Fractional order control of the Thomson’s ring magnetic levitation system

MANUEL A. DUARTE-MERMOUD
Department of Electrical Engineering and Advanced Mining Technology Center
University of Chile
Av. Tupper 2007, Santiago
CHILE
mduarm@ing.uchile.cl

NORELYS AGUILA-CAMACHO
Department of Electrical Engineering and Advanced Mining Technology Center
University of Chile
Av. Tupper 2007, Santiago
CHILE
naguila@ing.uchile.cl

RAFAEL CASTRO-LINARES
Department of Electrical Engineering, Mechatronic Section
CINVESTAV
Av. IPN No. 2508, Col. San Pedro Zacatenco, 07340
MEXICO
rastro@cinvestav.mx

Abstract: This work presents the position control of a magnetic levitation system, known as Thomson’s ring. The design and implementation of two control strategies are presented; the first one corresponds to a fractional order proportional-integral-derivative controller (FOPID) and the second corresponds to a sliding mode control. In both cases, a fractional order observer is used to estimate the ring speed. A particle swarm optimization (PSO) technique is used in the case of the FOPID, in order to select the optimal parameters of the controller. Stabilization and tracking experiments are carried out in order to show the behavior of the controlled system, and the effect of external disturbances on the output of the system it is also addressed.

Key–Words: Magnetic levitation systems, Fractional order PID controllers, Sliding mode control, Fractional order observers, PSO, Regulation, Tracking.

1 Introduction

The materials transport is an important problem in the manufacturing industry. In some cases, the complexity of the transport requirements has led to the need of designing specific transport systems, e.g. minimizing the friction between working surfaces.

The levitation systems (MagLev) have low or non-existent friction side effects, and also have the advantage of operating with low noise levels and the possibility of operating in high vacuum environments. Besides their advantages for the materials transport, the MagLev systems can be applied to other important areas such as microrobotics [8], photolithography [7] and launching systems [4].

In general, a MagLev system can be classified, according to the forces considered on it, as repulsive systems or attractive systems [11]. These systems are also highly nonlinear and unstable in open loop, and for that reason they require control systems to achieve a closed loop stable operation.

Particularly, the MagLev known as “Thomson’s ring” was created by Elihu Thomson (1853–1937), and it is composed by an induction coil with a ferromagnetic core and a ring placed in the core (see Figure 1, where a detailed diagram of the system is shown). When an alternate current flows through the coil, a current is induced in the ring. The magnetic field due to this induced current opposes to the magnetic field induced by the coil, producing a repulsive force between the two elements, and in that way the ring rises above the coil core [1].

As it was mentioned before, the Thomson’s ring is an open loop unstable system, and for that reason a control strategy to stabilize the ring in a desired position and to follow a desired trajectory is needed. Some control strategies for this system have been reported in the literature. We can mention the work by García-Antonio et al. [3], where the synchronization of two Thomson’s ring modules is addressed, using synchronization techniques proposed in mobile robotics and sliding mode control. More recently, in the work by Ramírez-Neira et al. [10], a linear control of the ring
position is addressed, using an active disturbance rejection technique, based on generalized proportional-integral observers.

This paper presents the control of a Thomson’s ring, using fractional order control techniques. Particularly, two control schemes are proposed: a FOPID and a sliding mode controller. A fractional order observer is also proposed, in order to estimate the ring speed, which is used in the control techniques. The parameters of the FOPID are optimized using particle swarm optimization (PSO), and the behavior of both controllers is showed by numerical simulations, comparing the obtained results. Some robustness experiments are also addressed, where the influence of external disturbances in the system output is considered.

The paper is organized as follows. Section 2 describes the Thomson’s ring system and its dynamical model. Next, some concepts of fractional calculus are presented in Section 3. The design of the fractional order observer, the FOPID and the sliding mode controller are presented and discussed in Section 4, as well as the behavior through numerical simulations. Section 5 shows the behavior of the two controllers in the presence of external disturbances; and finally, Section 6 presents the conclusions of the work.

2 Thomson’s ring model

Let us consider the Thomson’s ring shown in Figure 1, where the ring is a ferromagnetic material and the ring is made of an electrical non-magnetic conductor (e.g. aluminum). A sinusoidal input voltage $V_c$ produces an input current $I_c$ in the coil. As a result, a levitation force is generated, which opposes the effect of the gravity. It can be shown ([1] and [2]) that the dynamic of the corresponding ring motion is described by the following second order differential equation:

$$\ddot{z} = -g + K \frac{V_c^2}{m |Z'_c|^2} \dot{z} \tag{1}$$

where $m$ is the ring mass, $g$ is the gravity acceleration and $V_c$ is the input voltage, acting as the control input to the system. $Z'_c$ is the ring impedance given by

$$Z'_c = \sqrt{(R'_c)^2 + (\omega L'_c)^2} \tag{2}$$

where $\omega$ is the frequency of the sinusoidal voltage signal applied to the coil and

$$R'_c = R_c - \frac{M^2 \omega^2}{|Z'_c|} \cos (\phi_R) \tag{3}$$

$$L'_c = L_c + \frac{M^2 \omega}{|Z'_c|} \sin (\phi_R)$$

In these equations, $R_c$ and $L_c$ represent the resistance and the inductance of the coil, respectively. $M$ is the mutual induction coefficient of the coil-ring system for a fixed distance $z = Z$; $\phi_R$ is the offset produced by the ring’s impedance given by $\phi_R = \arctan \left( \frac{\omega L_c}{R_c} \right)$, being $R_r$ and $L_r$ the resistance and the inductance of the ring. $Z_r = \sqrt{R_r^2 + (\omega L_r)^2}$ represents the impedance of the ring.

The parameter $K$ in Equation (1) is given by

$$K = \frac{M^2 \omega}{2 |Z'_c|} \sin (\phi_R) \tag{4}$$

It can be seen that this parameter has a rather complex dependence on the magnetic field, the core and ring electrical circuits and the ring position respect to the upper side of the coil. In this work, it is considered that $K$ is a constant determined by the nominal equilibrium conditions. Nevertheless, in a real operation and specially in time varying tracking reference problems, the parameter $K$ shows a notorious variation that can’t be easily measured.

In this work, the parameters of a real Thomson’s ring system will be used, with the main objective to apply, in a second stage, the proposed control techniques at laboratory level. The values of the real parameters are given in Table 1 [2].
Choosing the state variables $x_1 = z$, $x_2 = \dot{z}$, the state variable representation of the system (1) is given by

$$
\dot{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -g + \frac{x_2}{m |Z_c|^2} u \\ 0 \end{bmatrix}
$$

(5)

where $x = [x_1 \ x_2]^T$ is the state and $u = V_c^2$ is the input to the system. The output of the system corresponds to the ring position i.e. $y = x_1$. If a constant input $u = \bar{u}$ is considered, the equilibrium point of system (5) is given by:

$$
\bar{x} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{K \bar{u}}{mg |Z_c|^2} \\ 0 \end{bmatrix}
$$

(6)

### 3 General concepts of fractional calculus

The fractional calculus is the field that studies integrals and derivatives of orders that can be real or complex numbers [6].

In the time domain, the fractional derivative and fractional integral are defined by a convolution operation, and that is the reason why they are specially useful to describe some phenomena involving storage or memory. In the Laplace domain, those operations correspond to the operator $s^\alpha$, $\alpha \in \mathbb{R}$ [6].

The fractional calculus has gained considerably popularity during the last years in the field of applications in science and engineering. This is because some concepts of the traditional calculus that are used inside control strategies have been generalized, and in that way designers have found more general solutions having a better performance. It should be mentioned that in FOPID, besides the proportional, integral and derivative gains, the order of the integral and the derivative parts are also to be chosen.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_r$</td>
<td>$0.20816 \times 10^{-3}$ $\Omega$</td>
</tr>
<tr>
<td>$L_r$</td>
<td>$2.07023 \times 10^{-6}$ $H$</td>
</tr>
<tr>
<td>$M_z$</td>
<td>$36.2 \times 10^{-6}$ $H$</td>
</tr>
<tr>
<td>$V_c$</td>
<td>$53.768$ $V$</td>
</tr>
<tr>
<td>$m$</td>
<td>$1.4482 \times 10^{-3}$ $Kg$</td>
</tr>
<tr>
<td>$R_c$</td>
<td>$14.5$ $\Omega$</td>
</tr>
<tr>
<td>$L_c$</td>
<td>$128 \times 10^{-3}$ $H$</td>
</tr>
</tbody>
</table>

Table 1: Nominal values of the Thomson’s ring parameters

In what follows, some definitions of the fractional calculus are presented, which are used along this work.

**Definition 1 ([6])** The Riemann-Liouville fractional integral of a function $f(t)$, defined in a finite interval in $\mathbb{R}^+$ is given as follows:

$$
I^\alpha_a f(t) = \frac{1}{\Gamma (\alpha)} \int_a^t \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau
$$

(7)

where $t > a$, $\Re (\alpha) > 0$ and $\Gamma (\alpha)$ corresponds to the Gamma function [6].

Regarding the fractional derivative, there exist several definitions in the literature. This work uses the Caputo fractional derivative, because it uses initial conditions of the function and its integer order derivatives, which can be physically interpreted in the traditional way.

**Definition 2 ([6])** The Caputo fractional derivative of order $\alpha \in \mathbb{R}^+$ of a function $f(t)$, defined in a finite time in $\mathbb{R}^+$, is given as follows:

$$
C D^\alpha_a f(t) = \frac{1}{\Gamma (n-\alpha)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau
$$

(8)

where $t > a$ and $n = \min \{ k \in \mathbb{N} / k > \alpha \}$, $\alpha > 0$.

### 4 Design, implementation and behavior of the control techniques

This section first addresses first the design of a fractional order observer, in order to estimate the speed of the ring to be used inside the control techniques. After that, the design of a FOPID controller and a sliding mode controller is presented. The corresponding behavior for stabilization problem and reference tracking problem are simulated.

#### 4.1 Design of a fractional order observer for the speed of the ring

The first problem that arises when proposing a control strategy for the system (5), is the fact that the only state variable that can be measured is the ring position $x_1$. For that reason, this subsection presents the design of a state observer, with the main objective of having an estimation of the state variable $x_2$ (ring speed) to be used later in the design of control techniques.
Since the system parameters and model structure are assumed to be known, it is relatively simple to construct an observer. From the non linear model of system (5) with output \( y = x_1 \), the following fractional order state observer is proposed

\[
\begin{align*}
C^D a \dot{x}_1 &= \dot{x}_2 + l_1 (\dot{x}_1 - x_1) \\
C^D a \dot{x}_2 &= -g + \frac{K}{m |Z_c|^2} u + l_2 (\dot{x}_1 - x_1)
\end{align*}
\]
\( \dot{y} = \dot{x}_1 \)

where \( 0 < \alpha < 2 \) and the parameters \( l_1, l_2 \) will be defined later. Defining the state estimation errors as

\[
\begin{align*}
e_1 &= \dot{x}_1 - x_1 \\
e_2 &= \dot{x}_2 - x_2
\end{align*}
\]

the equations describing the evolution of the errors (10) can be written as

\[
\begin{bmatrix}
C^D a^{\alpha} \dot{x}_1 - \dot{x}_1 \\
C^D a^{\alpha} \dot{x}_2 - \dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
l_1 & 1 \\
l_2 & 0
\end{bmatrix}
\begin{bmatrix}
e_1 \\
e_2
\end{bmatrix}
\]

(11)

The equation (11) does not have a known structure, since a fractional order observer has been proposed for an integer order system. On the other hand, there is no analytical proof reported in literature for the stability and convergence of the estimator (9). However, through simulation studies it has been observed that if the parameters \( l_1 \) and \( l_2 \) are selected such that the matrix

\[
\begin{bmatrix}
l_1 & 1 \\
l_2 & 0
\end{bmatrix}
\]

(12)

has real negatives eigenvalues, then the estimation errors (10) converge to zero as \( t \) tends to infinity. The roots of the characteristic equation of matrix (12) are given by

\[
r_{1,2} = \frac{l_1 \pm \sqrt{l_1^2 + 4l_2}}{2}
\]

(13)

In order to have real negative eigenvalues, it must be guaranteed that \( l_1^2 + 4l_2 \geq 0, l_1 < 0 \) and \( \sqrt{l_1^2 + 4l_2} < |l_1| \). In this study the following values were selected \( l_1 = -40, l_2 = -400 \), corresponding to eigenvalues of matrix (12) located at \( -20 \).

4.2 Design of the fractional order proportional integral and derivative controller (FOPID)

The proportional integral and derivative controller (PID) is one of the most used in process control, since it is simple and useful for linear and non linear systems. With the introduction of the fractional operators in the control field, the FOPID controller arises, where not only the proportional, integral and derivative gains are to be chosen by the designer, but also the order of the fractional integral and the order of the fractional derivative. It can be mentioned as one of many examples, the work by Zamani et al. [12].

As first control strategy for the Thomson’s ring, a FOPID is proposed. The FOPID input is the control error, corresponding to \( e(t) = r(t) - x_1(t) \), where \( r(t) \) is the desired reference for the ring position, and it generates a control signal \( u(t) \) from equation

\[
u(t) = k_P e(t) + k_I \Gamma e(t) + k_D C^D a \beta e(t)
\]

(14)

In order to find the gains \( k_P, k_I, k_D \), the integration order \( \gamma \) and the derivative order \( \beta \), an optimization procedure is performed using particle swarm optimization (PSO) [9]. The objective function to minimize in this problem is chosen as

\[
J = M_p + E_{ss} + tr + ts + \int_0^T |r(t) - x_1(t)| dt
\]

(15)

where \( M_p \) corresponds to the overshoot, \( E_{ss} \) is the steady state error, \( tr \) is the rising time and \( ts \) is the settling time. The computation of the parameters in the objective function was done by applying a 15 mm step reference to the control scheme, in a time window of 100 seconds. The integration and derivative orders were selected in the interval (0, 2). The integral term in (15) corresponds to the integral of the absolute value of the control error, with \( T = 100 \) seconds.

The most relevant parameters used for the PSO algorithm were:

- Swarm size: 100
- Number of iterations: 300
- Initial inertia weight: 0.9
- Final inertia weight: 0.4

As a result of this optimization process, the following controller parameters were obtained:

\[
\begin{align*}
k_P &= 7.6690 \times 10^4 \\
k_I &= 12.013 \times 10^4 \\
k_D &= 7.801 \times 10^4 \\
\gamma &= 1 \\
\beta &= 0.97
\end{align*}
\]

(16)

As can be seen, the optimization process gave as the best result a controller with an integer integral part and a fractional derivative component, but pretty close to 1. Thus, the fractional derivative of the error is needed in order to implement the control scheme. To do this, one option could be to derive directly the error, but the negative influence of the derivation process in presence of noises is well known. To avoid
that, the fractional derivative was implemented using the output of the fractional order observer designed in Section 4.1. According to the definition of the control error, $\dot{e}(t) = \dot{r}(t) - \dot{x}_1(t)$. Since $\dot{x}_1(t) = x_2(t)$, then the first derivative of the error can be constructed using the first derivative of the reference signal and the signal $x_2$, which is obtained from the state observer. This is the approach followed in this study.

### 4.3 Design of the sliding mode controller

The second control technique implemented for the Thomson’s ring was a sliding mode controller, since this is a control technique specially developed for nonlinear systems. To design the controller, the following change of variable is considered for the model of the system (5)

$$
e_1 = x_1 - x_{1d}$$

$$
e_2 = x_2 - x_{2d}$$

(17)

where $x_{1d}, x_{2d}$ correspond to the desired values for the position and the speed of the ring, respectively. The system can be now described as

$$
\dot{e}_1 = e_2
$$

$$\dot{e}_2 = -g + \frac{K}{m|Z_c|^2} \frac{u}{e_1 + x_{1d}} - \dot{x}_{2d}
$$

(18)

$$
y_c = e_1
$$

In order to design a sliding mode control to stabilize system (18), we choose first a function $f(e_1)$ such that for the system

$$
e_2 = f(e_1)
$$

(19)

results that $\lim_{t \to \infty} e_1 = 0$. To do this, let us consider the following Lyapunov function candidate

$$
V = \frac{1}{2} e_1^2
$$

(20)

The first derivative of $V$ along the system (18) results

$$
\dot{V} = e_1 \dot{e}_1 = e_1 e_2
$$

(21)

Then if we choose $e_2 = f(e_1) = -k_1 e_1$ with $k > 0$, the first derivative of the Lyapunov function becomes

$$
\dot{V} = -k_1 e_1^2
$$

(22)

which is negative definite, and it can be concluded that $\lim_{t \to \infty} e_1 = 0$.

Let us now design the sliding surface, in such a way that the convergence of the system to the sliding surface can be guaranteed in a finite time. To do that, let us propose the following sliding surface

$$
s = e_2 + k_1 e_1
$$

(23)

This implies that

$$
\dot{s} = \dot{e}_2 + k_1 \dot{e}_1
$$

(24)

Using (18) in (24) results

$$
\dot{s} = -g + \frac{K}{m|Z_c|^2} \frac{u}{e_1 + x_{1d}} - \dot{x}_{2d} + k_1 e_2
$$

(25)

In order to select the control signal $u$ to make $\lim_{t \to \infty} s = 0$, we choose the following Lyapunov function candidate

$$
V = \frac{1}{2} s^2
$$

(26)

Taking its first derivative along (25) results

$$
\dot{V} = s \dot{s} = s \left[ -g + \frac{K}{m|Z_c|^2} \frac{u}{e_1 + x_{1d}} - \dot{x}_{2d} + k_1 e_2 \right]
$$

(27)

Then if we choose the control signal as

$$
u = (e_1 + x_{1d}) m|Z_c|^2 \left[ g - k_1 e_2 + \dot{x}_{2d} - B \text{sgn}(s) \right]
$$

(28)

where $B > 0$ is a design parameter to handle the convergence speed, resulting

$$
\dot{V} = -B|s|
$$

(29)

As can be seen, $\dot{V}$ is negative definite and it implies that $\lim_{t \to \infty} s = 0$.

With the control signal defined in (28), the control scheme was implemented. However, given that $e_2$ is needed to build the control signal, then the fractional observer designed in Section 4.1 was used to implement $e_2$, in the same way explained in Section 4.2 for the FOPID controller. The design parameters $k_1, B$ were selected as $k_1 = B = 25$, in order to obtain a good convergence speed of the errors to zero. The hyperbolic sine function was used instead of the sign function, in order to avoid the chattering effects in the control signal that are usual in this kind of controller [5].

### 4.4 Numerical simulations

In order to verify the behavior of the proposed control schemes, a first experiment was performed, where the control goal is to stabilize the ring in a desired position. Figure 2 shows the behavior of the FOPID controller and Figure 3 shows the behavior of the sliding mode controller, when a 15 mm step reference is applied. The results of several experiments were plotted,
Figure 2: Behavior of the controlled system using FOPID.

Figure 3: Behavior of the controlled system using sliding mode control (28).

for different values of the order \( \alpha \) for the fractional observer.

As can be seen from Figures 2 and 3, the stabilization is achieved in both cases, no matter the order of the fractional observer. In the case of the FOPID, all the responses are very similar. It is important to point out that for the FOPID, only fractional observers with orders \( \alpha \leq 1 \) were used, since using orders \( \alpha > 1 \) lead to a very oscillating transient behavior in the control signal.

In the case of the sliding mode controller, the settling time is shorter than in the case of the FOPID, although the transient response is different depending on the fractional order observer used.

A second experiment was carried out, where the control goal was tracking a sinusoidal reference signal centered in 15 mm, with amplitude 5 mm and frequency \( \pi/6 \text{ rad/s} \). Figure 4 shows the behavior of the system using FOPID, and in this case the control error has been plotted instead of the system output.

As can be seen from Figure 4, even tough the error remains bounded, it has a considerable magnitude for the FOPID, since the parameter optimization process was carried out for a step reference. If a new parameter optimization procedure is carried out using a sinusoidal reference signal, then different parameters will be found for the FOPID controller. This was done using in this case a different objective function, since most of the parameters in (15) has no meaning if the reference is different from a step. In this new case, the objective function (30) was used, and the resulting parameters are shown in (31).

\[
J = \int_0^T t |r(t) - x_1(t)| dt. \tag{30}
\]
Figure 5: Behavior of the controlled system using FOPID, when a sinusoidal reference signal is applied.

\[
\begin{align*}
 k_P &= 1.7754 \times 10^6 & k_I &= 0.1352 \times 10^6 \\
 k_D &= 0.2213 \times 10^6 & \gamma &= 1 & \beta &= 0.97 \\
\end{align*}
\] (31)

With this new set of parameters, a better tracking of the sinusoidal reference signal is achieved, as can be seen in Figure 5. Although there is still an error its magnitude is rather small. This second set of parameters (31) also offers good results in the case of a step reference, but the control effort is higher than that obtained for the previous set of parameters.

Figure 6 shows the evolution of the position error when the reference signal is sinusoidal, using sliding mode control. As can be seen from Figure 6, the error remains bounded, and it does not converge to zero neither, as in the case of the FOPID. However, we should mention that in this case the magnitudes of the error are lower than for the case of the FOPID, and also with no modifications in the design of the control scheme.

5 Behavior of the controllers in the presence of external disturbances

As it is seen from previous experiments, both control strategies can achieve a satisfactory performance in stabilization problems. In the case of tracking sinusoidal references, there is always a steady state error, although the magnitudes of this error can be decreased if some design parameters are suitably manipulated. This behavior was similar for every order \( \alpha \) used in the fractional observer.

However, some robustness experiments under disturbances were carried out, in order to test the behavior of the controllers depending on the order \( \alpha \) used in the fractional observer. In one of this experiments, once the system is stabilized in 15 mm, a sinusoidal disturbance is applied to the system output, with amplitude 1 mm and frequency \( \pi/2 \), specifically at the time instant \( t = 60 \) seconds. In the case of the FOPID the parameter values in (16) were chosen, and in the case of the sliding mode controller the design parameters were \( k_1 = B = 25 \).

In order to evaluate the behavior of the controlled systems under this external disturbance, the integral of the squared error is plotted in Figure 7, for both control strategies and for every order \( \alpha \) used in the fractional observer.

As can be observed, in the case of using FOPID the ISE is pretty similar for every order \( \alpha \) used for the fractional observer. However, in the case of the sliding mode control, the lower ISE is achieved with the fractional order \( \alpha = 1.1 \), that is to say, the controlled system is more robust in the presence of this external disturbance when the observer is of fractional order.

From this simple analysis, it can be deduced that the introduction of the fractional operators in the control schemes can effectively make the system more robust under external disturbances, although the choice of this fractional order depends not only on the sys-
Figure 7: Integral of the squared error when a sinusoidal external disturbance is applied to the system output.

tem under control but also on the specific control technique used.

6 Conclusions

This work presents the position control in a Thomson’s ring magnetic levitation system. In order to achieve this goal, two control strategies were designed, implemented and evaluated. The first one corresponds to a FOPID and the second one to a sliding mode controller. In both cases, a fractional order observer is used to estimate the speed of the ring. The parameters of the FOPID were obtained through an optimization problem using PSO, with an objective function depending on the problem addressed (stabilization or tracking a sinusoidal reference). Both control strategies resulted in a good performance in stabilization problems, and for the case of tracking a sinusoidal reference, a steady state error was obtained, though the magnitude can be diminished by handling some design parameters of the controllers. The use of a fractional order in the observer lead to a more robust controlled system, when a sinusoidal external disturbance was applied to the system output. All the obtained results motivate implementing these control techniques in the real MagLev system, which will be our next step.

Acknowledgements: The results reported in this work have been supported by CONICYT-Chile under the Fondecy Project 1120453 "Improvements of Adaptive Systems Performance by using Fractional Observers and Particle Swarm Optimization", under the Basal Project FB0809 "Centro de Tecnologia para la Minera and under the CONICYT-CH/CHA/National PhD scholarship program, 2013-21130004.

References:

