Association Rules of Web Mining with Background Knowledge of Web Structure

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Abstract: This paper proposed discovering sound, complete and non-redundant association rules from web transaction in the term of formal context. Furthermore, the association rules are also non-redundant regarding to the web structure since there are possibly some association rules which can be inferred from the web structure. We view the web structure as background knowledge. In this paper, the web transaction is represented by a formal context where as the background knowledge is represented by constraint.

Key–Words: Association rules, web transaction, web structure, attribute exploration, background knowledge.

1 Introduction

In web mining, one of patterns which are discovered from web data is association rule. Mostly association rule is used in mining of web transaction. Web transaction consists of user transactions which are semantically a group of visited pages in one session of a user. An association rule of web transaction shows that if a user visited some certain pages then also visited some other ones. [1]

Some works used the discovered association rules for an effective personalization system [2], a web recommendation [3], and an improvement of the quality of bussines strategies in a commerce [4]. The other works proposed some kinds of association rules, e.g. indirect association rules [3] and association rules of cross transaction web [5]. However, there is no works considering web structures in discovering the association rules where there are possibly some association rules which can be inferred from the web structures.

This paper proposes a method to discover another set of association rules, which is an implicational base of formal context. Furthermore, we will use the result of the research in [6, 7] to discover the association rules by considering the web structures as background knowledge. In that research, background knowledge is used to discard some attribute implications of the implicational base which can be inferred from the other implications together with the background knowledge. Therefore, the discoverd association rules should be free from some association rules which can be inferred from the web structures.

2 Foundations

2.1 Formal Context

Definition 1 (Formal Context) *A **formal context** (G, M, I) consists of two non-empty sets G and M, and a relation $I \subseteq G \times M$. We call the set G a set of objects, whereas the set M a set of attributes. For $g \in G$ and $m \in M$, $(g, m) \in I$ or gIm is read as the object g has the attribute m. [6–9].*

A cross table can represent a formal context. The rows of the cross table represent the objects, and the columns represent the attributes. The headers of the rows are object names, whereas the headers of the columns are attribute names. If an object g has an attribute m, then we cross the table in row g and column m. Fig. 1 is a formal context in cross table.

Definition 2 (Derivation Operator) *If $A \subseteq G$ is a set of objects, then we define [8]:*

$$A^I = \{m \mid (g, m) \in I \text{ for all } g \in A\} \qquad (1)$$

Reversely, if $B \subseteq M$ is a set of attributes, then we define:

$$B^I = \{g \mid (g, m) \in I \text{ for all } m \in B\} \qquad (2)$$

Notation A^{II} refers to $(A^I)^I$.

2.2 Attribute Exploration

Let M a set of attributes in (G, M, I). $A \Rightarrow B$ where $A, B \subseteq M$ is an attribute implication over the formal

	Odd	Even	Square	Prime	2s	3s	4s
1	×		×				
2		×		×	×		
3	×			×		×	
4		×	×		×		×
5	×			×			
6		×			×	×	
7	×			×			
8		×			×		×
9	×		×			×	

Figure 1: Formal context of small natural number.

context. The attribute implication holds in the formal context if each object of the formal context respects the attribute implication. An object $g \in G$ respects the attribute implication iff its attributes set is a model of the implication [6, 7, 9].

Definition 3 (Model of Attribute Implication) *Let $A, B, T \subseteq M$. T is a **model of attribute implication** $A \Rightarrow B$ iff $A \nsubseteq T$ or $B \subseteq T$.*

Definition 4 (Respecting Object) *An object $g \in G$ **respects to** $A \Rightarrow B$ over (G, M, I) iff $\{g\}^I$ is a model of the attribute implication. An object $g \in G$ **respects to** a set $\mathcal{L}$ of attribute implications iff g respects all attribute implications in $\mathcal{L}$.*

Definition 5 (Holding Attribute Implication) *An attribute implication $A \Rightarrow B$ holds in a formal context (G, M, I) iff all $g \in G$ respect the attribute implication. A set $\mathcal{L}$ of attribute implications holds in a formal context (G, M, I) iff all attribute implications in $\mathcal{L}$ holds in (G, M, I).*

Definition 6 (Inference) *An implication $A \Rightarrow B$ can be **inferred** from $\mathcal{L}$, denoted by*

$$\mathcal{L} \vDash A \Rightarrow B,$$

iff all models of $\mathcal{L}$ are also models of $A \Rightarrow B$.

Definition 7 (Implicational Base) *A set $\mathcal{L}$ of attribute implications is an **implicational base** of a formal context, if the followings hold:*

- ***sound**, if $\mathcal{L}$ holds in the formal context.*
- ***complete**, if the following holds. If there is an attribute implication which holds in the formal context, it can be inferred from $\mathcal{L}$.*
- ***non-redundant**, if there is no attribute implication in $\mathcal{L}$ that can be inferred from the others.*

Algorithm: Implicational Base
Input : A formal context (G,M,I)
Output: The implicational base, $\mathcal{L}$

begin
 $X \leftarrow \emptyset$
 $\mathcal{L} \leftarrow \emptyset$
 repeat
  if $(X \neq X^{II})$ ***then***
   $\mathcal{L} \leftarrow \mathcal{L} \cup \{X \Rightarrow X^{II}/X\}$
  $X \leftarrow$ Next_Closure(X) from $\mathcal{L}$
 until $(X = M)$
 return $\mathcal{L}$
end

Figure 2: Implicational Base Algorithm [6, 9]

Fig. 2 shows an algorithm to generate an implicational base of a formal context. **Next_Closure(X) from** $\mathcal{L}$ is the lexically smallest model of $\mathcal{L}$ which is lexically larger than X. Let $A, B \subseteq M = \{m_1, m_2, \ldots, m_n\}$ and $m_1 < m_2 < \ldots < m_n$. We define $A < B$, which means "A smaller than B" or "B larger than A", iff $A <_i B$, which is defined as follows, there is i such that

- $i \notin A$ and $i \in B$, and
- for all $j < i$, $j \in A$ iff $j \in B$.

Example 1 *Recall a formal context in Fig. 1. The implicational base of the formal context generated by algorithm in Fiq. 2 contains the following attribute implications:*

- *{4s} ⇒ {Even, 2s}*
- *{2s} ⇒ {Even}*
- *{Prime, 3s} ⇒ {Odd}*
- *{Square, 3s} ⇒ {Odd}*
- *{Square, Prime} ⇒ {Odd, Even, 2s, 3s, 4s}*
- *{Even} ⇒ {2s}*
- *{Even, 2s, 3s, 4s} ⇒ {Odd, Square, Prime}*
- *{Even, Prime, 2s, 4s} ⇒ {Odd, Square, 3s}*
- *{Even, Square, 2s} ⇒ {4s}*
- *{Odd, Even, 2s} ⇒ {Square, Prime, 3s, 4s}*

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Algorithm: Implicational Base
with Background Knowledge
Input : A formal context (G,M,I)
and background knowledge $\mathcal{H}$
Output: The implicational base, $\mathcal{L}'$

begin
$X \leftarrow \emptyset$
$\mathcal{L} \leftarrow \emptyset$
$\mathcal{L}' \leftarrow \emptyset$
repeat
if $(X \neq X^{II})$ ***then***
$B \leftarrow X^{II} \setminus X$
if not $((\mathcal{L}' \cup \mathcal{H}) \models X \Rightarrow B)$ ***then***
$\mathcal{L}' \leftarrow \mathcal{L}' \cup \{X \Rightarrow B\}$
$X \leftarrow$ Next_Closure(X) from $\mathcal{L}$
until $(X = M)$
return $\mathcal{L}'$
end

Figure 3: Implicational Base Algorithm with Background Knowledge [6, 9]

2.3 Attribute Exploration with Background Knowledge

In [10], attribute exploration with background knowledge is introduced. Instead of grabbing all attribute implications in a implicational base, some attribute implications which can be inferred from the other ones together with the background knowledge are discarded. Let $\mathcal{L}$ a set of attribute implication holding in a formal context, $A \Rightarrow B$ an attribute implication holding also in the formal context, and $\mathcal{H}$ a representation of background knowledge. The attribute implication is discarded if $(\mathcal{L} \cup \mathcal{H}) \models A \Rightarrow B$. Fig. 3 shows a modified algorithm from Fig. 2 by considering the background knowledge. To check whether $(\mathcal{L} \cup \mathcal{H}) \models A \Rightarrow B$ is discussed in [6, 7, 9]

2.4 Constraints as Background Knowledge

Let (G, M, I) a formal context. A constraint $C_{\{x_P\}}$ over the formal context restricts some attributes $P \subseteq M$ for all $g \in G$.

Definition 8 (Constraint) *A **constraint** $C_{\{x_P\}}$ over a formal context (G, M, I) is defined as follows [7]:*

$$C_{\{x_P\}} = \{(< x_P, v_{P_1} >) \mid 1 \leq i \leq n\} \qquad (3)$$

where $v_{P_i} \in 2^P$ and $P \subseteq M$.

Definition 9 (Satisfying Object) *An object $g \in G$ of formal context (G, M, I) **satisfies** a constraint $C_{\{x_P\}}$ where $P \subseteq M$ iff a compound label $(< x_P, g^I \cap P >) \in C_{\{x_P\}}$.*

Definition 10 (Satisfying Formal-Context) *A formal context (G, M, I) **satisfies** a constraint $C_{\{x_P\}}$ iff for all $g \in G$, g satisfies the constraint.*

A constraint $C_{\{x_P\}}$ can be represented as a formal context (G_P, M_P, I_P) which is defined as follows [7]:

- $G_P = C_{\{x_P\}}$
- $M_P = P$
- $(g, m) \in I_P$ where $g = (< x_P, v_P >) \in G_P$ and $m \in M_P$ iff $m \in v_P$

Example 2 *Recall a formal context in Fig.1. From our knowledge, there are some constraints which the formal context satisfies. The constraints are for attributes P_1={Odd, Even}, P_2={Even, 2s}, and P_3={2s,4s}. The following formal contexts represent the constraints, respectively:*

	Odd	*Even*
$(< x_{P_1}, \{Odd\} >)$	×	
$(< x_{P_1}, \{Even\} >)$		×

	Even	*2s*
$(< x_{P_2}, \emptyset >)$		
$(< x_{P_2}, \{Even, 2s\} >)$	×	×

	2s	*4s*
$(< x_{P_3}, \emptyset >)$		
$(< x_{P_3}, \{2s\} >)$	×	
$(< x_{P_3}, \{2s, 4s\} >)$	×	×

We discard some attribute implications which can be inferred from other ones together with the constraints. The remaining attribute implications are follows:

- {*Prime,3s*} $\Rightarrow$ {*Odd*}
- {*Square,3s*} $\Rightarrow$ {*Odd*}
- {*Square,Prime*} $\Rightarrow$ {*Odd,Even,2s,3s,4s*}
- {*Even,2s,3s,4s*} $\Rightarrow$ {*Odd,Prime,Square*}
- {*Even,Square,2s,4s*} $\Rightarrow$ {*Odd,Prime,3s*}
- {*Even,Square,2s*} $\Rightarrow$ {*4s*}

Session id	Pages	Session id	Pages
1	d_1, d_2, d_4, d_6	6	d_2, d_4
2	d_1, d_4, d_6	7	d_4, d_5, d_6
3	d_1, d_2, d_4, d_6	8	d_2, d_4, d_5, d_6
4	d_1, d_3	9	d_1, d_4, d_6
5	d_2, d_4, d_5, d_6	10	d_1, d_3

Table 1: User sessions example

3 Association Rules with Background Knowledge

A set of association rules is a representation of knowledge extracted from web transaction. Association rules show that if a user visited some certain pages then the user also visited some other ones.

3.1 Web Transaction and Association Rules

Data of web transaction is a history of visited pages of users. There are some methods to identify a unique user. One of the methods is identifying by user session [3]. Table 1 shows an example of web transaction where a unique user identified by user session. The table is a modification of an example in [3]. The pages column shows all visited pages of a session. However, we do not consider which method is used in identifying since in general we can transform every data of web transaction into that table.

Let a transaction of web transaction be in the form of $< id, pages >$ where id is natural number of user sesssion id and $pages$ is a set of visited pages. The data of web transaction is a set $\mathcal{S} = \{< i, D_i >| \ i = 1, 2, \ldots, n\}$. $\mathcal{S}$ can be represented by a formal context (G, M, I) which is defined as follows:

- $G = \{1, 2, \ldots, n\}$,
- $M = D_1 \cup D_2 \cup \ldots \cup D_n$, and
- $(g, m) \in I$ where $g \in G$ and $m \in M$ iff $m \in D_g$.

In the other words, G, M, and I represent user session id, web pages, and relation between user session id and web pages, respectively. $(g, m) \in I$ means "user session id g visited page m"'. Fig. 4 shows a formal context representing the web transaction in Table 1.

With this representation, an association rule will be an attribute implication of the formal context. In this case, an attribute implication $A \Rightarrow B$ means "all users that visited all pages in A also visited all pages in B"'. Therefore, all sound, complete, and non-redundant association rules of web transaction can be obtained from the implicational base generated by the algorithm in Fig. 2.

	d_1	d_2	d_3	d_4	d_5	d_6
1	×	×		×		×
2	×			×		×
3	×	×		×		×
4	×		×			
5		×		×	×	×
6		×		×		
7				×	×	×
8		×		×	×	×
9	×			×		×
10	×		×			

Figure 4: Formal context of user sessions in Table 1

Example 3 *Sound, complete, and non-redundant association rules from the formal context in Fig.4 are follows:*

- $\{d_6\} \Rightarrow \{d_4\}$
- $\{d_5\} \Rightarrow \{d_4, d_6\}$
- $\{d_3\} \Rightarrow \{d_1\}$
- $\{d_2\} \Rightarrow \{d_4\}$
- $\{d_1, d_4\} \Rightarrow \{d_6\}$
- $\{d_1, d_4, d_5, d_6\} \Rightarrow \{d_2, d_3\}$
- $\{d_1, d_3, d_4, d_6\} \Rightarrow \{d_2, d_5\}$

3.2 Web Structure as Constraints

A web structure is a structure of a typical web graph consisting of web pages as nodes and hyperlinks as edges connecting between two related pages. In some situation, a web structure possibly restricts how a user visits some web pages. Thus, a web structure is possibly a constraint of visiting some pages. There are 5 interesting web structures [11]. The web structures and their representations as constraints in formal contexts are follows:

- Endorsement

With this structure, we know that page q can be reached from page p. Thus, this web structure shows a restriction between page p and page q. The constraint for the restriction is follows:

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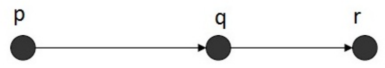
	p	q
$(< x_{\{p,q\}}, \emptyset >)$		
$(< x_{\{p,q\}}, \{p\} >)$	$\times$	
$(< x_{\{p,q\}}, \{p,q\} >)$	$\times$	$\times$

- Transitive endorsement



We can split the structure into two endorsement web structures. Therefore, we represent this structure by two constraints. The first constraint is between page p and q, whereas the second one is between q and r. The followings are both constraints:

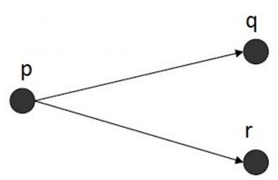
	p	q
$(< x_{\{p,q\}}, \emptyset >)$		
$(< x_{\{p,q\}}, \{p\} >)$	$\times$	
$(< x_{\{p,q\}}, \{p,q\} >)$	$\times$	$\times$

	q	r
$(< x_{\{q,r\}}, \emptyset >)$		
$(< x_{\{q,r\}}, \{q\} >)$	$\times$	
$(< x_{\{q,r\}}, \{q,r\} >)$	$\times$	$\times$

- Mutual reinforcement

Since we can reach an page from the other one, this web structure does not give a restriction between both.

- Co-citation

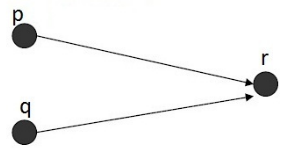


We can also split this web structure into two endorsement web structures. The first web structure is between p and q, whereas the second one is between p and r. The followings are the constraints of both:

	p	q
$(< x_{\{p,q\}}, \emptyset >)$		
$(< x_{\{p,q\}}, \{p\} >)$	$\times$	
$(< x_{\{p,q\}}, \{p,q\} >)$	$\times$	$\times$

	p	r
$(< x_{\{p,r\}}, \emptyset >)$		
$(< x_{\{p,r\}}, \{p\} >)$	$\times$	
$(< x_{\{p,r\}}, \{p,r\} >)$	$\times$	$\times$

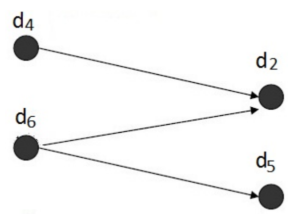
- Social choice



This web structure shows that page r can be reached from either page p or page q. Thus, the constraint for this web structure is follows:

	p	q	r
$(< x_{\{p,q,r\}}, \emptyset >)$			
$(< x_{\{p,q,r\}}, \{p\} >)$	$\times$		
$(< x_{\{p,q,r\}}, \{q\} >)$		$\times$	
$(< x_{\{p,q,r\}}, \{p,q\} >)$	$\times$	$\times$	
$(< x_{\{p,q,r\}}, \{p,r\} >)$	$\times$		$\times$
$(< x_{\{p,q,r\}}, \{q,r\} >)$		$\times$	$\times$
$(< x_{\{p,q,r\}}, \{p,q,r\} >)$	$\times$	$\times$	$\times$

Example 4 *Recall the web transaction in Table 1. Suppose we have a following web structure related to the web transaction:*



We can split the web structure into two web structures, i.e: an endorsement web structure of page d_5 and page d_6, and a social-choice web structure of pages d_2,d_4, and d_6. Both web structures are represented by two constraints as follows:

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	d_5	d_6
1		
2		$\times$
3	$\times$	$\times$

a. Constraint for $\{d_5, d_6\}$

	d_2	d_4	d_6
1			
2			$\times$
3		$\times$	
4		$\times$	$\times$
5	$\times$		$\times$
6	$\times$	$\times$	
7	$\times$	$\times$	$\times$

b. Constraint for $\{d_2, d_4, d_6\}$

3.3 Web Structures as Background Knowledge

By definition, web structures are information which already exist for a web. Therefore, we can consider them as background knowledge of web transaction. Furthermore, we can view them as background knowledge in the form of constraints since web structures possibly restrict user in visiting some pages.

However, a web transaction either satisfies the constraints or not. If a web transaction satisfies some constraints of web structures, we will discard some generated association rules which can be inferred from other association rules together with the constraints. We apply algorithm in Fig. 3 to solve this.

Recall Example 3 and Example 4. There is an association rule $\{d_2\} \Rightarrow \{d_4\}$ which means "all user that visited page d_2 also visited page d_4". Although from the related web structures we know that d_2 can be reached from either page d_4 or d_6, the association rule is still interesting since the association rule shows that all user visited page d_2 from d_4 instead of from page d_6. However, we also have an association rule $\{d_6\} \Rightarrow \{d_4\}$, which means "all user that visited page d_6 also visited page d_4". Together with the constraints, it implies that possibly some users visited d_2 from d_6 also. Thus, we can not conclude that all user visited page d_2 from d_4 instead of from page d_6. Therefore, association rule $\{d_2\} \Rightarrow \{d_4\}$ is not interesting and should be discarded.

Another problem is for association rule $\{d_5\} \Rightarrow \{d_4, d_6\}$. From the constraints we know that page d_5 is reached from page d_6. It implies that if a user visited page d_5 then the user also visited page d_6. Together with another association rule, which is $\{d_6\} \Rightarrow \{d_4\}$, the constraints implies the association rule $\{d_5\} \Rightarrow \{d_4, d_6\}$. Thus, the association rule is not real information from the web transaction. Therefore, it should be discarded.

Example 5 shows the remaining association rules after discarding some association rules inferred from the others together with constraints.

Example 5 *It is trivial to check that the formal context in Fig. 4 satisfies all constraints in Example 4. By applying the algorithm in Fig. 3, we obtain this following association rules:*

- $\{d_6\} \Rightarrow \{d_4\}$
- $\{d_3\} \Rightarrow \{d_1\}$
- $\{d_1, d_4\} \Rightarrow \{d_6\}$
- $\{d_1, d_4, d_5, d_6\} \Rightarrow \{d_2, d_3\}$
- $\{d_1, d_3, d_4, d_6\} \Rightarrow \{d_2, d_5\}$

3.4 Support and Confidence

Two issues of association rules are support and confidence. Confidence denotes the strength of implication and support indicates the frequency of the patterns occurring in the rule [12]. Support and confidence of association rule $A \Rightarrow B$ in web transaction $\mathcal{S}$ represented by formal context (G, M, I) are defined as follows:

$$support(A \Rightarrow B) = \frac{\left|\{g \mid (A \cup B) \subseteq \{g\}^I\}\right|}{|G|}$$

$$confidence(A \Rightarrow B) = \frac{\left|\{g \mid (A \cup B) \subseteq \{g\}^I\}\right|}{|\{g \mid A \subseteq \{g\}^I\}|}$$

Example 6 *Recall Example 5 and Table 1. Support and confidence of the association rules are follows:*

- $\{d_6\} \Rightarrow \{d_4\}$*, support = 60%, confidence = 100%.*
- $\{d_3\} \Rightarrow \{d_1\}$*, support = 20%, confidence = 100%.*
- $\{d_1, d_4\} \Rightarrow \{d_6\}$*, support = 40%, confidence = 100%.*
- $\{d_1, d_4, d_5, d_6\} \Rightarrow \{d_2, d_3\}$*, support = 0%, confidence =* ∞*.*
- $\{d_1, d_3, d_4, d_6\} \Rightarrow \{d_2, d_5\}$*, support = 0%, confidence =* ∞*.*

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From Example 6, there are some association rules whose support are 0%. We have to discard the such association rules. However, from the example, we can conclude that there are possibly some association rules whose support are 0%.

Proposition 11 *Let* $A \Rightarrow B$ *an association rule holding in a formal context* (G, M, I). *If* $support(A \Rightarrow B) > 0$ *then* $confidence(A \Rightarrow B) = 100\%$.

Proof: From definition of derivation operator, we obtain:

$$\begin{aligned}&\{g \mid (A \cup B) \subseteq \{g\}^I\}\\&= \{g \mid (g, m) \in I \text{ for all } m \in (A \cup B)\}\\&= (A \cup B)^I\\&\{g \mid A \subseteq \{g\}^I\}\\&= \{g \mid (g, m) \in I \text{ for all } m \in A\}\\&= A^I\end{aligned}$$

From algorithm either in Fig. 2 or in Fig. 3, we know that $B = A^{II} \setminus A$. Since function $.^{II}$ is a closure operator in formal context, $A \subseteq A^{II}$ [8, 9]. Thus, $A \cup B = A^{II}$. Therefore,

$$confidence(A \Rightarrow B) = \frac{\left|(A \cup B)^I\right|}{|A^I|} = \frac{\left|A^{III}\right|}{|A^I|}$$

It is proved in [13] that $A^I = A^{III}$. Hence,

$$confidence(A \Rightarrow B) = \frac{\left|A^I\right|}{|A^I|}$$

Since $support(A \Rightarrow B) > 0$, it implies $\left|A^I\right| = \left|(A \cup B)^I\right| > 0$. Consequently, $confidence(A \Rightarrow B) = 100\%$ □

4 Conclusion

We have presented a new method to generate a set of association rules, which are sound, complete, and non-redundant, from web transaction. Each association rule in the set can also not be inferred from the other ones together with background knowledge of web structures. However, if support of an association rule is greather than 0, then it's confidence is 100%. It will become a new challenge in this research area since in web mining we sometime accept an association rule whose confidence is less than 100% and greater than a predetermined minimum treshold.

For the future work, it is interesting to modify the method such that it is able to generate some association rules whose confidence are greater than a predetermind minimum treshold. The other future work is conducting some experiments of real worlds related to web mining with background knowledge of web structure.

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