(i_1+\mu_2\right)}\cdot$$

$$\cdot x^{2i_1+2\mu_2-1}\cdot e^{-\frac{\mu_2\left(k_2+1\right)}{\Omega_2}x^2}\cdot$$

$$\cdot\left(1-\frac{2\mu_1\left(k_1+1\right)^{\frac{\mu_1+1}{2}}}{k_1^{\frac{\mu_1-1}{2}}e^{\mu_1 k_1}\Omega_1^{\frac{\mu_1+1}{2}}}\cdot\right.$$

$$\cdot\sum_{i_2=0}^{\infty}\left(\mu_1\sqrt{\frac{k_1\left(k_1+1\right)}{\Omega_1}}\right)^{2i_2+\mu_1-1}\frac{1}{i_2!\Gamma\left(i_2+\mu_1\right)}\cdot$$

$$\left.\cdot\frac{1}{2}\left(\frac{\Omega_1}{\mu_1\left(k_1+1\right)}\right)^{i_2+\mu_1}\cdot\gamma\left(i_2+\mu_1,\frac{\mu_1\left(k_1+1\right)}{\Omega_1}x^2\right)\right) \quad (12)$$

Previous expression is probability density function of the minimum of two *k-μ* random variables. This result can be applied in the performance analysis of wireless relay communication system with two sections operating over *k-μ* multipath fading channel. By using the obtained expression, the outage probability of proposed relay system can be evaluated.

The random variable evaluated as minimum of two *k-μ* random variables is marked with min(*k-μ*, *k-μ*). The min(*k-μ*, *k-μ*) random variable is general random variable and min(Nakagami-*m*, Nakagami-*m*), min(Rician, Rician), min(Rician, Nakagami-*m*) and min(Rayleigh, Rayleigh) random variables may be carried out from min(*k-μ*, *k-μ*) random variable. By placing k_1=0 and k_2=0, the min(*k-μ*, *k-μ*) random variable reduces to min(Nakagami-*m*, Nakagami-*m*) random variable; the min(Rician, Rician) random variable can be derived from min(k-μ, k-μ) random variable by setting μ_1=1 and μ_2=1; min(Rician, Nakagami-m) random variable can be calculated from min(k-μ, k-μ) random variable for μ_1=1and k_2=0; and the min(Rayleigh, Rayleigh) random variable approximates min(k-μ, k-μ) random variable for k_1=0, k_2=0, μ_1=1 and μ_2=1.

4 Numerical Results

In Fig. 1, the histogram of k-μ random process is shown. The abscissa of the histogram presents the amplitude value of k-μ random process; the ordinate is the number of samples in the interval of abscissa.

In Fig. 2, the cumulative distribution function of k-μ random variable is given. The histogram of product of two k-μ random variables is displayed in Fig. 3.

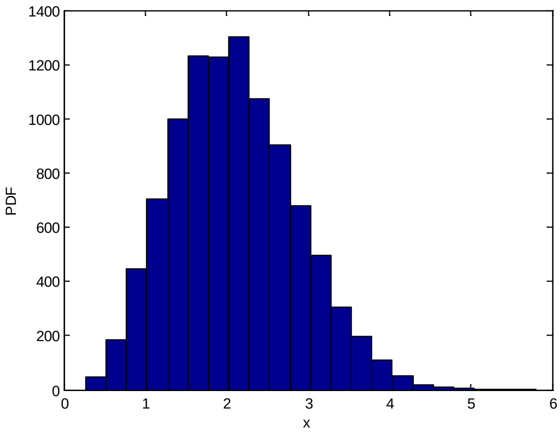

Fig.1. Histogram of k-μ random variable

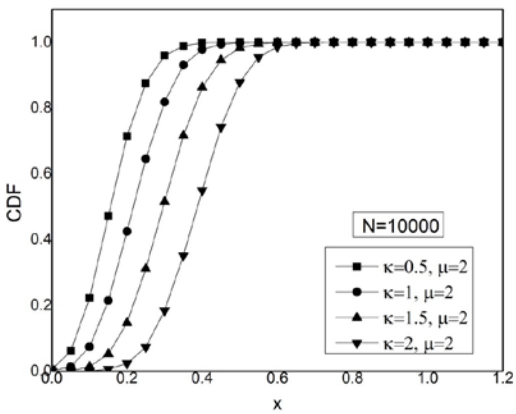

Fig.2. Cumulative distribution function of k-μ random variable

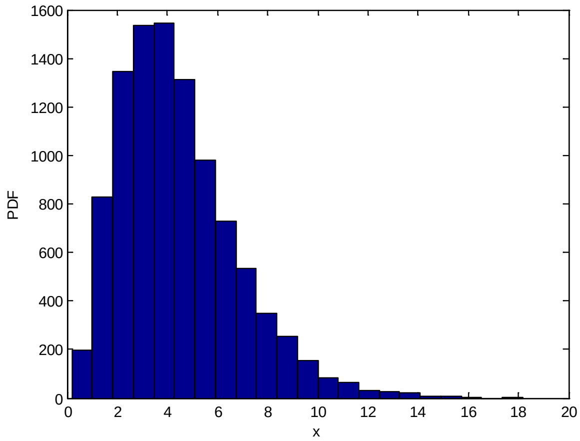

Fig.3. Histogram of the product of two k-μ random variables

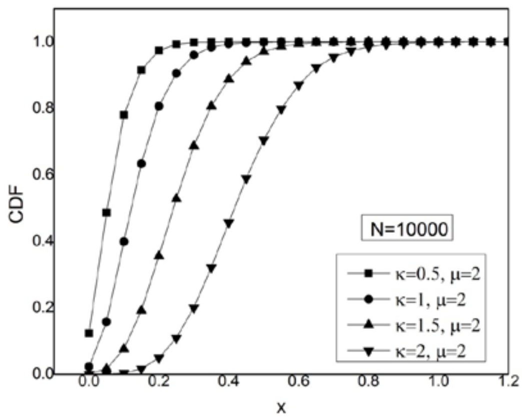

Fig.4. Cumulative distribution function of the product of two k-μ random variables

The cumulative distribution function of the product of two k-μ random variables is provided in Fig. 4. The product of two k-μ random variables is presented for several values of parameters k and μ=2. The cumulative distribution function of product of two k-μ random variables is outage probability of wireless relay communication system with two sections operating over k-μ multipath fading. The outage probability increases as output signal envelope increases.

The output signal envelope has higher influence on the outage probability for lower values of the output signal envelope. This figure shows that the outage probability increases when parameter k decreases for lower values of signal envelope. The outage probability stagnates for higher values of the output signal envelopes, reaches value 1 and stay with it for all magnitudes of the output signal envelope.

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The histogram of the ratio of two *k*-*μ* random variables is introduced in Fig. 5. The cumulative distribution function of the ratio of two *k*-*μ* random variables is shown in Fig. 6.

The cumulative distribution function of the ratio of two *k*-*μ* random variables is the outage probability of wireless communication system operating over *k*-*μ* short term fading in the presence of co-channel interference subjected to *k*-*μ* multipath fading. The influence of the output signal envelope is higher for lower values of the output signal envelope *x*.

The histogram of the maximum of two *k*-*μ* random variables is presented in Fig. 7. The cumulative distribution function of the maximum of two *k*-*μ* random variables is given in Fig. 8. The cumulative distribution function of the maximum of two *k*-*μ* random variables is actually the outage probability of wireless communication system with selection combining diversity receiver with two inputs in the presence of *k*-*μ* multipath fading.

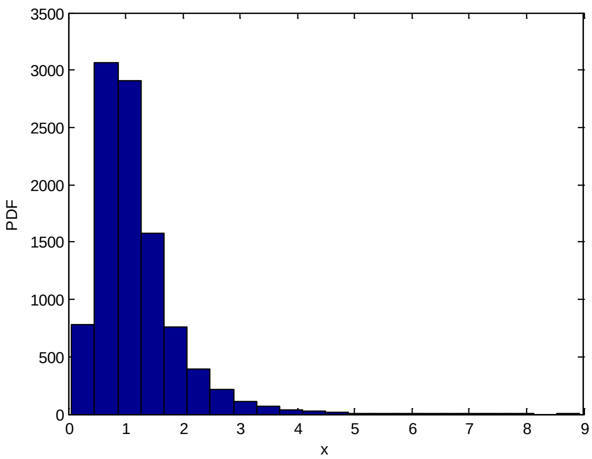

Fig.5. Histogram of the ratio of two *k*-*μ* random variables

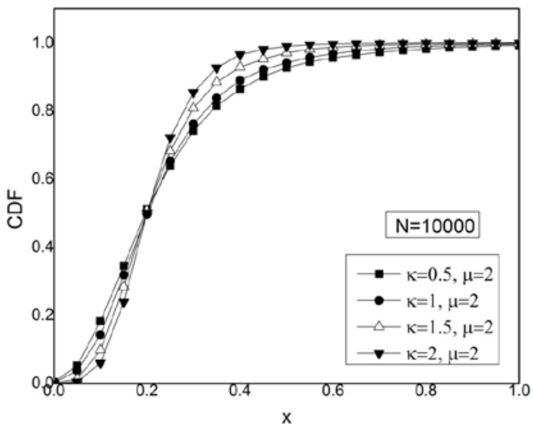

Fig.6. Cumulative distribution function of the ratio of two *k*-*μ* random variables

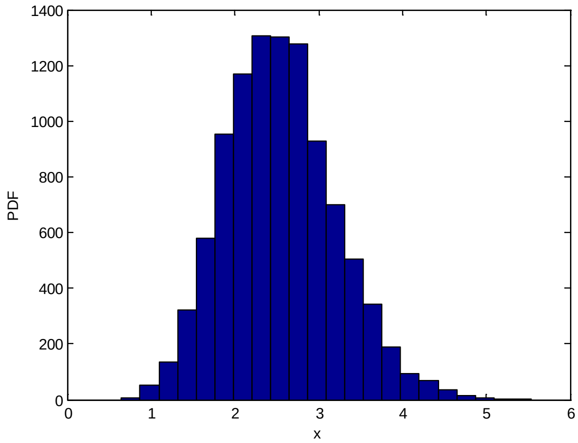

Fig.7. Histogram of the maximum of two *k*-*μ* random variables

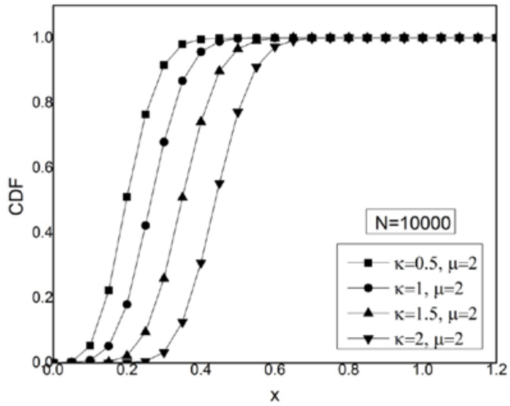

Fig.8. Cumulative distribution function of the maximum of two *k*-*μ* random variables

One can see from Fig. 8 that the outage probability increases as output signal envelope increases. The output signal envelope has higher influence on the values of the outage probability for lower values of the output signal envelope. This figure also shows that the outage probability has higher values for smaller values of the Rician parameter *k*. For lower values of the signal envelope the outage probability rises, achieves a maximum, and further normally has constant value of 1 for all other values of the output signal envelopes.

The histogram of the minimum of two *k*-*μ* random variables is displayed in Fig. 9. The cumulative distribution function of the minimum of two *k*-*μ* random variables is plotted in Fig. 10.

It is visible from Fig.10 that influence of Rician factor *k* is bigger for smaller values of output signal envelope and CDF is bigger for smaller values of parameter *k*.

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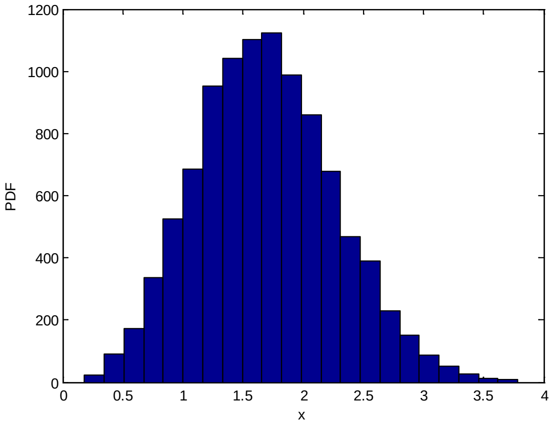

Fig.9. Histogram of the minimum of two *k-μ* random variables

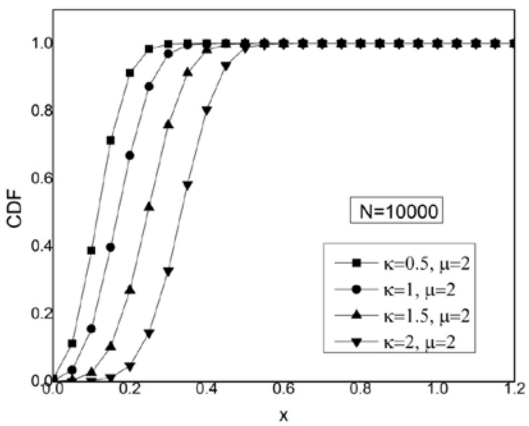

Fig.10. Cumulative distribution function of the minimum of two *k-μ* random variables

The obtained results can be applied in performance analysis of multi-hop systems operating over *k-μ* multipath fading channel. By using obtained expressions, the outage probability of proposed system can be evaluated.

Specifically, submitted results help the designers of wireless communication systems to simulate different wireless environments and configure system parameters in order to meet the QoS demands using the outage probability as important and widely accepted performance measure. Subsequently, PDF of minimum of ratios of k-μ random variables presented here, can be used to derive expression for moments-generating function [19] to evaluate lower bounds for the average bit error probability, which is another important system performance measure.

4 Conclusion

In this paper, *k-μ* random variable is considered and closed form expressions for probability density function, cumulative distribution function and moments of *k-μ* random variable are derived. These formulas can be used for calculation the outage probability, the bit error probability and the channel capacity of wireless communication system operating over *k-μ* multipath fading channel. Also, in this paper, the product, ratio, maximum and minimum of two *k-μ* random variables are considered and probability density function of these variables are calculated in the closed form.

As well, (*k-μ*)*(*k-μ*), (*k-μ*)/(*k-μ*), max(*k-μ*, *k-μ*) and min(*k-μ*, *k-μ*) random variables are formed and analyzed. The (*k-μ*)*(*k-μ*) random variable can be calculated as product of two *k-μ* random variables, the (*k-μ*)/(*k-μ*) random variable can be computed as the ratio of two *k-μ* random variables, max(*k*-μ, *k-μ*) random variable can be determined as a maximum of two *k-μ* random variables and min(*k-μ*, *k-μ*) random variable is performed as a minimum of two *k-μ* random variables.

PDF of product of two *k-μ* random variables is applying in performance analysis of wireless relay communication system with two sections. The ratio of two *k-μ* random variables can be utilized in analysis of wireless communication system operating over *k-μ* fading channel in the presence of *k-μ* co-channel interference. The maximum of two *k-μ* random variables is applicable in the analysis of operating characteristics of the wireless communication system which use SC receiver to reduce *k-μ* fading effects on the system performance. The minimum of two *k-μ* random variables may avail in performance examination of wireless communication system with two sections operating over *k-μ* multipath fading channel.

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