Dispersion effects in the Falkner-Skan problem and in the kinetic theory

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Abstract

The conservation laws of continuum mechanics and the kinetic theory with the influence of the angular momentum and the associated with it rotation of the elementary volume are written, the variant of accounting lag is investigated to discrete media. The equations for gas are found from the modified Boltzmann equation and the phenomenological theory. For a rigid body the equations were used of the phenomenological theory, but changed their interpretation. The asymmetric stress tensor was obtained. The Boltzmann equation is written with an additional term. This situation is typical for discrete media as the transition from discrete to continuous environment is a key to issue of mechanics. Summary records of all effects lead to a cumbersome system of equations and therefore require the selection of main effects in a particular situation. The Hilbert paradox is solved. The simplest problem of the boundary layer continuum (the Falkner-Skan task) and the kinetic theory are discussed. A draw attention at the delay process would be suggested for the description of discrete media. Results are received for the some limiting cases.

Keywords: angular momentum conservation laws, unbalanced stress tensor, the Boltzmann equation, delay.

1. Introduction.

There were previously obtained modified energy equation of motion, continuity and momentum for structureless particles, taking into account the effects of the change of the angular momentum in the elementary volume. They result from modified Boltzmann equation, which was received from modified Liouville equation. The classical phenomenological theory was used for a solid body but interpretation was varied in this case [1-3]. Another approach proposed in [4]. For the construction of continuum mechanics equations conditions of forces equilibrium are used. In general mechanics we used the laws of force and angular momentum equilibrium. Using conditions of equilibrium of forces leads to a symmetric stress tensor and a violation of "continuity" of the environment that a rigorous analysis requires additional conditions. The laws provide a balance of power conservation of mass, momentum and energy. The fundamental conservation law in addition is especially for turbulent flows. Analysis of the gap was made by T.G. Elizarova [8]. In symmetric relatively time and space. It is correct when integral method for construction of continuum mechanics equations is used. It is obtained that constructing equations through delta-function gives us the formulation of conservation laws as is at present. Probably, more correct the law of conservation of angular momentum. It should be noted that for the kinetic theory (the Boltzmann equation) the law of conservation of angular momentum does not hold. Besides for the Chapman-Enskog distribution function formally we have values (density, linear moment and energy) with the first-order error. This fact was noted by Hilbert without further use and correction [5-7]. So

$$f(t,x,\xi) \equiv f^0(t,x,\xi) = n\left\{-\frac{m}{2kT}c^2\right\},$$

$$\left(c^2 = c_1^2 + c_2^2 + c_3^2\right) = (\xi - \boldsymbol{u})^2$$

and

$$f = f^0\left[1 + \frac{p_{ij}m}{2pkT}c_i c_j - \frac{q_i m}{pkT}c_i\left(1 - \frac{mc^2}{5kT}\right)\right]$$

have the same macroparameters in f^0.

The importance of these effects is observed for fluid mechanics, plasma gas and solid body. It should be noted that in general case, ergodicity is not observed, which is very important,

conservation laws for space averaging is fulfilled but for time is not as laws are not

representation for the kinetic theory would be formulation of the equations through an integral with average in space and time that should have average time among molecule collisions.

In the Navier-Stokes equations the boundary layer non-linearity and dissipation interact with each other. Non-linearity causes distortion of the original signal. The dissipation reduces the amplitude of the signal. However, we know that in addition to the above factors for a number of tasks dispersion effects are important, i.e. splitting the signal into individual harmonics. Classical equation, which is characterized by the presence and interaction of non-linearity and dispersion, is the Korteweg - de Vries – Burgers equation. As is known, dispersion leads to mixing of the individual harmonics. Waveform is changing. If in the equations of motion of the system of Navier - Stokes equations we introduce an additional term with the third derivative, then it will turn into the Korteweg - de Vries – Burgers equation. Usually non-linearity and dissipation for large gradients of laminar flows change flows to turbulent. In the Reynolds model actually stand fast and slow variables. In the resulting averaged equations establish the connection between stress and the Reynolds-averaged flow parameters, but the answer to the question of the form of the closing ratio yet. The process of building relationships based or on empirical evidence or written out of the equation for higher order moments, such as turbulent (fluctuating) kinetic energy. In these equations include new unknowns, and the process is repeated for the specified circuit scenario. For the inertial flow in the equilibrium case, a well-known law of N.A. Kolmogorov's theory of the dimension is suggested. At the heart of all theories are the Navier-Stokes equations. Even if involved Boltzmann equation in order to obtain from it the equations of turbulent flow, the output method is focused on validation of the model Reynolds, built on the basis of the Navier-Stokes equations. It is hardly possible to derive the equation of continuity for the tube of flow to turbulent flow. The main directions of current research include:

1. The increasing complexity of the structure of the dynamic viscosity coefficient
2. The increasing complexity of the equations (the introduction of the term with the third derivative, a third-order tensor, etc.)
3. The wording of the new system of equations for the functions of the new system
4. Introduction of the equation of the motion of random or accidental forces viscosity
5. Solution of the full three-dimensional unsteady Navier-Stokes equations on detailed grids with high-order schemes
6. Isolation of large eddies with the addition of the flow pattern inside the grid cell by further study the model selected inside the cell

We aim to show that the proposed system of equations can be derived from the first principles, and in this case, at least part of the known properties, which are presented in the textbooks and observed in experiments for turbulent flows, can be explained without additional assumptions about the form of the turbulent viscosity, and that one can restrict the molecular viscosity.

Before proceeding to the solution of specific problems, we present a quote from the Loitsyanskii book [9]: "The current lines of pulsating movement of cross streamlines of the mean flow, penetrate from one layer to another, and create the stirring - it is called a molar or turbulent mixing - accompanied by the transport through the boundary between the layers of momentum, energy, heat, and other mechanical or thermodynamic parameters of the mean flow liquid. Given for the turbulent velocity profile are averaged. As the theory of stability in the areas of origin of the turbulence observed first regular vibrational structure." Log infinite plate profiles and tubes, generally obtained final product length, though very elongated. The theoretical profile is defined by introducing the Reynolds stress $\frac{d^2u}{dy^2} = \frac{d\tau}{dy} = 0$.

Log infinite plate profiles and tubes, generally obtained final product length, though very elongated. The theoretical profile is defined by introducing the Reynolds stress. The velocity distribution in a laminar flow $u = \frac{\tau_w}{\mu}$, $\mu\frac{du}{dy} = const = \tau_w$, y-distance in the vertical direction to the surface, $\mu -$ viscosity, u – velocity, τ_w – friction. For the turbulent motion of turbulent shear stress $\tau = -\rho\overline{u'}v'$ at the wall is zero. Reynolds equation $\frac{d^2u}{dy^2} = \frac{d\tau}{dx}$

If the shear stress given by the formula Prandtl $\tau = \rho l^2 (\frac{du}{dx})^2 = \tau_w, l = ky, k = const$

$$u = \frac{1}{k\sqrt{\frac{\tau_w}{\rho}}} \, lny + C, \;\; C = const$$

Thus, after obtaining the velocity profile by classic theory certain assumptions are used and did not solve of the Navier-Stokes equations and the boundary

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layer, but new equations by Prandtl. The initial equations are not satisfied with any external boundary or surface; no transition asymptotic solutions problem for the semi-infinite plate to solve the problem for infinite plate. Historical (classical) formulation of conservation laws based on a closed elementary volumes for exchange on the tangential component of the physical variables that led to the formulation of the conditions for the equilibrium of forces. Being open system, the elementary volume exchanged components physical quantities in all directions.

Very importance is the role of the inertia centre [3]. In this paper we numerically investigate the influence of small perturbations of vertical velocity on the longitudinal velocity in the modified problem Falkner-Skan to reflect changes in the angular momentum in the elementary volume. The delay in the kinetic theory is necessary to study due to the finiteness of the interaction time between the molecules and the definition of a derivative.

2.Elements of general modified theory.

Here I –collision integral, F –force, f is the one-particle distribution function, $\boldsymbol{x}$ - coordinate of the point and according to definition of f in element of physical volume $d\boldsymbol{x}$ near the point $\boldsymbol{x}$ in moment t probable number of molecules with velocity in element $d\boldsymbol{c}$ near the $\boldsymbol{c}$ is $f(t, \boldsymbol{x}, \boldsymbol{c})$, t is the time, ρ is the density, are x_i –the Cartesian coordinates of a particle, X_i are the projections of a volume force, q_j is the heat flow, R is the gas constant, and T is a temperature. Another problem for the solving of the Boltzmann equation is the asymptotical methods. It is essential that selecting the local equilibrium distribution function f^0 as the basis in solution of the Boltzmann equation by the Chapman-Enskog method exploits macroscopic parameters in f^0 from the Euler equations [10]. Macroscopic parameters are determined by the Chapman-Enskog distribution function leads to the Euler equations parameters and tensor P is symmetric. Formally in that way we have values (density, linear moment, energy) with mistake of the first order. This is the Hilbert paradox.

As follows from the previous work, taking into account the laws of conservation of angular momentum we have follows equations [1-3]

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} + \frac{\partial}{\partial x_i}\left(x_i \frac{\partial \rho u_i}{\partial x_i} \right) = 0. \quad (1)$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial}{\partial x_i}\left(\rho u_i u_j + P_{ij} + x_i \frac{\partial P_{ij}}{\partial x_i} \right) - \frac{X_i}{m}\rho = 0.$$

$$\frac{\partial}{\partial t}\, \rho\, \left(\frac{3}{2} RT + \frac{1}{2} u^2 \right) + \frac{\partial}{\partial x_j}\left[\rho u_j \left(\frac{3}{2} RT + \right.\right.$$
$12\ u2{+}uk\ Pkj{+}\ qj{+}\ uk\ Pkj{+}\ qj]{+}$

$$+ \frac{\partial}{\partial x_i} x_i \frac{\partial}{\partial x_j}\left[\rho u_j \left(\frac{3}{2} RT + \frac{1}{2} u^2 \right) + u_k P_{kj} + \right.$$
$qj\ {=}0$

t- time here, x_i- coordinates, u_i-speed, µ - viscosity, ρ-density, T-temperature, q –thermal-flow, P_{ij}-tensor of viscous pressure, X-force.
The system is complemented by the law of conservation of angular

$$\frac{\partial \vec{r}}{\partial x} \times \vec{p}_x + \frac{\partial \vec{r}}{\partial y} \times \vec{p}_y + \frac{\partial \vec{r}}{\partial z} \times \vec{p}_z + x_j \frac{\partial}{\partial x_j}\left(\vec{P}_j \right) = M_I \quad (2)$$

This equations were received from modified the Boltzmann equation [3].

$$\frac{df}{dt} = \frac{\partial f}{\partial y} + c_i \cdot \frac{\partial f}{\partial \boldsymbol{x_i}} + c_i \cdot \frac{\partial}{\partial \boldsymbol{x_i}} r_i \frac{\partial f}{\partial \boldsymbol{r_i}} - \frac{F}{m}\frac{\partial f}{\partial x_i} = I.$$

$$\int \varphi(\xi) f^0 d\,\xi = \int \varphi(\xi) f d\,\xi = \beta,$$

here β-macroscopic parameter, $\varphi(\xi)$-function.

$$n(t,\vec{\mathbf{x}}) = \int f(t,\vec{\mathbf{x}},\vec{\xi})d\vec{\xi}, \quad \boldsymbol{u}(t,\vec{\mathbf{x}}) = \frac{1}{n}\int \vec{\xi} f(t,\vec{\mathbf{x}},\vec{\xi})d\vec{\xi},$$

$$P_{ij} = m\int c_j c_i f(t,\vec{\mathbf{x}},\vec{\xi})d\vec{\xi}, \quad q_i = \frac{m}{2}\int c^2 c_i f(t,\vec{\mathbf{x}},\vec{\xi})d\vec{\xi}. \quad (3)$$

So for the equilibrium in collision integral

$$f(t, \boldsymbol{x}, \boldsymbol{\xi}) \equiv f^0(t, \boldsymbol{x}, \boldsymbol{\xi}) = n\left\{ -\frac{m}{2kT} c^2 \right\}, \quad (4)$$

$$\left(c^2 = c_1^2 + c_2^2 + c_3^2 \right) = (\boldsymbol{\xi} - \boldsymbol{u})^2$$

and for nonequilibrium distribution function

$$f = f^0 \left[1 + \frac{p_{ij} m}{2pkT} c_i c_j - \frac{q_i m}{pkT} c_i \left(1 - \frac{mc^2}{5kT} \right) \right] \quad (5)$$

we have the same macroparameters in f^0. The nonequilibrium distribution function is such that integral of it contains only the integral of equilibrium function f^0 and gives containing in f^0 macroscopic parameters. The remaining term gives null. We suggest to do another iteration for macroparameters selecting them from the Navier-Stokes equations. Elementary volume can be rotated around the axis of inertia or to be involved in the rotational movement. In both cases the density of the flow across the border changes by $\frac{d(\rho u)}{dr}.(r'-r)+\cdots$ (fig.1) by rotation of the elementary volume. The contribution of other components is small,

3. Falkner-Scan task with modified boundary conditions.

Consider the boundary layer with the forward motion of the cylinder at a speed at the outer edge ($\mathbf{U_e}=\mathbf{ax^m}$)) with input torque terms. This problem contains as a special case of the decision to the plate with a uniform external flow and interesting as an example of accelerated (m> 0) or delayed (m <0) motion in the external flow $\mathbf{U_e}$

$$U_e = U_\infty + \varepsilon\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)$$

(6)

$$u\frac{\partial u}{\partial y}+v\frac{\partial u}{\partial y} = U_e\frac{\partial U_e}{\partial x}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial y}\left(\mu y\frac{\partial u^2}{\partial u^2}\right), \qquad (7)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

(8)

or

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial}{\partial y}y\frac{\partial v}{\partial y}=0$$

(9)

With boundary conditions

$u=0,\quad v=0,\quad \mu\frac{\partial u}{\partial y}=\tau_w=\alpha/\sqrt{x^{(m-1)/2}}\ ,\ y=0.$

$u=U_e,\quad y\to\infty,\ x>0;\quad u=U_e\ ,\ x=0,$

$U_e=ax^m, a=const, m=const.$

Managed to count 300 steps, then there was an increase the vertical velocity component and require a different mathematical model. Solution self-similar problem was carried out in [11].

At fig. 2 horizontal line-stretch coordinate

$$u=cx^m\,\Phi(\eta),\ \eta=\sqrt{\frac{c}{\mu}}\,yx^{(m-1)/2},\quad v=\sqrt{\mu c}\,x^{\frac{m-1}{2}}V(\eta).$$

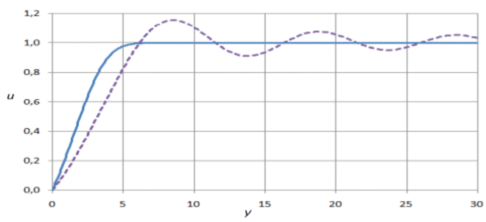

Fig. 1. $m=-0.05;\tau_w=0.2202$

taking into account the smallness of the volume and the absence of rotation on axis.

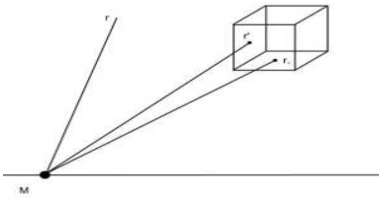

Fig.1

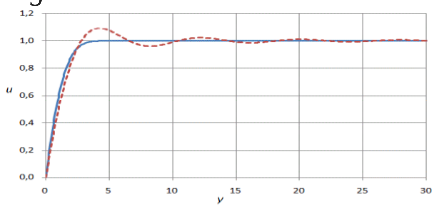

Fig.2 . $m=0.33;\ \tau_w=0{:}7575$

Results eddy constant boundary conditions at the outer edge of the boundary layer.

Figures 2-6 illustrate the effects of the angular momentum on the velocity profiles in the boundary layer (equation 8). For In the presence of the vortex perturbations around an axis z, the structure inside the boundary layer are there. It is known [12] that vortex perturbations located for parallel flows, new structures are not allowed, but for turbulent flows there are various another structures within the boundary layer [12-16]. On vertical coordinate is u, on horizontal coordinate is y, u and v

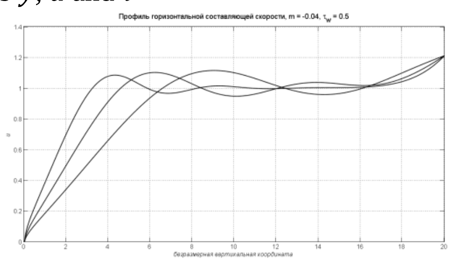

*Fig.*3. Profile of the horizontal velocity component along the vertical axis: m= -0.04, $\tau_w=0.5$;

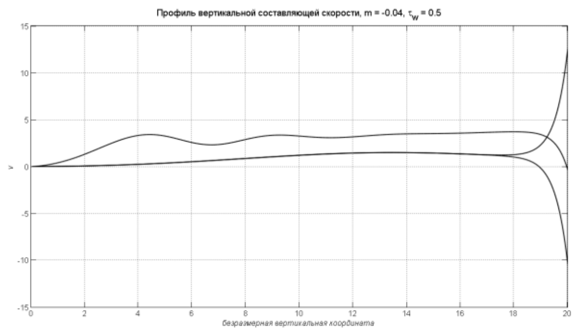

Fig.4, Profile v, m=-0.04 , $\tau_w=$ *0.5,*

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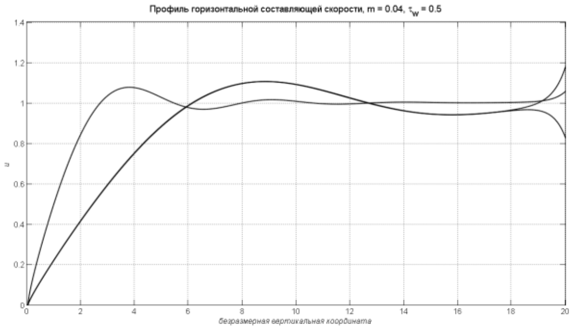

Fig5 Profile of the horizontal velocity component along the vertical axis: m=0, $\tau_w = 0.5$;

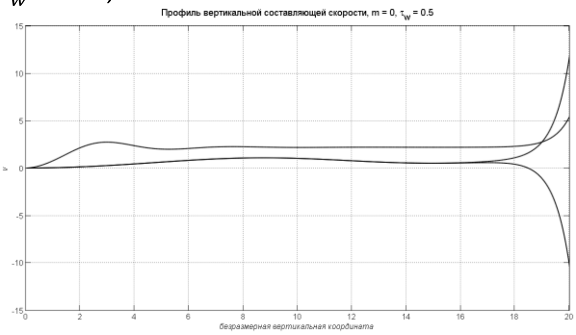

Fig.6, Profile v, m= 0, $\tau_w =$ *0.5*

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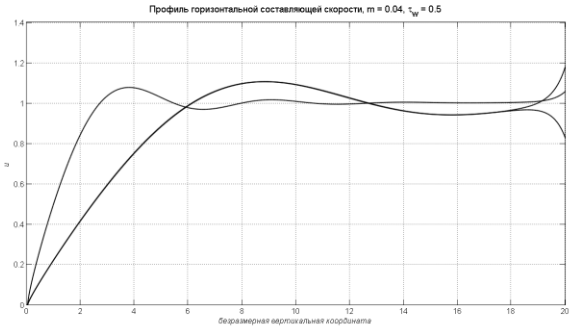

*Fig.*7 Profile of the horizontal velocity component along the vertical axis: m= 0.04, $\tau_w = 0.5$;

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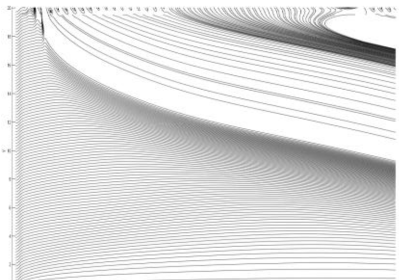

Fig.8, The stream function for the latter option

4. Exact solution for kinetic theory. The barometric Boltzmann formula.

Gas is at stationary condition in field of force which have potential **φ** (this is analogy tasks from [10]):

As before solution we shall be look for as $f = A(x)e^{-B(x)\xi^2}$. We receive the old result $B = Const.$ For $A(x)$ we have equation

$$\frac{dA}{dx} + \frac{d}{dx_i} x_i \frac{dA}{dx_i} + 2\frac{A\cdot B}{m} \quad \frac{\partial\varphi}{\partial x_i} = 0. \quad (9)$$

We have old result that is common for one-dimension

tasks: $f = n_0 \, (\frac{m}{2\pi kT})^{3/2} \, e^{-\frac{\varphi}{kT}} \, e^{-\frac{m}{2kT}\xi^2}$. (10)

General local-Makswell distribution is $f = n(\frac{m}{2\pi kT})^{3/2} \exp\left\{-\frac{m}{2kT}c^2\right\}$, $c = \xi - u$. (11)

Modification the Boltzmann equation is

$$\xi_i \frac{\partial f}{\partial x_i} + \xi_i \frac{\partial}{\partial x_i} x_i \frac{\partial f}{\partial x_i} - g_i \frac{\partial f}{\partial \xi_i} = J(f, f). \quad (12)$$

$g = X/m$ -acceleration of molecules.

Let local-Makswell solution of the equation will be considered as in old algorithm

$$lnf = \gamma_0 + \gamma_i\xi_i + \gamma_4\xi^2. \quad (13)$$

Then we receive old equation and the changing

$$\frac{\partial\gamma_0}{\partial t} + g_i\gamma_i = 0, \quad (14)$$

$$\frac{\partial\gamma_i}{\partial t} + 2g_i\gamma_4 + \frac{\partial\gamma_0}{\partial x_i} + \frac{\partial\gamma_0}{\partial x_i} + \frac{1}{2}x_i \frac{\partial\gamma_0}{\partial x_i^2}^2 + \frac{\partial}{\partial x_i} x_i \frac{\partial\gamma_0}{\partial x_i} = 0, \quad (15)$$

$$\frac{\partial\gamma_4}{\partial t}\delta_{ij} + \frac{1}{2} \quad (\frac{\partial\gamma_i}{\partial x_j} + \frac{\partial\gamma_j}{\partial x_i}) + \frac{1}{2} \quad (\frac{\partial\gamma_i}{\partial x_j} + \frac{\partial\gamma_j}{\partial x_i}) + \quad (16)$$

$$+ \frac{1}{2} * \frac{1}{2}\left(x_i + x_j\right)\left(\frac{\partial\gamma_i}{\partial x_j} + \frac{\partial\gamma_j}{\partial x_i}\right)\frac{\partial\gamma_0}{\partial x_i} + \frac{1}{2}\left(x_i + x_j\right) \frac{1}{2} \left(\frac{\partial}{\partial x_j}\left(\frac{\partial\gamma_i}{\partial x_j} + \frac{\partial\gamma_j}{\partial x_i}\right) + \frac{\partial}{\partial x_i}\left(\frac{\partial\gamma_i}{\partial x_j} + \frac{\partial\gamma_j}{\partial x_i}\right)\right) = 0,$$

As before

$$\frac{\partial\gamma_4}{\partial x_i} = 0, \; T = const. \quad (17)$$

As the result we shall receive modified gas-dynamic equations but without viscosity and thermal conductivity

5. The role of angular momentum and delay.

In general case three positions are important for understanding the causes of modification the theory [1-3]. Another type of including angular momentum is contained in [5].

1. It is necessary to take for equilibrium conditions more general condition-condition of angular momentum, although equilibrium of force is needed to retain but with non-symmetric stress tensor
2. Replacement of velocity decomposition near point of elementary volume to the decomposition near axis of inertia

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3. Delay

For kinetic theory is need to investigate the question what is measured in experiment: the role of delay: at present moment or for middle results. If in experiment are measured middle results then is essential to choose time and scale. If the time is in coordination then the delay is no need to take into consideration besides the cases for relaxation time where we are need to coordinate the delay. Then the Boltzmann equation is

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + c_i \cdot \frac{\partial f}{\partial r_i} + c_i \cdot \frac{\partial}{\partial r_i} r_i \frac{\partial f}{\partial r_i} - \frac{F}{m} \frac{\partial f}{\partial c_i} = I \qquad (18)$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \overline{\tau} \frac{\partial^2 f}{\partial^2 t} + \mathrm{c}_i \cdot \frac{\partial f}{\partial \boldsymbol{r}_i} + \mathrm{c}_i \cdot \frac{\partial}{\partial \boldsymbol{r}_i} r_i \frac{\partial f}{\partial \boldsymbol{r}_i} - \frac{\mathbf{F}}{m} \frac{\partial f}{\partial c_i} = I\,, \qquad (19)$$

$$\Delta^{-} = dt\, d\vec{x}\, d\vec{\xi} \quad f(t, \vec{x}, \vec{\xi} \quad) \int \left[f_1(t, \vec{x}, \vec{\xi}) + O\left(\Delta t \vec{\xi} \frac{\partial f_1}{\partial x}\right) \right] gbdbd\varepsilon\, d\vec{\xi}_1 \qquad (20)$$

$$\Delta^{+} = dt\, d\vec{x}\, d\vec{\xi'} \quad \int \left[f(t, \vec{x}, \vec{\xi}')\, f(t, \vec{x}, \vec{\xi}'_1) + O\left(\Delta t \vec{\xi} \frac{\partial f_1}{\partial x}\right) \right] g'\, b'\, d\varepsilon' d\vec{\xi}_1 \qquad (21)$$

$$I = \Delta^{-} - \Delta^{+} \qquad (22)$$

$$\frac{\partial f}{\partial t} \leftrightarrow \frac{\partial f}{\partial t} + \overline{\tau} \frac{\partial^2 f}{\partial^2 t} \qquad (23)$$

$$f'(t, x, \xi') \leftrightarrow f'(t, x, \xi') \qquad (25)$$

$$f(t, x, \xi) \leftrightarrow f\left(t + \overline{\tau}, x + \overline{\lambda}, \xi\right) \leftrightarrow f(t, x, \xi) + \overline{\tau} \frac{\partial f}{\partial t} + \overline{\lambda} \frac{\partial f}{\partial x} + \cdots \qquad (26)$$

$$f_1(t, x, \xi_1) \leftrightarrow f_1\left(t + \overline{\tau}, x + \overline{\lambda}, \xi_1\right) \leftrightarrow f_1(t, x, \xi_1) + \overline{\tau} \frac{\partial f_1}{\partial t} + \overline{\lambda} \frac{\partial f_1}{\partial x} + \cdots \qquad (27)$$

In the formulas selected averages, although one can select to individual speeds and consider all their sums. Similarly, calculated values with the molecule that is flying during the free path, the mean free path of the molecules and the travel time to and after the collision can be different (may be different values of the incident and impinging molecules (with index one))

$$ff_1 - f'\, f_1' \leftrightarrow ff_1 - f'\, f_1' + \tau \frac{\partial f}{\partial t} f_1 + \tau_1 f \frac{\partial f_1}{\partial t} + \lambda \frac{\partial f}{\partial x} f_1 + \lambda_1 f \frac{\partial f_1}{\partial x} + \cdots - \tau' \frac{\partial f'}{\partial t} f_1' - -\tau_1' f \frac{\partial f_1'}{\partial t} - \cdots - \lambda' \frac{\partial f'}{\partial x} f_1' - \lambda_1' \frac{\partial f_1'}{\partial x} f' - \ldots \qquad (28)$$

In general, this formula should be writing in this form, but for small gradients of simple gas can be write to single time and single long of the run. However, for structural gas, for example, at altitudes above 120 km the free time of run on three Mach numbers, i.e. lag time 10^{-8} c and more that can be comparable with the relaxation time. In fact, the expression can be simplified by considering the order of magnitude. Then we have

$$ff_1 - f'\, f_1' \leftrightarrow ff_1 - f'\, f_1' + \tau \frac{\partial f^0}{\partial t} f_1^0 + \tau_1 f^0 \frac{\partial f_1^0}{\partial t} + \lambda \frac{\partial f^0}{\partial x} f_1^0 + \lambda_1 \frac{\partial f_1}{\partial x} f^0 + \cdots - \tau' \frac{\partial f'^0}{\partial t} f_1'^0 - \tau_1' f^0 \frac{\partial f_1'^0}{\partial t} - \cdots - \lambda' \frac{\partial f'^0}{\partial x} f_1'^0 - \lambda_1' \frac{\partial f_1'^0}{\partial x} f' - \ldots$$

Then the integrals are computed and we obtain the corresponding kernel of the Navier-Stokes equations. For small and medium gradients free time is single and single mean free path. But significant differences are in the interaction of gases with different properties. Will be discussed the situation when we have the density for the two interaction elementary volume which are much different. Another situation when properties of molecules are close. In order to evaluate we are using single mean free time and single mean free path. The density after collision is counted to equal to density of impinging of particles.

The preceding value of right part will be $(\mathrm{n} \gg \mathrm{n}_1)$

$$\left(\tau - \frac{n}{n_1} \tau\right) \frac{\partial f^0}{\partial t} f_1^0 + \left(\frac{n}{n_1}\tau - \frac{n}{n_1}\tau\right) f^0 \frac{\partial f_1^0}{\partial t} + \left(\lambda - \frac{\xi'}{\xi}\lambda\right) \frac{\partial f^0}{\partial x} f_1^0 + \left(\frac{n_1}{n}\lambda - \frac{n}{n_1}\lambda\right) \frac{\partial f_1}{\partial x} f^0 \qquad (29)$$

Derivative on x is given us self diffusion, termo-diffusion and baro-diffusion about which S.V. Vallander told. Direct influence on delay the first item has $\mathrm{C}\frac{\mathrm{n}}{\mathrm{n}_1}\tau\frac{\partial \mathrm{f}^0}{\partial \mathrm{t}}\, \mathrm{f}_1^0$. If count up that n ≫ we shall receive the main item $\frac{n}{n_1}\tau\frac{\partial f^0}{\partial t}$ It is additive core of the modified Boltzmann equation.

6. Conclusion.

Influence of the angular momentum is investigated and, as consequence, non-symmetry of stress tensor is received for elementary volume. The role of delay is investigated in kinetic theory. Shows the influence of these effects in continuum mechanics and kinetic theory, some the numerical results are demonstrated (the Falkner-Skan problem). The striped structure was received. Vertical component of velocity plays more role if the angular momentum is take into account. The exact solution of kinetic theory that is well known in classic are considered.

Reference

[1]. E.V. Prozorova. Influence of mathematical models in mechanics. Problems of nonlinear analysis in engineering systems. ", №2 (42), т.20, 2014, 78-86

[2]. E.V. Prozorova. The effect of dispersion in nonequilibrium continuum mechanics problems environment, Moscow State University, Electronic Journal" Physico-chemical kinetics in gas dynamics. 2012. Volume 13, URL: http://www.chemphys.edu.ru/pdf/2012-10-30-001.pdf
(in Russian)

[3]. Evelina V. Prozorova. Influence of the Delay and Dispersion In mechanics.
Journal of Modern Physics, 2014, 5, 1796-1805

[4]. E. A Bulanov. The momentum tension on mechanics of solid, free flowing and liquid medium. College book. 2012

[5]. C. Cercignani, Mathematical methods in kinetic theory Macmillan. 1969

[6]. J.H. Ferziger, H.G. Kaper, Mathematical theory of transport processes in gases./ Amsterdam-London.1972.

[7] J.O. Hirschfelder, C.F. Curtiss, R.B. Bird, The molecular theory of gases and liquids. New-York, 1954.

[8]. T.G. Elizarova. Guasi-gasdynamic equations and numerical methods for viscous flow simulation. M.:Scientific word.2007. 352p.

[9]. L.G.Loyotinskiy. Mechanics of fluids and gas. M.: Nauka. 1970 (Russion)

[10]. M.N. Kogan, The dynamics of the rarefied gases. M.: Nauka, 1967.(Russian)

[11]. V.A.Kononenko, E. V. Prozorova, A.V. Shishkin . Influence dispersion for gas mechanics with great gradients. 27-th international symposium on Shock waves. St. Peterburg. pp.406-407, 2009.

[12]. M.M. Katasonov, V.V. Kozlov, N.V. Nikitin, D.C. Sboev. Beginnings and development of localization indignation in circle tube and boundary layer. Educational text-book, Novosibirsk. State university, 2015.

[13].VA Babkin, VN Nicholas. Turbulent flow in a circular tube and a flat channel and model mesoscale turbulntnosti. Engineering Physics magazine. T. 84. N.2

[
14].G.E.Elsinga, R.J. Adrian, B. W. Van Oudhensden and F. Scarano. Three-dimensional vortex organization in a high-Reynolds-number supersonic turbulent boundary layer. J. Fluid Mech. (2010), vol.644, pp. 35-60

[15]. N.V. Priezjev&s.M. Trouan. Influence of periodic wall Roughness on slip behaviour at liquid/solid interfaces: molecular-scale simulations versus continuum predictions. J. Fluid Mech. (2006), vol. 554, pp. 25-47

[16].Yukio Kameda, Junga Yoshino, Takashi Ishihara. Examination of Kolmogorov's 4/5 Law by High-Resolution Direct Numerical Simulation Data of Turbulence. Journal of the Physical Society of Gapan. Vol. 77, No. 6, 2008, 064401