MSA algorithms for solving the combined assignment-control problem

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Abstract: - In this paper we focus on the combined assignment-control problem that arises when signal control parameters of an urban road network are locally optimised and they have to be consistent with equilibrium traffic flows. This problem can be formulated as an (asymmetric) equilibrium assignment problem. In this paper we compare some solution algorithms for solving the combined assignment-control problem, based on the MSA (Method of Successive Averages) framework; some algorithms, proposed in the literature for solving the (symmetric) assignment problem, are adapted to the combined assignment-control problem. All the algorithms are tested on a real-scale network. Numerical results show that these algorithms are able to reduce computing times significantly, respect the classic MSA algorithm, in order to solve the asymmetric assignment problem.

Key-Words: - Combined assignment-control problem, signal settings, MSA, fixed-point problems, asymmetric assignment.

1 Introduction

Traffic-light systems play a fundamental role in urban network management and effective optimisation of signal settings may produce significant improvements in network performance.

The problem of optimising only the signal settings, assuming as fixed and invariable the physical configuration of the urban network (topology and link capacities), can be seen as a particular case of the more general Equilibrium Network Design Problem (ENDP), where signal settings assume the role of decision variables; this problem is also known as the Signal Setting Design Problem (SSDP). For solving the SSDP, two different approaches can be identified: a global approach and a local approach.

In the first case, the problem is actually an ENDP, formulated with a (non-linear constrained) optimisation model, and it is also known as Global Optimisation of Signal Settings (GOSS). In this problem, signal settings of all junctions are jointly designed so as to minimise the total travel times on the network (or other performance indexes) and the equilibrium traffic flows represent the descriptive variables that have to be calculated by an equilibrium assignment algorithm.

In the second case, instead, it is assumed that the signal settings of each junction are designed so as to minimise only the total delay at the same junction, according to a specific local control policy. This problem, known also as Local Optimisation of Signal Settings (LOSS), can be formulated as a fixed-point problem, where there is a circular dependence between flows, costs and signal settings, or, equivalently, as an asymmetric equilibrium problem. For a more detailed description of GOSS and LOSS problems see [1]. In [2] the GOSS problem is solved using meta-heuristic algorithms.

In this paper, we focus on the solution of the LOSS problem, comparing several algorithms proposed in the literature; all the compared algorithms are based on the MSA general framework [3, 4, 5].

The general LOSS problem was studied, among others, in [1, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18].

The remaining part of the paper is organised as follows: Section 2 introduces the model formulation; the solution algorithms are analysed in Section 3, and Section 4 summarises the numerical results; finally, Section 5 concludes the paper.

2 Model formulation

On a road network, the link flow vector, $\boldsymbol{f}$, once the transport demand has been fixed, depends on link costs and hence on the link cost vector, $\boldsymbol{c}$:

$$\boldsymbol{f} = \boldsymbol{f}(\boldsymbol{c}) \quad (1)$$

where

$$\boldsymbol{f}(\boldsymbol{c}) = \boldsymbol{A}\ \boldsymbol{P}(\boldsymbol{A}^{\mathrm{T}}\ \boldsymbol{c})\ \boldsymbol{d}$$

with

- $\boldsymbol{A}$ the link-path incidence matrix, whose components, a_{lp}, are equal to 1 if link l belongs to path p and 0 otherwise;
- $\boldsymbol{P}$ the path choice probability matrix, with a column for each *od* pair and a row for each path p; the generic element, $P_{p,od}$, of this matrix represents the probability that a user will use path p from o to d (if path p does not connect the *od* pair, $P_{p,od}$ is equal to 0);
- $\boldsymbol{d}$ the demand vector, whose components are the demand values d_{od} for each O-D pair.

The generic cost, c_l, of link l is, for urban networks, the sum of two terms: the running cost, that is usually assumed dependent only on the flow on the same link, f_l, and the delay (waiting time at the intersection) which, for signalised intersections, depends on both traffic flows, $\boldsymbol{f}$, and signal settings, $\boldsymbol{g}$. Formally, it is possible to write:

$$\boldsymbol{c} = \boldsymbol{c}(\boldsymbol{f}, \boldsymbol{g}) \quad (2)$$

Substituting eqn (2) in eqn (1) we obtain the eqn:

$$\boldsymbol{f} = \boldsymbol{f}(\boldsymbol{c}(\boldsymbol{f}, \boldsymbol{g})) \quad (3)$$

that relates flows, $\boldsymbol{f}$, costs, $\boldsymbol{c}$, and signal settings, $\boldsymbol{g}$.

Having fixed the $\boldsymbol{g}$ vector, we can calculate the equilibrium traffic flows, $\boldsymbol{f}^*$, with one of the algorithms available in the literature (see for instance [19]). Formally, we can write:

$$\boldsymbol{f}^* = \boldsymbol{f}(\boldsymbol{c}(\boldsymbol{f}^*, \boldsymbol{g})) \quad (4)$$

In this case, since the transportation supply is given, it may be stated, under some assumptions, that the solution of the fixed-point problem (4) exists and is unique [19]. Therefore, eqn (4) is an *application*: one, and only one, equilibrium traffic flow vector, $\boldsymbol{f}^*$ corresponds to each signal settings vector, $\boldsymbol{g}$. In Fig. 1(a) the relations between the variables are represented.

Now let us introduce a second application between signal settings, $\boldsymbol{g}$, and traffic flows, $\boldsymbol{f}$. Having fixed the traffic flows on the network, this yields uniquely the signal settings:

$$\boldsymbol{g} = \boldsymbol{\omega}(\boldsymbol{f}) \quad (5)$$

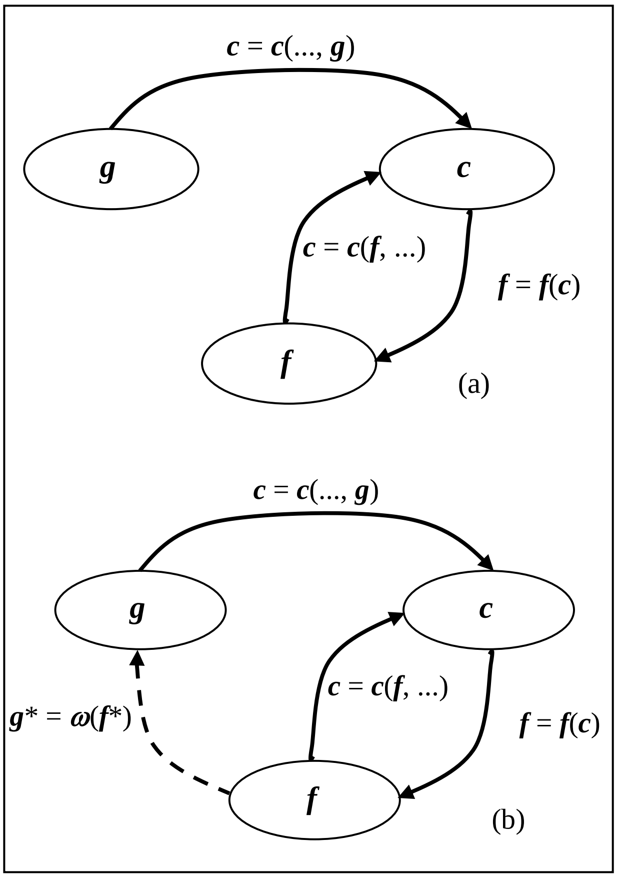

Fig. 1 - Relations between flows, costs and signal settings: (a) fixed signal settings; (b) local optimal signal settings.

This relation usually represents the local control policies that optimise the signal settings of each intersection as a function of only the flows at the same intersection. Given eqn (5), eqns (2) and (3) become:

$$\boldsymbol{c} = \boldsymbol{c}(\boldsymbol{f}, \boldsymbol{\omega}(\boldsymbol{f})) \quad (6)$$

$$\boldsymbol{f} = \boldsymbol{f}(\boldsymbol{c}(\boldsymbol{f}, \boldsymbol{\omega}(\boldsymbol{f}))) \quad (7)$$

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and the fixed-point problem (4) becomes:

$$\boldsymbol{f}^* = \boldsymbol{f}(\boldsymbol{c}(\boldsymbol{f}^*, \boldsymbol{\omega}(\boldsymbol{f}^*))) \quad (8)$$

Therefore, the problem is to find the equilibrium traffic flows, $\boldsymbol{f}^*$, that are congruent with the local control policy; the corresponding signal settings, $\boldsymbol{g}^* = \boldsymbol{\omega}(\boldsymbol{f}^*)$, are the solution of the Local Optimisation of Signal Settings (LOSS) problem. This solution is also a feasible but suboptimal solution of the Global Optimisation of Signal Settings (GOSS) problem. In this case, it is possible to state under some assumptions (see [1]) the existence of the fixed point solution but not the uniqueness. Fig. 1(b) reports the relation between variables.

In this paper we focus on solving the problem (8), proposing several algorithms based on the general framework of MSA (Method of Successive Averages) [3, 4, 5].

3 Solution algorithms

In order to solve the fixed-point problem (8) we propose to use some algorithms based on an MSA general framework. The first formulation of MSA [3] generates a succession of feasible link flow vectors, $\boldsymbol{f}^k$, starting from an initial solution (usually $\boldsymbol{f}^0 = \boldsymbol{0}$). At each iteration k the solution $\boldsymbol{f}^k$ is generated by combining traffic flows obtained by a Stochastic Uncongested Network (SUN) assignment, $\boldsymbol{f}_{\text{SUN}}^k = \boldsymbol{\varphi}_{\text{SUN}}(\boldsymbol{c}^k)$, with the previous solution, $\boldsymbol{f}^{k-1}$. At each iteration k, in order to generate the solution $\boldsymbol{f}^k$, the results of the SUN assignment, $\boldsymbol{f}_{\text{SUN}}^k$, are averaged, with weight $1/k$, with the results of the previous iteration $\boldsymbol{f}^{k-1}$, with weight $(k-1)/k$ (see Fig. 2). Since the algorithm works directly on flows, it is also known as MSA-FA (Flow Averaging) [20].

```
# MSA-FA #
k = 0
f^0 = 0
        do while |f_SUN^k − f^(k−1)|/|f^(k−1)| ≥ ε
                k = k + 1
                c^k = χ(f^(k−1))
                f_SUN^k = φ_SUN(c^k)
                f^k = f^(k−1) + 1/k (f_SUN^k − f^(k−1))
        loop
f* = f^k
end
```

Fig. 2 - Code of MSA-FA algorithm.

Cascetta et al. [1] proposed an algorithm for solving the fixed-point problem (8) based on the MSA-FA; this algorithm updates, according to local control policies, the signal settings at each iteration (see Fig. 3). In this code, $\boldsymbol{g}^{start}$ is a feasible solution for $\boldsymbol{g}$ (for instance, the solution where effective green times are the same for all phases and the cycle time is a prefixed value of 90 s). This algorithm will be referred to as $\text{MSA-FA}^{\text{acp}}$.

```
# MSA-FA^acp #
k = 0
f^0 = 0
g^0 = g^start
        do while |f_SUN^k − f^(k−1)|/|f^(k−1)| ≥ ε
                k = k + 1
                c^k = χ(f^(k−1), g^(k−1))
                f_SUN^k = φ_SUN(c^k)
                f^k = f^(k−1) + 1/k (f_SUN^k − f^(k−1))
                g^k = ω(f^k)
        loop
f* = f^k
g* = g^k
end
```

Fig. 3 - Code of $\text{MSA-FA}^{\text{acp}}$ algorithm.

Moreover, Cascetta et al. [1] proposed to modify the MSA algorithm substituting the averaging factor $1/k$, where k represents the number of the current iteration, with a different sequence of numbers. In general, it is possible to substitute to the ratio $1/k$ a general function $1/\xi(k)$, where $\xi(k)$ can be defined in several ways. It can be a numerical series (a succession of integers) or an actual function (succession of real numbers). Obviously, for $\xi(k) = k$ we obtain the *classic* MSA algorithms.

In the literature, over the method proposed in [1], devoted to the combined assignment-control problem, other authors proposed to modify the averaging factor of MSA algorithm, even if for the assignment problem. In this paper, we test and compare these methods proposed in the literature, adapting them to the solution of the combined assignment-control problem. The functions that we will test are the follows.

Refresh Memory (RM) function [1]*:*

$\xi = 1, 2, \ldots, \zeta, 2, 3, \ldots, 2\,\zeta, 4, 5, \ldots, 4\,\zeta, \ldots, x, x+1, \ldots, x\,\zeta, \ldots$ $\quad x$ integer $\quad x = 1, 2, 4, 8, \ldots$

where ζ is a parameter that Cascetta et al. [1] proposed to assume equal to 10. In the following,

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the corresponding algorithm will be indicated by MSA-FA-RM.

Polyak method [21]*:*

$\xi(k) = k^{2/3}$

In the following, the corresponding algorithm will be indicated by MSA-FA-POL.

Nagurney and Zhang method [22]*:*

$\xi(k)$ = 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, …, x, x, ..., x (x times), ... x integer $x \rightarrow \infty$

In the following, the corresponding algorithm will be indicated by MSA-FA-NAZ.

Bar-Gera and Boyce method [23]*:*

$\xi(k) = \zeta$

where ζ is a constant parameter between 1 and 50. According to the results reported in [23] we will test the method assuming ζ equal to 5. In the following, these algorithms will be indicated by MSA-FA-BGB.

Fig. 4 shows a comparison between $1/\xi$ values for the averaging functions (k from 1 to 100).

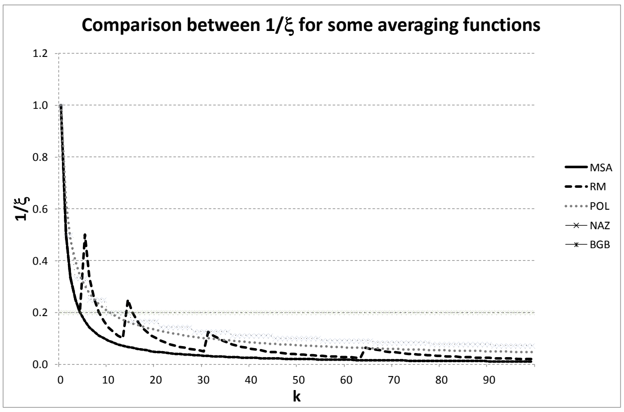

Fig. 4 - Comparison between averaging functions.

It can be noted that the RM method tends to increase the values of $1/\xi$, with respect to $1/k$, in successive waves, the POL methods give higher values of $1/\xi$, vis-à-vis $1/k$, for all iterations, without waves and, finally, NAZ and BGB methods assume constant values in some iteration intervals (BGB always constant values).

4 Numerical results

Numerical tests were conducted on the urban network of Benevento, a town in the south of Italy with about 61,000 inhabitants. The network, as represented in Fig. 5, has 1,577 oriented links, comprising about 216 km of roads, 678 nodes and 80 centroids (66 internal and 14 external). The peak-hour OD matrix was made available by the studies developed while drawing up the Urban Traffic Plan. The route choice model is Multinomial Logit with implicit enumeration of paths according to the algorithm proposed in [24].

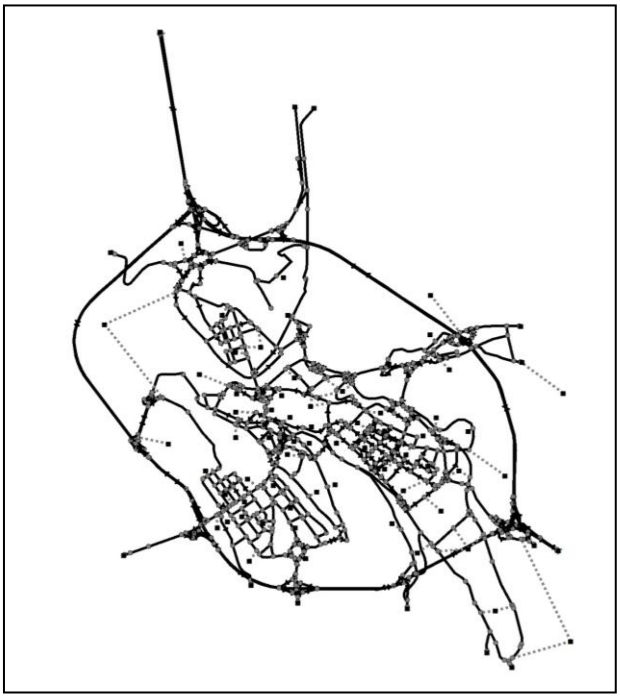

Fig. 5 - Real-scale test network.

The network contains only eight signalised intersections, but in order to expand the tests also to networks with higher percentages of such intersections, we built four other cases, increasing the number of signalised intersections. We thus tested five networks with: 8 (real case), 18, 28, 38 and 48 signalised intersections, the networks being labelled SIG08, SIG18, SIG28, SIG38 and SIG48, respectively.

In order to test the proposed algorithms under several network congestion levels, over the available OD matrix, four other matrices were generated, multiplying all cells of the OD matrix by the following factors: 0.8, 1.2, 1.4 and 1.6. We therefore have five matrices available, which we identify with OD08, OD10 (real case), OD12, OD14 and OD16, respectively.

Combining networks and OD matrices, we generated 25 different test cases, all based on the urban network of Benevento. All tests were conducted using a PC Intel core i5. For all tests we adopted as local control policy that proposed in [25] and tested the classic MSA-FA algorithm adapted for solving the combined assignment-control problem. In Table 1 the results are summarised in terms of computing times and iteration number.

	Iterations				
	OD08	**OD10**	**OD12**	**OD14**	**OD16**
SIG08	7	16	31	49	51
SIG18	7	16	32	49	51
SIG28	7	12	28	46	49
SIG38	6	9	28	46	50
SIG48	6	9	28	45	50
	Computing times (s)				
	OD08	**OD10**	**OD12**	**OD14**	**OD16**
SIG08	12	25	47	72	94
SIG18	11	25	48	73	95
SIG28	13	19	42	68	91
SIG38	14	15	42	68	92
SIG48	13	15	42	67	91

Table 1 - MSA-FA results.

The MSA-FA algorithms described in Section 3 were tested on the same network for the cases where it may be actually useful to reduce the number of iterations and hence computing times (case SIG48-OD16). Table 2 summarises the results in terms of iterations and computing times, compared with classic MSA-FA.

Algorithm	Iterations	Computing time (s)	Comparison of computing time vs. MSA
MSA-FA	50	91	-
MSA-FA-RM	15	27	- 70.33 %
MSA-FA-POL	14	26	- 72.53 %
MSA-FA-NAZ	13	24	- 73.63 %
MSA-FA-BGB	20	38	- 62.64 %

Table 2 - Comparison among MSA algorithms.

It may be noted that all algorithms different by the classic MSA perform better in terms of iterations and computing times; in particular, RM, POL and NAZ methods are able to reduce computing times by over 70 % with respect to the MSA.

Figs. 6-9 report the convergence of the tested algorithms compared with the corresponding MSA in the test case SIG48-OD16.

Reducing the computing time of the combined assignment-control problem is useful when it is a subroutine of a Network Design Problem [26, 27, 28, 29]. Indeed, in the NDP at each iteration it is necessary to calculate signal settings and equilibrium traffic flows. For instance, in [26] the algorithm used for solving the NDP examined up to 171,000 solutions, spending up to 370 hours in computing times; in this case, the 70 % saving in computing time results in a total of 111 hours, with 259 hours (more than 10 days) saved.

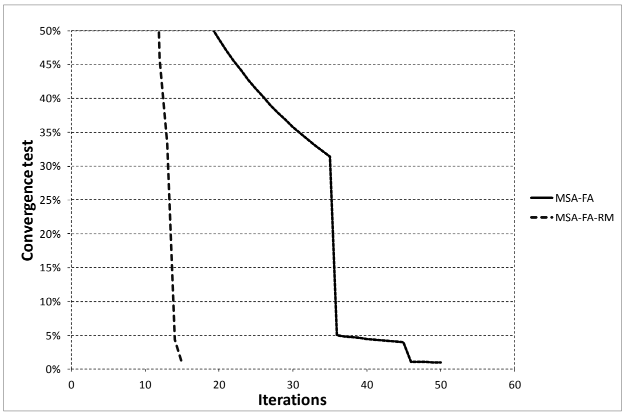

Fig. 6 - Convergence of the algorithm MSA-FA-RM.

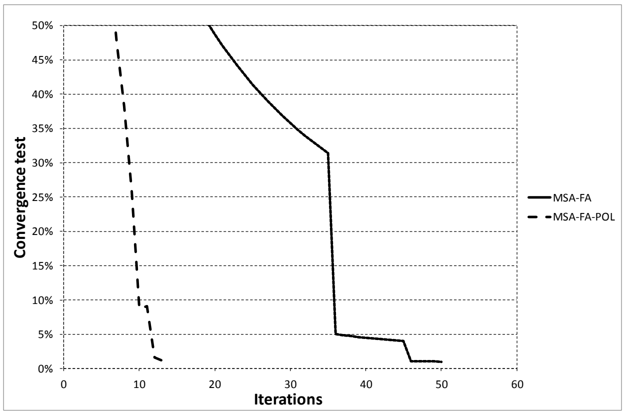

Fig. 7 - Convergence of the algorithm MSA-FA-POL.

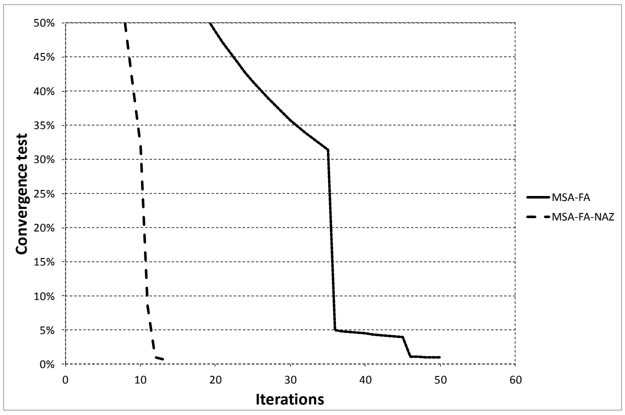

Fig. 8 - Convergence of the algorithm MSA-FA-NAZ.

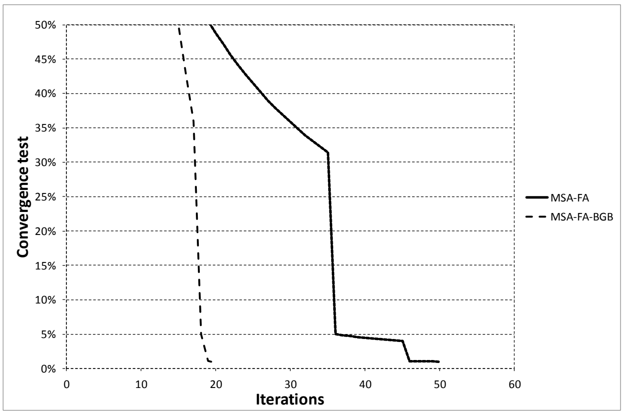

Fig. 9 - Convergence of the algorithm MSA-FA-BGB.

5 Conclusion and research prospects

In this paper we adapted and tested some algorithms based on the MSA framework in order to increase the convergence speed. Numerical results on a real-scale network highlighted that the computing times can be reduced until over 70% respect the classic MSA.

Future research will be addressed to generalise the MSA algorithm adopting different averaging functions in order to reduce computing times.

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