Mathematical Model of Wave Perturbations in the Solar Atmosphere near Resonance Frequencies

O.N. SAVINA
National Research University Higher School of Economics
25/12 Bolshaia Pecherskaja Ulitsa 603155 Nizhny Novgorod
RUSSIA
ONSavina@mail.ru

Abstract: - Analytically and numerically calculations according to the original effective algorithms for large-scale acoustic-gravity wave perturbations in the chromosphere from sources at the level of the photosphere are analyzed. Limitations to the energy flux of acoustic-gravity waves from the photosphere through the chromosphere are formulated. Structure of a narrow region with elevated pressure at the resonance altitude where the horizontal phase wave velocity is equal to the sound velocity is examined.

Key-Words: - Sun, chromosphere, corona, acoustic-gravity wave, temperature profile, analytical and numerical simulation

1 Introduction

Vertical oscillations observed in the solar atmosphere are important from the point of view of the possibility of the energy transfer to the low corona [1-3]. This question remains open, until now [4]. Five-minute oscillations very confidently are observed during the experiment [5].

The passage conditions of the energy flow of low-frequency wave perturbations from the photosphere to the corona are closely related to the special features of the acoustic-gravity wave propagation in the chromosphere with the realistic high-altitude temperature profile. For explaining of qualitative regularities and testing of numerical algorithms it is important to search analytical solutions for waves, which length is comparable with the characteristic scales of an altitude temperature profile via the selection of convenient models for the high-altitude temperature profile. So with the approximation of the temperature by linear altitude dependence the analytical solution is written in the special functions [6]. The propagation of internal gravity waves in the layer of the atmosphere with the piecewise-linear temperature profile was analyzed [7]. The sample task about the nonreflective vertical propagation of the rapid acoustic-gravity waves was examined [8]. Authors of this paper indicated the model altitude dependence of atmospheric temperature, which provides the precise analytical solutions for the wave disturbances. Authors found the frequency band of the extended waves, in which their propagation according to the geometric optic relationship is impossible. The so-called adjastment- effect was examined for the model, in which an altitude dependence of the temperature was the linear layer [9].

In papers [10-12] an original model of atmospheric wave propagation in the Earth's atmosphere with a realistic high-altitude temperature profile is analyzed. Shaping of a narrow domain with elevated pressure in the resonance region where the horizontal phase wave velocity is equal to the sound velocity is examined theoretically within the framework of the linearized equations. Numerical simulations for the model profiles of atmospheric temperature and viscosity confirm analytical result for the special feature of wave fields.

In this paper, one of the aspects of the problem in question is examined. We carry out an analysis of acoustic-gravity wave behavior near the resonance level, at which the condition of equality of horizontal phase wave velocity is equal to the local value of the sound velocity. We have shown that the Solar atmosphere temperature profile is such that there is a range of wave phase velocities (or a frequency range with fixed horizontal dimensions of the source) in which the wave does not pass through a resonance domain.

2 Basic equations for acoustic-gravity waves in the nonisothermal atmosphere

The existence of low frequency oscillations in the Solar atmosphere is propably related to acoustic-

ISBN: 978-1-61804-333-7

gravity waves. The properties of these waves are essential for understanding of the processes in the chromosphere and the lower corona, since the magnetic-field forces in these regions of the Solar atmosphere can be relatively small on the corresponding spatio-temporal scales. In a book [1] mentioned out many years ago that taking into account nonisothermicity of the Solar atmosphere for waves analysis is very important.

The linearized system of equations of gas dynamics for the pressure disturbance *p*, the horizontal velocity *u* and the vertical velocity *w* is well known [13]. We select axis *z* in the vertical direction against the gravity acceleration and axis *x* in the horizontal direction, ρ_0 is the basic state density, c_s is a sound velocity

$$\begin{aligned} &\rho_0 \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x}; \\ &\rho_0 \frac{\partial w}{\partial t} = -\frac{\partial p}{\partial z} - \rho g\,; \qquad (1) \\ &\frac{\partial p}{\partial t} + w\frac{dp_0}{dz} = c_s^2\left(\frac{\partial \rho}{\partial t} + w\frac{d\rho_0}{dz}\right). \end{aligned}$$

Really, the background temperature T depends on vertical coordinate. The regular pressure p_0 depends on altitude according to a condition of a thermodynamic equilibrium

$$p_0(z) = p_{00}\exp\left[-\int_0^z \frac{dz'}{H(z')}\right].$$

The regular density is described by a relation

$$\rho_0(z) = \frac{p_0(z)}{gH(z)}.$$

The differential equation for the vertical component of the velocity of medium has the form [7]:

$$\begin{aligned} &c^2\left(\frac{\partial^2}{\partial t^2} - c^2\frac{\partial^2}{\partial x^2} - c^2\frac{\partial^2}{\partial y^2}\right)\frac{\partial^4 w}{\partial t^2 \partial z^2} + \gamma g\left[c^2\left(\frac{dH}{dz}+1\right)\times\right. \\ &\left.\times\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) - \frac{\partial^2}{\partial t^2}\right]\frac{\partial^3 w}{\partial t^2 \partial z} - \left[\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2}{\partial t^2} - c^2\frac{\partial^2}{\partial x^2} - \right.\right. \\ &\left. - c^2\frac{\partial^2}{\partial y^2}\right)^2 + c^2\gamma g^2\frac{dH}{dz}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 + \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\times \\ &\left.\times g^2(1-\gamma)\left(\frac{\partial^2}{\partial t^2} - c^2\frac{\partial^2}{\partial x^2} - c^2\frac{\partial^2}{\partial y^2}\right)w\right] = 0 \qquad (2) \end{aligned}$$

The exact solution of this equation is possible only in some special cases (see, for example, [14]). It is not difficult to see that with the nontrivial value of a temperature gradient a resonance level must exist at the height where horizontal phase wave velocity is equal to the sound velocity.

We assume that the source is based on the bottom chromosphere. The simplest solution of this problem is founding the geometrical-optics approximation. In Fig.1 we can see transformation of the dispersion dependence between a wave frequency and a horizontal wave number when the atmospheric temperature increases with altitude.

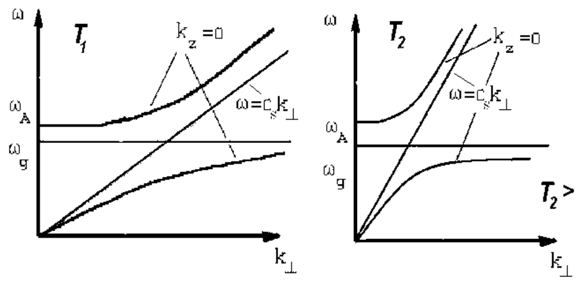

Fig.1.

According this approximation the up propagating infrasonic wave reflects from higher-temperature domain, but some energy leaks up and the level $z = z_*$, where $\omega - c_s(z_*)k_\perp = 0$. During the vertical propagation resonance level can be achieved only on an altitude growing temperature profile (Fig.1). The geometrical-optics approximation is not correct near the resonance level; then we need to examine the more general task.

3 Wave perturbations near the resonance level

We analyze the model of acoustic-gravity waves up propagation with a realistic high-altitude temperature profile. The linearized system of equations for the wave perturbation can be reduced to the following form:

$$\begin{aligned} &\left[-\omega^2 + \omega_g^2(z)\right]W - i\frac{\omega}{\rho_E}\left[\frac{\partial}{\partial z} + \Gamma(z)\right]P = 0, \\ &V = \left(k_\perp/\omega\right)P \qquad (3) \\ &\left[c_s^2(z)k_\perp^2 - \omega^2\right]P - i\omega\rho_E c_s^2(z)\left[\frac{\partial}{\partial z} - \Gamma(z)\right]W = 0. \end{aligned}$$

Here, $V = (\rho/\rho_{00})^{1/2}v$, $W = (\rho/\rho_{00})^{1/2}w$, and $P = (\rho_{00}/\rho_0)^{1/2}p$, where ρ_0 and ρ_{00} are the basic state densities in the current layer and at the bottom, respectively. Field variables are proportional to $\exp(-i\omega t + ik_\perp x)$ for a monochromatic signal with frequency ω in plane atmospheric layers. In this model, the horizontal wave number is altitude independent, and its main value is defined by the scale of the sources.

ISBN: 978-1-61804-333-7

Let us consider in greater detail the processes near the resonance level $z = z_*$, which are described by the system of equations

$$[-\omega^2 + \omega_g^2(z)]W - i\frac{\omega}{\rho_{00}}\left[\frac{\partial}{\partial z} + \Gamma(z)\right]P = 0\,; \qquad (4)$$

$$\left[-\omega^2 + c_s^2(z)k_\perp^2\right]P - i\omega\rho_{00}c_s^2(z)\left[\frac{\partial}{\partial z} - \Gamma(z)\right]W = 0.$$

As it was shown in the works [10-12] at the level $z = z_*$ the conditions $W = 0$ and $\frac{dW}{dz} = 0$ are fulfilled. The absence of disturbances of both the vertical velocity and its derivative leads to the conclusion that above the level $z = z_*$ the solutions both for W and for P are identically equal to zero.

Consequently, in order to counter balance the pressure jump at the level in question, the finite mass should be concentrated at the level $z = z_*$, which is taken into account in the solution using a delta function. Hence, if for the wave perturbation in the nonisothermal atmosphere at some level $z = z_*$ a condition $\omega = c_s(z_*)k_\perp$ is satisfied, then the averaged vertical energy flux is equal to zero. Above the first of such levels, wave perturbation are absent along the vertical propagation path.

Under the actual conditions, the resonance in the form of a delta function in the pressure disturbance, as well as in the horizontal velocity perturbation that is proportional to it ($V = P/c_s(z = z_*)\rho_0$), is smeared due to the molecular viscosity and the nonlinearity.

4 Full-wave calculations for resonance perturbation in the Solar atmosphere

In this section, we determine perturbations of the pressure and vertical velocity by means of full-wave numerical calculations. We assume that on the photosphere level there is a monochromatic source of vertical velocity and that at the altitudes higher than transition region the atmosphere is isothermal. Numerical calculation of (1) made it possible to find the high-altitude distribution of the wave perturbations for a monochromatic source of vertical velocity on the bottom.

The wave fields are conveniently calculated in dimensionless variables, as which we selected $\tilde{\omega} = \omega / \omega_{g0}$, $\tilde{k}_\perp = k_\perp c_{s0} / \omega_{g0}$, $\tilde{c}_s = c_s(z) / c_{s0}$, $\tilde{W} = W / W_0$, $\tilde{P} = P / \rho_{00}c_{s0}^2$. Here, subscript “0” indicates the value of the variable on the bottom chromosphere surface. We selected the altitude temperature profile following the experimental models of the solar atmosphere [15]. We determine the analytical function for this temperature profile using the model values marked by stars in Fig.2. The temperature profiles was approximated by a polynomial of the tenth order (a solid curve in Fig.2).

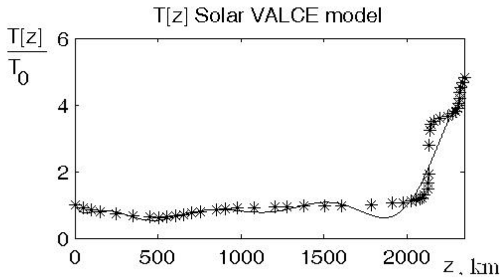

Fig.2.

The results of numerical calculation are shown on Fig. 3, corresponds to $\tilde{\omega} = 1.6$ and $\tilde{k}_\perp = 0.9$. On the upper panels there are the altitude dependence $\tilde{c}_s(z)\tilde{k}_\perp$, the straight line is the frequency $\tilde{\omega}$. On the bottom panels, the solid curves are the altitude dependence of pressure amplitude $|\tilde{P}(z)|$ and the dashed curves are the altitude dependence of the vertical velosity amplitude $|\tilde{W}(z)|$.

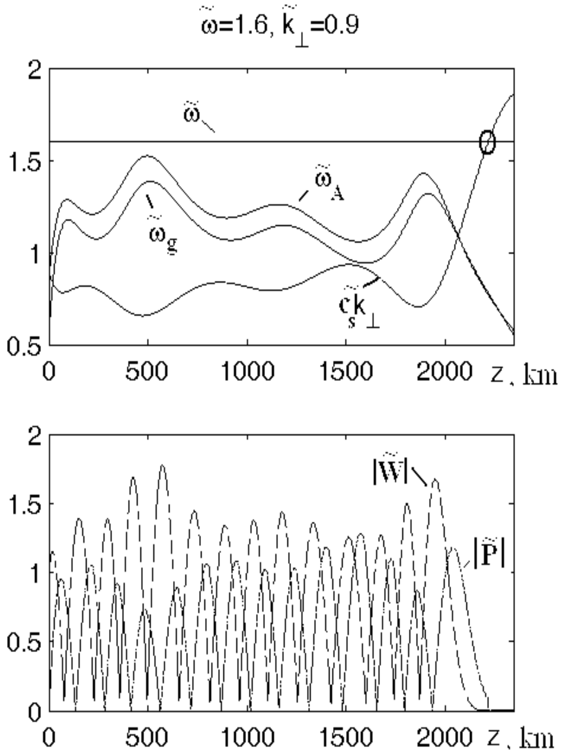

Fig.3.

According to the obtained results, the wave component of pressure has singularities near the altitude z_*, where the equality of the horizontal phase velocity of perturbation and sound velocity is fulfilled. The vertical velocity in perturbation turns to zero at this altitude. The wave perturbations do not propagate higher.

5 Conclusion

Acoustic-gravity disturbance in a nonisothermal atmosphere near resonance frequencies has been studied. According to the analytical results, the pressure wave amplitude has a local wave singularity near the layer at which the horizontal phase velocity is equal to the sound velocity and the vertical velocity in the disturbance becomes zero. In real conditions, many resonance layers (for different ω and $k_\perp$) can exist. The atmospheric viscosity and nonlinearity limit the pressure wave singularity. The vertical dimensions of singularity domain will be of the order of the mean free path of the molecules at the resonance level.

Acknowledgments. This research carried out in 2015 - 2016 was supported by “The National Research University „Higher School of Economics Academic Fund Program” grant No. 15-01-0039. The financial support from the Government of the Russian Federation within the framework of the implementation of the 5/100 Program Roadmap of the National Research University – Higher School of Economics is acknowledged.

References:

[1] S. A. Kaplan, S.B. Pikelner and V.N. Tsytovich, *Plasma Physics of the Solar Atmosphere*, Moscow: Nauka, 1977 (in Russian).

[2] E.G. Gibson, *The Quiet Sun*, Washington: NASA, 1973.

[3] M.J. Aschwanden, *Physics of the Solar Corona*, Berlin, N.Y.: *Springer*, 2004.

[4] A. Fossum and M. Carlsson, High-frequency acoustic waves are not sufficient to heat the solar chromospheres, *Nature*, Vol.435, 2005, pp. 919–921, DOI:10.1038/nature03695.

[5] R.J. Rutten and J.M. Krijger, Dynamics of the solar chromosphere IV. Evidence for atmospheric gravity waves from TRACE. *Astron. Astrophys.*, Vol.407, 2003, pp. 735–740.

[6] R.J. Greenfield and D.G. Harkrider, Acoustic-gravity waves calculations in a layer with a linear temperature variation. *Geophys. J. Roy, Astron. Soc.*, Vol.26, No.1-4, 1971, pp. 323-339.

[7] G.I. Grigor'ev and O.N. Savina, Acoustic-gravity waves in the atmosphere with a piecewise-linear temperature profile, *Radiophysics and Quantum Electronics*, Vol.45, No.8, 2002, pp. 606-611.

[8] N. Petrukhin, E. Pelinovsky and E.K. Batsyna. Reflectionless propagation of acoustic waves in the solar atmosphere, *Astronomy Letters*, Vol.38, No.6, 2012, pp. 388-393.

[9] D.G. Duffy, Adjustment in nonisothermal atmosphere. *J. Atm. Sci*., Vol. 60, 2003, pp. 339-353.

[10] O.N. Savina and P.A. Bespalov, Filtering features of long acoustic-gravity waves in a windless atmosphere. *Radiophysics and Quantum Electronics*, Vol.57, No.2, 2014, pp. 117-124.

[11] P.A. Bespalov and O.N. Savina, Exponential and local Lamb waves in the nonisothermal atmosphere as an obstacle to the acoustic-gravity disturbance propagation up to the ionosphere. 2015. *J. Atm. Solar-Terr. Phys.*, Vol.123, 2015, pp. 137–143.

[12] P.A. Bespalov and O.N. Savina, Ionospheric response to the acoustic gravity wave singularity. *Acta Geophysica*, Vol.63, No.1, 2015, pp. 319-328, DOI: 10.2478/s11600-014-0246-1.

[13] E. Gossard and W. Hooke., *Waves in the Atmosphere*. Amsterdam-Oxford-New York: Elsevier Scientific Publishing Company, 1975.

[14] O.N. Savina, Acoustic gravity waves in the atmosphere with a realistic temperature distribution, *Geomagn. Aeron*. Vol. 36, No.2, 1996, pp. 218-224.