The dimensioning of pillars in the mining rooms and pillars method through a detailed evaluation of the stress conditions in the rock

FAHIMIFAR A.
Department of Civil and Environmental Engineering,
Amirkabir University of Technology,
Hafez Ave., Tehran,
Iran
fahim@aut.ac.ir

ORESTE P.
Department of Environment, Land and Infrastructure Engineering,
Politecnico di Torino,
Corso Duca degli Abruzzi 24, Torino,
Italy
pierpaolo.oreste@polito.it

RANJBARNIA M.
Department of Civil Engineering,
Tabriz University,
Iran
m.ranjbarnia@aut.ac.ir

Abstract: - The determination of the minimum side length of rock pillars in the room and pillar method is of fundamental importance in mining engineering. In the last decades some simple analytical equations were used for this aim: the area of influence method and different empirical formulations for obtaining the pillar strength. This kind of approach can lead to overdimensioning or critical stability conditions, with the risk of the miners being involved, should the pillars and the room collapse.

In order to conduct a detailed analysis of the stress conditions inside pillars, a parametric analysis with tri-dimensional numerical modelling was carried out. This made it possible to identify a critical point, where the minimum local safety factor is reached, at the corners of the pillar close to the roof of the mining room.

Through the estimation of the major principal stress at the critical point, obtained in function of the width/height ratio of the pillar, it was possible to evaluate the local safety factor in the aforementioned critical point, the geometric and geomechanical parameters of influence being known.

The dimensioning of the pillars through the local safety factor at the critical point makes it possible to avoid overdimensioning and static problems, which instead can occur when simplified calculation methods are used.

Key-Words: - Rock pillars, stress state, safety factor, rock strength, tributary area, local stress, pillar dimensioning.

1 Introduction
The collapse of rock from the roof of excavations or from the lateral surfaces of pillars is the cause of 15% of the accidents that involve workers inside mining voids created with the room and pillar method (Esterhuizen et al., 2011).

The dimensioning of rock pillars in underground mines, excavated by void methods, is one of the most interesting problems of rock engineering. Many studies were developed over the last century in order to go more in the detail on this subject, and to be able to conduct a dimensioning that would, on one hand, lead to the stability of underground voids and on the other to the maximum possible recovery of the mineralized rock.

The area of influence method was generally used in the last decades to estimate the mean axial stress in the pillar. Many researchers have developed formulations in order to determine the pillar strength and therefore the safety factor of the pillar.
However, the simplicity of the method has lead to great approximations being made in the evaluation of the stability of pillars. Esterhuizen et al. (2011), in studies on mines in the East of the United States, mined with the room and pillar method, revealed the collapse of pillars that showed safety factors of between 1 and 4, and with mean values of even 2.35. For this reason, the dimensioning of pillars in rock with the area of influence method requires relatively high safety factors (usually above 3).

Mortazavi et al (2009) have found, from a numerical calculation, that pillars behave in different ways during the post-rupture phase, in function of the width/height ratio: narrow pillars (low width/height ratio) show brittle elastic-plastic behaviour, with a remarkable decay of the mean axial stress of the pillar on reaching rupture; squat pillars (relatively high width/height ratio, above the unit) show ideal elastic-plastic behaviour, with the a constant mean axial stress value in the post-rupture phase as the mean axial deformations progress; very squat pillars (very high width/height ratio, above 1.5) show hardening elastic-plastic behaviour, with appreciable increases in the mean axial stress as the mean axial deformation progresses in the post-rupture phase.

On the basis of these considerations, the necessity emerges of requesting higher safety factors for narrow pillars, which could collapse suddenly once rupture is reached, while it is possible to assume lower safety factors for squat and very squat pillars, which could however show elevated strength, even in the presence of evident signs of rupture of the pillar.

Kaiser and Tang (1998) showed that when the elastic modulus of the rock on the roof of a void is much lower than that of the pillars, the rupture phase of a pillar is of a brittle elastic-plastic type, as the great energy accumulated by the rock on the roof and on the floor is suddenly discharged onto the pillar until it ruptures. Jaeger and Cook (1969) sustained that the rupture of the pillar can be violent in these cases, and rock blocks can even be thrown from the side walls. Again in these cases, given the great risks for the underground workers, connected to the rupture of the pillar, it is necessary to foresee high safety factors.

Today the area of influence method can not be used to dimension a rock pillar. Different methods, like tridimensional numerical modelling (Do et al., 2013; 2014a; 2014b), can be adopted to determine the stress strain state inside the pillar and therefore the local safety factors in the rock mass.

The work presented in this article was carried out with the aim of conducting a detailed analysis of the stress state induced inside pillars and of developing more sophisticated dimensioning techniques in order to drastically reduce the risk of the fall of rock blocks from the side surfaces of rock pillars and to avoid the risk of the collapse of a pillar, with the consequent collapse of the rooms and of the pillars nearby.

The stability of pillars without important internal natural discontinuities or layers of poor rock with a tendency to extrude is dealt with in the following sections: the presence of such anomalies can in fact lead to collapse phenomena concerning pillars with specific characteristics (Esterhuizen et al., 2011); this requires detailed analyses of the characteristics of the discontinuities (roughness, alterations, fillings, persistence, aperture, direction and dip) or of the layer of poor rock inside the pillar.

2 The area of influence method

A simplified picture of the stresses induced in a pillar can be obtained from a simple analysis of the equilibrium of forces. Figure 1 shows a horizontal section of an underground mining panel with pillars arranged in a regular pattern. The width of the room and pillars is I and wp, respectively.

Because of equilibrium of forces:

$$\sigma_p \cdot A_p = \sigma_{v0} \cdot A_c$$

Where:

- $\sigma_p$: mean vertical stress in the pillar;
- $A_p$: transversal section area of the pillar ($A_p = w_{px} \cdot w_{py}$);
- $A_c$: competence area of the pillar;
- $\sigma_{v0}$: lithostatic vertical stress at the depth of the chamber roof;
w_{px} and w_{py}: width of the pillars in the two horizontal directions x and y; 
\( l_x \) and \( l_y \): width of the chamber in the two horizontal directions x and y.

From which it is obtained:
\[
\sigma_p' = \sigma_{v0}' \cdot A_p' = \sigma_{v0}' \cdot \frac{(w_{px} + l_x)(w_{py} + l_y)}{(w_{px} \cdot w_{py})} 
\]
(2)

Figure 1. Simplified geometric scheme of an underground mining panel in the room and pillar method. a) Horizontal section; b) Vertical section.

The original natural vertical stress \( \sigma_{v0}' \) is not generally known, except for when certain determinations are conducted on site. In order to have a preliminary estimation, it is possible to refer to the simple expression \( \sigma_{v0}' = \gamma' \cdot z \), where z is the mean depth of the room roof. The estimation of the natural horizontal stress \( \sigma_{h0}' \) is instead much more problematic, but it is important above all when separation slabs are foreseen between the different mining levels. Even in the case of mining on only one level, the natural horizontal stress influences the stress-strain behaviour of the pillar, in particular close to the end zones at the top and bottom (Innaurato et al., 2000).

Eq. 2 shows that the mean axial stress induced in a pillar \( (\sigma_p') \) can be calculated by starting directly from the dimensions of the room and of the pillar and from the vertical stress found at the height of the room roof in natural conditions (before creating the mining voids).

The extraction ratio \( r \), that is, the ratio between the volume of rock extracted divided by the total volume of rock in the mining panel, is given by the following expression:
\[
r = \frac{A_c - A_p}{A_c} = 1 - \frac{A_p}{A_c} \quad (3)
\]

From equations 2 and 3, it is possible to write:
\[
\sigma_p' = \frac{\sigma_{v0}'}{1 - r}
\]
from which it is possible to see how the mean stress in the pillar grows hyperbolically to infinity with \( r \) as it comes closer to 1. The mean stress in the pillar increases considerably for small increases of \( r \) for \( r \) above the value of 0.75.

For square section pillars \( (w_p = w_{px} = w_{py}) \) and for rooms of the same width \( (l = l_x = l_y) \), equation 2 can be simplified in the following manner:
\[
\sigma_p' = \sigma_{v0}' \cdot \frac{(w_p + l)^2}{w_p^2} \quad (4)
\]

When the area of influence method is used to calculate the axial stress in the pillar, it is advisable to consider the following drawbacks:

- The mean axial stress in the pillar is purely a convenient quantity that is used to represent the loading state of a pillar in the vertical direction: in reality, the axial stresses can vary to a great extent inside the transversal section and can differ remarkably, according to the position of the transversal section of the pillar that is considered.
- The stresses in the two directions perpendicular to the axial one and the three shear stresses that exist in the tri-dimensional space inside the pillar are not considered; only the mean value of one of the six stress components that exist inside the pillar is considered.
- As the mean value of only the vertical stress is considered, the effects produced by the horizontal lithostatic stresses, and those of any possible difference in stiffness (elastic modulus) of the rock on the roof and on the foot compared to the stiffness of the rock of which the pillar is made are neglected.
- The effect of the position of the pillar inside the mining panel is ignored.
3 Considerations on the strength of pillars

The evaluation of the strength of a pillar, which should be compared with mean axial stress $\sigma_p$ to obtain the safety factor of the pillar, is the most important aspect of the problem, when the simplified approach of the area of influence method is adopted. The formulations that are available to obtain the strength of rock pillars have all been obtained from an analysis of the behaviour of real structures, imposing a safety factor value equal to 1 (conditions for which the mean axial stress is equal to the strength of the pillars) for all those cases in which the pillar has shown extreme stability conditions, at the limit of collapse, or with collapse already underway (Martin and Maybee, 2000; Esterhuizen et al., 2011).

For this reason, most formulations on the strength of pillars consider not only the volume of the pillar, which can in fact have a certain direct influence on the strength value of the rock mass, but also the shape (which is usually described by the slenderness $h_p/w_p$ or by its inverse), which reflects on the distribution of the stresses inside the pillar. In this way, one attempts to compensate for the great simplification that is made evaluating the induced stress state, taking into consideration one of the most important aspects (the shape) that can condition the development of the stresses inside the pillars.

The effect of the volume on the strength can be understood immediately in terms of distribution of the natural discontinuities of the rock mass. As the volume increases, so does the probability that there are more discontinuities in the rock.

Hoek and Brown (1980) clearly showed that a reduction in strength of the intact rock occurred as the diameter of a laboratory sample increased. These data were made dimensionless by dividing the individual strength values by the strength of a 50 mm diameter specimen, which is the typical dimension of a laboratory sample. The same authors suggested that the uniaxial compressive strength $\sigma_{cd}$ of a rock specimen, with a diameter $d$ in mm, depends on the uniaxial compressive strength $\sigma_{c50}$ of a 50 mm diameter specimen by the following relation:

$$\sigma_{cd} = \sigma_{c50} \cdot \left( \frac{50}{d} \right)^{0.18}$$

(5)

where:

$\sigma_{cd}$ is the uniaxial compressive strength of the intact rock, measured on a specimen of diameter $d$ (mm);

$\sigma_{c50}$ is the uniaxial compressive strength of the intact rock, measured on a 50 mm diameter specimen.

Experimental tests on specimen in the laboratory show that the increase in the dimension of the specimen leads to a reduction of its strength. The important definition of “critical dimension”, defined as the dimension of a specimen for which a further increase in the width does not cause a significant increase in strength, belongs to this context. Different authors have attempted to determine this magnitude, in consideration of the type of material the pillar is made of.

The definition of the critical magnitude is important because the strength values relative to the critical magnitude can be applied directly to the whole pillar.

The effect of the slenderness of the pillar basically arises from the stress confinement that develops inside the pillar, as a result of the impossibility of deforming laterally, which is imposed by the rock on the roof and floor of the pillar in correspondence to its upper and lower extremes: a slender pillar is less influenced by these constraints in the central portion of the pillar, while a squat pillar is more influenced, with a positive contribution to its stability.

Lunder and Pakalnis (1997) estimated that the mean confinement in a pillar ($\sigma_{3m}$) can be expressed in function of its slenderness $h_p/w_p$:

$$\frac{\sigma_{3m}}{\sigma_p} = 0.46 \cdot \left[ \log \left( \frac{1}{\left( \frac{h_p}{w_p} \right)} + 0.75 \right) \right]^{1.4 \cdot \frac{h_p}{w_p}}$$

Moreover, Maybee (1999) showed that the lateral confining stresses are negligible inside slender pillars ($w_p/h_p<0.5$), and that the influence of the natural lateral thrust coefficient $k$ (ratio between the horizontal and vertical stresses in natural conditions) on the lateral confinement stresses inside a pillar only occur in squat pillars ($w_p/h_p>1$).

However, the influence of the coefficient $k$ is only felt for $w_p/h_p > 1.25$.

For all the reasons mentioned above, the strength of a pillar is generally evaluated in function of the two aforementioned parameters (volume and shape):
\[ \sigma_{str} = \sigma_{str,0} \cdot V_p \cdot \left( \frac{h_p}{w_p} \right)^a \left( \frac{w_p}{h_p} \right)^b \]  
(6)

where:

- \( \sigma_{str,0} \) is the reference uniaxial compressive strength (referring to a unitary volume);
- \( V_p \) and \( h_p \) are the volume and height of the pillar;
- \( w_p \) is the minimum dimension of the pillar.

As an alternative, simpler expressions can be used of this type:

\[ \sigma_{str} = \sigma_{str,0} \cdot h_p^a \cdot w_p^b \]  
(7)

The values of the exponents \( a \) and \( b \), or \( \alpha \) and \( \beta \), which different authors have found in their studies, are reported in Salamon and Munro (1967). The typical values of these parameters are: \( \alpha = 0.5 \pm 1; \beta = 0.46 \pm 0.50; a = 0.07 \pm 0.17; b = 0.4 \pm 0.8 \).

Hardy and Agapito (1977) compared the volume and slenderness of a pillar with the volume and slenderness of a laboratory specimen, in bituminous schists, in order to obtain the strength of the pillar \( \sigma_{str} \) as the percentage reduction of the uniaxial compressive strength of the intact rock \( \sigma_{ci} \):

\[ \frac{\sigma_{str}}{\sigma_{ci}} = \left( \frac{V_p}{V_s} \right)^{-0.118} \cdot \left( \frac{h_s}{h_p} \right)^{0.833} \left( \frac{w_s}{w_p} \right) \]  
(8)

where:

- \( V_p \) and \( V_s \) are the volume of the pillar and of the rock specimen for the uniaxial compressive test in the laboratory, respectively;
- \( h_s \) and \( w_s \) are the height and the diameter of the laboratory specimen, respectively, for which \( \sigma_{ci} \) is evaluated;
- \( \sigma_{ci} \) is the uniaxial compressive strength measured in the laboratory on intact rock.

Many other formulas have been developed over the years, and each of these refer to the specific situations for which they were obtained. Extrapolation to other situations (in relation to the type of rock, to the dimensions and shape of the pillars and to the lithostatic stress state) could even lead to relevant errors.

Martin and Maybee (2000), for example, evaluated the \( \sigma_{str} / \sigma_{ci} \) ratio for a 5 m high pillar on the basis of different empirical correlations, varying the \( w_p/h_p \) ratio (the inverse of slenderness). Pillar strength values that are very different from each other were found: \( \sigma_{str} \) varies in the 0.2-0.4 · \( \sigma_{ci} \) interval for \( w_p/h_p=0.5 \) (slender pillar) and varies in the 0.4-0.8 · \( \sigma_{ci} \) interval for \( w_p/h_p=3.0 \) (very squat pillars).

Gonzalez-Nicieza et al. (2006) obtained the strength of a pillar of 10 m x 10 m at the bottom, varying the height of the pillar, according to different empirical correlations taken from literature, considering a uniaxial compressive strength of the intact rock equal to 100 MPa. Again in this case, there was a great dispersion of the results obtained with the different empirical formulations. For a height of 5 m, the strength of the pillar varied as much as 20-80 MPa, while for a height of 15 m, it varied in the 8-48 MPa interval.

It is obvious that a designer, when faced with this large variability of strength values of pillars, would cautiously choose the minimum value from those proposed by various authors (the value proposed by Potvin et al., 1989 for \( w_p/h_p \) below 0.75 or that proposed by Krauland and Soder (1987) for \( w_p/h_p \) equal to or higher than 0.75; the value proposed by CMRI–Central Mining Research Institute in Dhanbad in India (Sheorey et al., 2000) for \( h_p \) below 7 m or that proposed by Hardy and Agapito (1977) for \( h_p \) equal to or higher than 7 m). This uncertainty can, in the best of cases, lead to an over-dimensioning (sometimes also excessive) of the pillars. Instead, if the minimum value of the strength estimated by the different empirical correlations is not assumed, it is possible to run the risk of foreseeing pillars that are not able to bear the loads produced by the rock on the roof, which would inevitably lead to their collapse.

This important variability of the strength values of pillars leads to the conclusion that the simplistic approach of the areas of influence can no longer be accepted, and that it is necessary to obtain a complete and reliable evaluation of the stress state inside pillars and to define exact safety factors inside the rock that makes up the individual pillars.

The observation of pillars in critical stability conditions allows us to establish the portions of the pillars that tend to break off first, when collapse does not occur suddenly. Gonzalez-Nicieza et al. (2006) have identified different degrees of criticality of a pillar, from the stress point of view: damage to one or more cornerstones and their tendency to
become round; damage to one or more lateral faces (such a degree of criticality is considered as the evolution of the previous one, when the stress state induced in the pillar is such that it leads to the rupture and breaking off of the rock between the already damaged cornerstones); rupture that occurs in the pillar which leads to the further detachment of rock from all the lateral walls until the typical hourglass shape is reached.

4 Analyses of the stress state and of the local safety factors in pillars with a tri-dimensional numerical method

An extended parametric analysis using the Flac 3D (Itasca, 2006) numerical method was performed in order to analyse the behaviour of single rock pillars (Guarascio and Oreste, 2012; Oreste, 2012), considering the most frequent geometries for square pillars and deep excavations.

An elastic behaviour of the rock mass was considered: the value of the elastic modulus, however, has not any effect on the results. The pressure applied on the upper surface of the model was equal to 1 MPa: the stress state induced in the rock pillar is linear dependent of this pressure, that is influenced by the depth of the excavations.

The adopted numerical elements were cubic with dimension of 0.25 m.

After having attributed the mechanical properties of the rock and the boundary conditions on the boarders, the elements in the chamber zones around the pillar were annulled simulating the mining excavation of the chambers.

The analysed pillar geometries are reported in table 1 (Oreste, 2012). A wp/hp ratio interval of 0.2-1.2 and l/wp ratio interval of 1.6-3.3 were considered in the parametric analyses. They are typical interval values for mining excavations in medium-high strength rocks.

All the stresses, in particular the major (σ1) and minor (σ3) principal stresses, were monitored for each element. These stresses are of fundamental importance to determine the exact local strength of the rock mass inside the pillar and the exact local safety factor concerning rupture of the rock mass.

The values of the major and minor principal stresses along different alignments inside the pillar for the case of wp/hp=0.70 and l/wp=2.43 are shown in figures 2-3 as examples.

It is possible to note how the major principal stresses at mid-height of the pillar reach a value, in the portion close to the pillar axis, of about 1.03 times the mean vertical stress σp, a value of 0.98·σp at half of one of its sides and a value of about 0.94·σp in the corners. At an intermediate height of ¾· hp (that is, at a distance of ¼·h from the top of the pillar), the major principal stress in the zone close to the pillar axis no longer show the maximum value and have a value of about 0.98·σp; this stress reduces slightly at mid-height of the side (0.97·σp) and rises to 0.98·σp in the corner. Finally, the major principal stress close to the pillar axis is very low (about 0.80·σp) at the summit of the pillar (at a distance of about 1/8·hp from the roof), while it grows considerably moving towards the peripheral zones of the pillar: 1.12·σp at half the side and 1.43·σp in the corner.

Table 1. Dimensions of the pillars and of the model and wp/hp and l/wp ratios values in the performed tri-dimensional numerical analyses.

<table>
<thead>
<tr>
<th>Width of the pillar (m)</th>
<th>l/wp ratio</th>
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Figure 2. Trend of the major principal stress in different alignments inside a square pillar (wp/hp=0.70 and l/wp=2.43). Key: mid height: at
½·hp from the pillar foot; ¾ height: at at ½·hp from the pillar foot; central: horizontal alignment passing through the pillar axis; periphery: horizontal alignment passing over the pillar side; intermediate: horizontal alignment between the central and periphery alignment.

From an analysis of the stress state, with particular reference to the major principal stresses (\(\sigma_1\)) and minor principal stresses (\(\sigma_3\)), it emerges that a point of great interest in the evaluation of the stability conditions of the rock is the corner in the upper portion of the pillar, close to the point of contact with the room roof (at a distance of 1/8·hp from the point of contact with the roof). The major principal stress in fact reaches its maximum value in this critical point and the minor principal stress is reduced to zero (absence of lateral confinement).

Figure 3. Trend of the major principal stress in different alignments inside a square pillar (\(w_p/h_p=0.70\) and \(l/w_p=2.43\)). Key: summit: close to the pillar top; central: horizontal alignment passing through the pillar axis; periphery: horizontal alignment passing over the pillar side; intermediate: horizontal alignment between the central and periphery alignment.

This circumstance has been verified in all of the performed numerical models. From an analysis of the obtained results, it has been possible to identify the influence of the \(w_p/h_p\) ratio on the major principal stress that develops at the critical point. The relation obtained through a linear regression of the results is as follows:

\[
\frac{\sigma_1'}{\sigma_p'} \approx 1.27 + 0.28 \cdot \frac{w_p}{h_p}
\]

where:

\[
\frac{\sigma_1'}{\sigma_p'}
\]

is the ratio between the major principal stress and the mean stress in the pillar at the previously described critical point.

Therefore, an increase in the major principal stress in the critical point can be observed for an increase in the \(w_p/h_p\) ratio.

For the Hoek and Brown strength criterium of the rock mass (Hoek and Brown, 1980, 1997; Hoek et al., 2002), the strength stress (\(\sigma_{1,str}\)) is dependent of the minimum principal stress (\(\sigma_3\)), the uniaxial compressive strength of the intact rock (\(\sigma_{ci}\)) and three strength parameters (Hoek et al., 2002; Hoek, 2007), which are function of GSI and the disturbance factor D (Hoek and Brown, 1997; Marinos and Hoek, 2000; Cai et al., 2004).

Cai et al. (2004) have stated that the GSI index can be applied for rock masses with a GSI<75. The discontinuities are not very frequent for rock masses with GSI>75, and the behaviour of the rock mass is conditioned by the presence of single discontinuities that represent zones of weakness inside the intact rock. In these cases, it is necessary to evaluate the stress and strain state that develops along the discontinuities in order to be able to obtain indications on the degree of stability of the pillar. When the GSI is close to 100 (the maximum value that can be reached with this index), the rock mass is affected by rare and not very persistent discontinuities, and can therefore be assimilated to the intact rock that is studied in the laboratory.

The local safety factor of a point inside a rock pillar can be obtained by the following equation:

\[
F_s = \frac{\sigma_{1,str}}{\sigma_1}
\]

where:

\(\sigma_{1,str}\) is the strength stress in the rock mass;
\(\sigma_1\) is maximum principal stress in a point inside the pillar.

In this way, it is possible to evaluate the trend of the local safety factors inside the rock mass and, in particular, to identify the zones with a minimum safety factor. The pillar can therefore be dimensioned by imposing that the minimum local safety factor in the pillar is above a certain threshold value.

Safety factors were evaluated in each numerical element into which the pillar was divided for the 18
geometrical cases analysed by means of the tri-dimensional numerical analysis and illustrated above, considering 27 different types of rock, obtained by permuting the following parameters:

- GSI index: 50, 75, 100;
- m index for the intact rock: 12, 20, 28;
- uniaxial compressive strength of the intact rock \( \sigma'_{ci} \): 20, 60, 100 MPa.

These values describe the typical field of variation of rock masses with medium-high strength, in which it is normal practice to proceed with the room and pillar method.

Moreover, three different values of \( \sigma'_{ci0} \) (vertical lithostatic stress at the depth of the mining room roof) were considered: 1, 5 and 9 MPa (values that roughly correspond to the following depths: 40, 200 and 360 m).

In this way, the local safety factors inside the pillar were evaluated, considering 27 (types of rock) x 3 (vertical lithostatic stresses) x 18 (geometrical cases analysed in the tri-dimensional models) for a total of 1458 types of pillar.

The minimum local safety factor inside the pillars was determined for each of these cases. After having discarding the minimum safety factors with a value below unity (unacceptable and therefore unrealistic condition), and those with a too high value (above 5), it was found that in all cases the minimum safety factor was located at the corners, close to the point of contact with the rock on the roof of the mining room (the critical point described in the previous section, found at a distance of about 1/8 the height of the pillar from the point of contact with the roof). The lateral confinement stress is null at this point. Moreover, the major principal stress (which coincides with \( \sigma_z \)) can be estimated in function of the \( w_p/h_p \) ratio and of the vertical lithostatic stress at the depth of the roof of the mining room (eq.9). Therefore, on the basis of eqs 4, 9 and 10, and of the Hoek and Brown strength criterium, it is possible to write the minimum local safety factor inside the pillar with the following simple relation:

\[
F_{s,\text{min}} = \frac{\sigma'_{ci} \cdot w_p^2}{\sigma'_{ci0} \cdot (w_p + l)^2} \left( \frac{GSI - 100}{9 - 3D} \right)^2 \left( \frac{1}{2} \right) \left( \frac{GSI}{100} \right)^{1/6} \left( \frac{w_p - h_p}{w_p + h_p} \right)^{2/3}
\]

(11)

Once the depth of the pillar (and therefore the lithostatic stress state \( \sigma'_{v0} \)), the mechanical characteristics of the intact rock \( \sigma'_{ci} \) and of the rock mass (GSI and the disturbance intensity parameter D), and the dimensions of the mining room (width I and height h,) are known, and imposing that the minimum local safety factor is above a certain threshold value \( F_s^* \), it is possible to calculate the minimum dimension of \( w_p \) for which \( F_{s,\text{min}} \geq F_s^* \).

5 Conclusion

The area of influence method and some empirical formulations were adopted in the last decades for the evaluation respectively of the mean axial stress in the pillar and the pillar strength. The application of these formulations from the literature to practical cases can lead to results that are very different from each other and can also lead to overdimensioning or critical stability conditions of the pillars.

Moreover, pillars, dimensioned with simplified analytical formulations, that even had safety factors equal to 4, have collapsed. This simplified design approach can therefore be considered no longer acceptable.

In this paper a detailed analysis of the stress state of pillars using tri-dimensional numerical modelling was developed. In this way, it has been possible to evaluate the stresses inside pillars in a precise manner, and in particular, the major principal stresses and minor principal stresses. An extensive parametric analysis has made it possible to analyse 18 different geometric conditions, for typical variations in the width/height and interaxis/width ratios of a pillar. Moreover, 3 different depths of the mining room and 27 types of rock mass, including rock masses with a mean-high strength for which the room and pillar mining method is usually adopted, were hypothesised. Reference was made to the well known Hoek and Brown strength criterion and the GSI index that describes the frequency and conditions of natural discontinuities in order to obtain the local strength of the rock mass. The local safety factor inside the pillars was evaluated for each analysed case as the ratio between the local strength of the rock mass and the existing major principal stress.

From the analyses of all the cases it was possible to show how the critical point inside the pillars, the one that has the minimum local safety factor, is
always located at the same point, at the corners close to the roof of the mining room (at a distance of about 1/8 the height of the pillar from the roof of the room). On the basis of these results, it has been possible to determine the major principal stress in the critical point in function of the width/height ratio of the pillar.

Thanks to the developed parametric study, the local safety factor has been calculated at the critical point, in function of the parameters of influence (geometry of the problem, characteristics of the rock mass, lithostatic stress state at the mining room depth).

The dimensioning of the pillars, on the basis of the local safety factor at the critical point, makes it possible to avoid the risk of overdimensioning or criticality of the pillars from the static point of view and to be sure that no point in the pillar can reach rupture. In this way, it is possible to avoid evolutionary phenomena of global collapse of the pillar, which can sometimes occur suddenly, and phenomena of localised detachment of rock.

References: