Analyses of truss structures via total potential optimization implemented with teaching learning based optimization algorithm

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Abstract: - According to the well-known principle of mechanics named minimum potential energy, if the total potential energy of a system is minimum, the system is in equilibrium state. Conventional methods use mathematical operations to find minimum potential energy of structures. Alternatively, metaheuristic algorithms that are frequently used for minimization or maximization of an objective function can be employed for this purpose. Thus, Total Potential Optimization using Meta-heuristic Algorithms (TPO/MA) technique has been proposed. In this paper, TPO/MA technique has been presented and used for the analyses of truss structures. The metaheuristic algorithms employed in the present study is teaching learning based optimization (TLBO) method. The named method was tested for three different systems and the results are compared with the documented methods. The proposed method is found to be effective, robust, powerful and accurate for analysing planar and space truss structures.

Key-Words: - meta-heuristics; teaching learning based optimization; total potential optimization method; truss structures.

1 Introduction

Metaheuristic methods such as genetic algorithm (GA) [1-2], particle swarm optimization (PSO) [3], ant colony (ACO) optimization [4], harmony search (HS) algorithm [5], firefly algorithm (FA) [6], bat algorithm (BA) [7] are becoming successful methods for solving optimization problems.

Rao at al. [8] has recently developed a metaheuristic algorithm from the inspiration of teaching-learning process in a classroom and it is called teaching learning based optimization (TLBO) algorithm. Although each metaheuristic algorithm has special parameters, TLBO is proposed as a simple algorithm without using any parameter. In a very short time, TLBO algorithm has been applied to a wide variety of engineering optimization problems including mechanical design, electrical power generator, robot gripper design, and structural design [9 - 14].

Total Potential Optimization using Meta-heuristic Algorithms (TPO/MA) technique has been recently proposed and successfully applied for the analyses of various structural systems including, linear and nonlinear trusses, cable structures and tensegrity structures. In this technique, the metaheuristic algorithms are used for finding the minimum potential energy of a structural systems, instead of mathematical expression that employed in the conventional analysis methods.

In this paper, the analyses of planar and space truss structural systems are presented by using TPO/MA technique. As metaheuristic algorithm TLBO algorithm has been used. The proposed approach has been performed on three numerical examples and the results of the analysis are compared with other applications that employs local search (LS) [15], GA [16], ACO [17], HS [18], PSO [19] and with FEM.

2 Methodology

Teaching learning based optimization (TLBO) algorithm is a metaheuristic algorithm inspired from the teaching-learning process [8]. In this process, teacher is a main character that has deep knowledge about the subjects and attendants to improve the knowledge of learners. Additionally, learners has important effect at the learning process by interaction, researcher, sharing information, communication between each other. Thus, TLBO algorithm searches the optimum results by two main parts called “teacher phase” and “learner phase”.

This algorithm determines the minimum potential energy of a structure in five steps.

Step 1- Data entering: In the first step, the design constraints such as material properties, cross-section dimensions, boundary conditions of joints, loading conditions, coordinates of joints are defined. Also, population size of the classroom (total number of learners) and the maximum iteration number (in order to stop the optimization process) are defined. As stated in the previous section, TLBO algorithm do not use any further parameters specific to the algorithm.

Step 2- Generation of initial class (solution matrix): In this step, initial solution matrix is generated. Size of this matrix is equal to population size (total number of learners or students) of the classroom. Each learner contains randomly generated joint coordinates of the structure. These coordinates correspond to the deformed shape of system under defined loading condition.

By using these coordinates, the strain energy of deformed system (StE), work done by external loads (WEL) and total potential of the system (TPs) can be calculated [15]. Thus, each generated solution vector (learner) has a specific TP value. Objective of the optimization process is to minimize the TP value. Consequently, the system is analyzed by determining joint coordinates (deformed shape of system) that makes the system with minimum potential energy under defined loading condition.

Step 3- Teacher phase: Iterative process begins in this step. First of all, because teacher is the person with deep knowledge, the variables with minimum objective is assigned as teacher ($X_{teacher}$).

$$X_{teacher} = X_{min\, f(X)} \tag{1}$$

Then, as teacher tries to improve mean knowledge (X_{mean}) of the all learners, each stored solution ($X_{old,i}$) is updated ($X_{new,i}$) according to best one ($X_{teacher}$).

$$X_{new,i} = X_{old,i} + rnd\left(0,1\right)\cdot\left(X_{teacher} - T_F \cdot X_{mean}\right) \tag{2}$$

where *rnd* is a random number within the range [0, 1] and T_F is teaching factor determined as

$$T_F = round\left[1 + rnd\left(0.1\right)\right] \rightarrow \left\{1-2\right\} \tag{3}$$

If the updated solution is better than old one, the old vector replaced with the new vector. This process is applied to each learner in the class (i=1 to population size)

Step 4- Learner phase: Learner phase simulates the contributions of the learners to knowledge level of the classroom. This contribution arise from interactions of learners. In the TLBO, learner phase can be written as

$$X_{new,i} = \begin{cases} X_{old,i} + r_i \cdot \left(X_i - X_j\right); & f\left(X_i\right) > f\left(X_j\right) \\ X_{old,i} + r_i \cdot \left(X_j - X_i\right); & f\left(X_i\right) < f\left(X_j\right) \end{cases} \tag{4}$$

where X_i and X_j are randomly selected learners that must not be the same. The new solution is accepted, if it is better than the old one.

Step 5- Control of stopping criteria: In the last step, the stopping criteria (maximum iteration number) defined in 1st step is checked. If this criteria is satisfied, the optimization process is ended. If not, the process is continued from the teacher phase (3rd step).

Pseudo code of the optimization process can be written as follows;

Randomly generate the initial students
Calculate objective function
While stopping criteria
    (Teacher Phase)
    Calculate the mean of each design variable
    Identify the best student as teacher
    For i=1:$N_{variable}$
        Calculate teaching factor Eq. 3
        Create a new solution based on teacher Eq. 2
        Calculate objective function for the new solution
    If X_{new} is better than X_{old}
            $X_{old} = X_{new}$
        End If
    End For
    (Learner Phase)
    For i=1:$N_{variable}$
        Select any two solution randomly [*i, j*]
        Create a new solution based on selected solutions Eq. 4
        Calculate objective function for the new solution
    If X_{new} is better than X_{old}
            $X_{old} = X_{new}$
        End If
    End For
End While

3 Numerical Examples

Three different numerical example are presented in this section. The results of presented method are compared with others given in the literature such as LS [15], GA [16], ACO [17], HS [18] algorithm, PSO [19] and with the results obtained by the well-known technique FEM. Also, in order to see effect population size of the class on the optimum results, optimization process was performed for 5, 10, 20, 30 and 40 learners. Thus, the sole parameter related with TLBO that may effect on the results was also investigated.

First example is a 2-bar plane truss structure (Fig. 1). The cross-sectional area of members and elasticity modulus of material are 9677 mm^2 and 68941 N/mm^2, respectively.

The analyses is carried out for 10 different concentrated load (*P*). The relationship between load and joint displacement can be seen in Fig. 2. Minimum total potential energy of system for each loading condition is given Table 1. Although, for smaller intensities of loading the energies and displacements values obtained for all methods seem compatible, for bigger loads FEM linear analyses diverge from other results, as expected. This means for bigger loads, geometrically nonlinearity behavior becomes an important factor on the results and this behavior must be taken into account during the analyses.

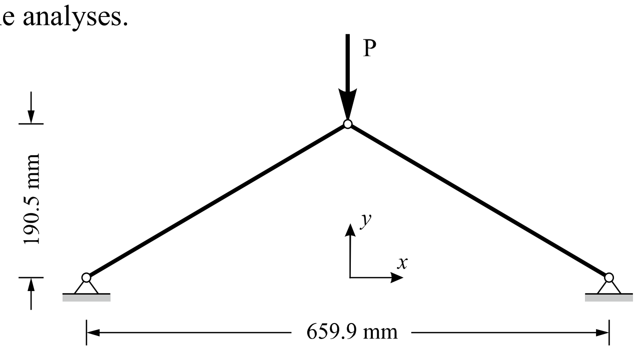

Fig. 1. 2-bar plane truss system

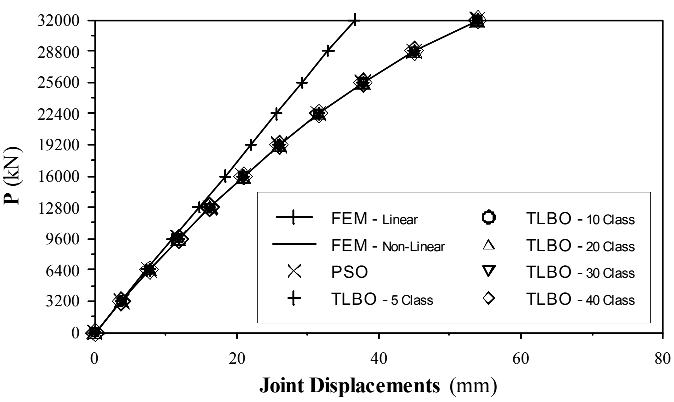

Fig. 2. Load-displacement diagram for example 1

Table 1: Minimum potential energy values for example 1

P (kN)	Total Potential Energy (kNm)							
	FEM Linear	FEM Nonlinear	PSO [18]	TLBO (5 Class)	TLBO (10 Class)	TLBO (20 Class)	TLBO (30 Class)	TLBO (40 Class)
3200	-5.93	-5.93	-5.93	-5.93	-5.93	-5.93	-5.93	-5.93
6400	-24.07	-24.11	-24.11	-24.11	-24.11	-24.11	-24.11	-24.11
9600	-54.93	-55.17	-55.17	-55.17	-55.17	-55.17	-55.17	-55.17
12800	-98.95	-99.83	-99.83	-99.83	-99.83	-99.83	-99.83	-99.83
16000	-156.81	-159.00	-159.00	-159.00	-159.00	-159.00	-159.00	-159.00
19200	-228.73	-233.77	-233.77	-233.77	-233.77	-233.77	-233.77	-233.77
22400	-315.66	-325.52	-325.52	-325.52	-325.52	-325.52	-325.52	-325.52
25600	-420.58	-436.10	-436.10	-436.10	-436.10	-436.10	-436.10	-436.10
28800	-535.55	-568.12	-568.12	-568.12	-568.12	-568.12	-568.12	-568.12
32000	-669.33	-725.72	-725.72	-725.72	-725.72	-725.72	-725.72	-725.72

For the second example, plane truss system given in Fig. 3, is investigated. Cross-sectional area of elements 1, 5 and 6 is 200 mm^2; cross-sectional area of the other members is 100 mm^2. Elasticity modulus of material and joint load (P) are 200000 N/mm^2 and 150 kN, respectively.

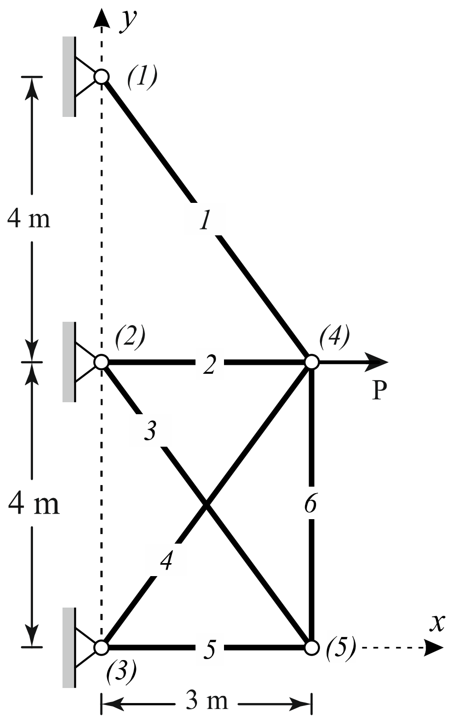

Fig. 3. 6-bar plane system

In Fig. 4 and Table 2, the results for different population sizes can be seen. Although, the same minimum energy values are obtained for all population sizes, convergence speed (computational cost) of 10 learners seems to be the most suitable one.

In Table 3, comparisons with the results from literature are presented. It can be seen that, except the FEM linear one, all results are nearly the same. As it stated in first example, this difference is due to the nonlinear behavior of system under given loads.

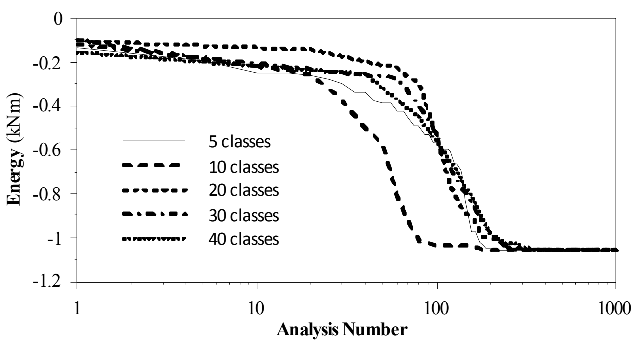

Fig. 4. Convergence speed of optimum results for different population sizes (example 2)

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Table 2: Analysis results of 6-bar plane truss structure system by TLBO different classes

		TLBO (5 Class)	TLBO (10 Class)	TLBO (20 Class)	TLBO (30 Class)	TLBO (40 Class)
Joint Displacements (mm)	u4	14.119	14.120	14.120	14.120	14.120
	v4	2.828	2.828	2.828	2.828	2.828
	u5	0.302	0.302	0.302	0.302	0.302
	v5	2.317	2.317	2.317	2.317	2.317
Member Forces (N)	1	49806.86	49811.68	49811.68	49811.68	49811.68
	2	94135.52	94142.17	94142.17	94142.17	94142.17
	3	-6688.53	-6688.53	-6688.53	-6688.53	-6688.53
	4	42971.97	42974.38	42974.38	42974.38	42974.38
	5	4038.60	4038.60	4038.60	4038.60	4038.60
	6	5348.61	5348.64	5348.64	5348.64	5348.64
Energy (kNm)	Min.	-1.059735	-1.059735	-1.059735	-1.059735	-1.059735
	Max.	-0.739129	-1.059735	-1.059735	-1.059735	-1.059735
	Avg.	1.056459	-1.059735	-1.059735	-1.059735	-1.059735
	St. Dev.	0.031897	0	0	0	0

Table 3: Analysis results of 6-bar plane truss structure system by different methods

		FEM		LS [15]	GA [16]	ACO [17]	HS [18]	PSO [19]	TLBO
		Linear	Nonlinear						
Joint Displacements (mm)	u4	14.150	14.120	14.12	-	14.12	14.118	14.12	14.120
	v4	2.843	2.828	2.83	-	2.83	2.830	2.83	2.828
	u5	0.300	0.302	0.30	-	0.30	0.304	0.30	0.302
	v5	2.309	2.317	2.32	-	2.32	2.319	2.32	2.317
Member Forces (N)	1	49725.13	49810.25	49811	51015.0	49811	49789.18	49811	49811.68
	2	94331.33	94140.09	94142	94140.6	94142	94131.58	94142	94142.17
	3	-6669.14	-6688.40	-6688	-6552.8	-6688	-6687.50	-6688	-6688.53
	4	43055.99	42973.59	42974	42029.5	42974	42977.97	42974	42974.38
	5	4001.49	4034.16	4035	3963.8	4035	4074.45	4035	4038.60
	6	5335.31	5351.45	5352	5284.3	5352	5354.77	5352	5348.64
Energy (kNm)		-1.059727	-1.059735	-1.059735	-1.0560	-1.059735	-1.059734	-1.059735	-1.059735

Third and the last example is a 25 bar space structure (Fig.5). Modulus of elasticity and cross-sectional area of all members are 200000 N/mm^2 and 10 mm^2, respectively. The system was analyzed under 3 different load cases given in Table 4.

Table 4: Load cases for 25-bar truss example (kN)

Node	Load Case 1			Load Case 2			Load Case 3		
	F_x	F_y	F_z	F_x	F_y	F_z	F_x	F_y	F_z
1	0	80	-20	0	800	-200	800	800	-200
2	0	-80	-20	0	-800	-200	0	0	0

The results obtained by using FEM (nonlinear), HS, PSO and presented approach (TLBO) are given in Table 4-6 for load cases 1-3, respectively. As seen from Tables 5-7, nearly the same minimum potential energies are obtained for all approaches. This conclusion can also be observed from the graphs (Figs. 6-8) showing the convergence behavior and statistical evaluation of 100 independent analyses of present approach (Table 8).

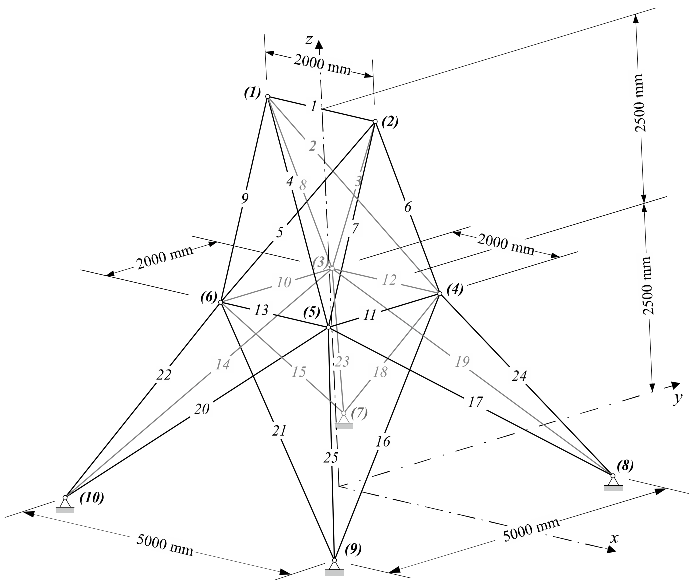

Fig. 5. 25-bar space truss system

Table 5: Analysis results of 25-bar space truss system by different methods (Loading 1)

		Loading 1							
		FEM Nonlinear	HS [18]	PSO [19]	TLBO (5 Class)	TLBO (10 Class)	TLBO (20 Class)	TLBO (30 Class)	TLBO (40 Class)
Joint Displacements (mm)	u(1)	0.000	-0.076	0.06	0.52	0	0	0	0
	v(1)	37.847	37.910	38.01	39.57	37.85	37.85	37.85	37.85
	w(1)	-37.199	-37.214	-37.27	-37.08	-37.2	-37.2	-37.2	-37.2
	u(2)	0.000	-0.064	0.06	0.1	0	0	0	0
	v(2)	-37.847	-37.766	-37.68	-35.99	-37.85	-37.85	-37.85	-37.85
	w(2)	-37.199	-37.153	-37.16	-35.09	-37.2	-37.2	-37.2	-37.2
	u(3)	0.867	0.892	0.86	0.91	0.87	0.87	0.87	0.87
	v(3)	-1.744	-1.731	-1.73	-1.96	-1.74	-1.74	-1.74	-1.74
	w(3)	-16.392	-16.342	-16.45	-16.34	-16.39	-16.39	-16.39	-16.39
	u(4)	0.867	0.924	0.88	0.81	0.87	0.87	0.87	0.87
	v(4)	1.744	1.750	1.76	1.54	1.75	1.74	1.75	1.75
	w(4)	-16.392	-16.355	-16.38	-15.22	-16.39	-16.39	-16.39	-16.39
	u(5)	-0.867	-0.822	-0.85	-0.83	-0.87	-0.87	-0.87	-0.87
	v(5)	1.744	1.751	1.78	2.03	1.74	1.75	1.74	1.74
	w(5)	-16.392	-16.402	-16.35	-15.07	-16.39	-16.39	-16.39	-16.39
	u(6)	-0.867	-0.844	-0.87	-0.77	-0.87	-0.87	-0.87	-0.87
	v(6)	-1.744	-1.746	-1.70	-1.31	-1.75	-1.74	-1.75	-1.75
	w(6)	-16.392	-16.430	-16.40	-16.01	-16.39	-16.39	-16.39	-16.39
Member Forces (kN)	1	75.693	75.676	75.690	75.55	75.69	75.69	75.69	75.69
	2	3.893	3.915	3.902	4.03	3.89	3.89	3.89	3.89
	3	3.893	3.883	3.869	4.04	3.89	3.89	3.89	3.89
	4	3.893	3.890	3.898	3.97	3.89	3.89	3.89	3.89
	5	3.893	3.885	3.881	4.14	3.89	3.89	3.89	3.89
	6	-13.883	-13.844	-13.879	-13.31	-13.88	-13.88	-13.88	-13.88
	7	-13.883	-13.875	-13.874	-13.34	-13.88	-13.88	-13.88	-13.88
	8	-13.883	-13.899	-13.909	-13.94	-13.88	-13.88	-13.88	-13.88
	9	-13.883	-13.894	-13.907	-13.93	-13.88	-13.88	-13.88	-13.88
	10	1.734	1.736	1.730	1.68	1.73	1.73	1.73	1.73
	11	1.734	1.745	1.730	1.64	1.73	1.73	1.73	1.73
	12	-3.489	-3.482	-3.490	-3.49	-3.49	-3.49	-3.49	-3.49
	13	-3.489	-3.497	-3.480	-3.34	-3.49	-3.49	-3.49	-3.49
	14	-3.395	-3.376	-3.413	-3.34	-3.39	-3.39	-3.4	-3.4
	15	-3.395	-3.412	-3.402	-3.4	-3.39	-3.4	-3.39	-3.39
	16	-3.395	-3.366	-3.385	-3.16	-3.39	-3.39	-3.39	-3.39
	17	-3.395	-3.412	-3.385	-3.05	-3.39	-3.39	-3.39	-3.39
	18	-4.656	-4.657	-4.660	-4.3	-4.66	-4.66	-4.66	-4.66
	19	-4.656	-4.643	-4.664	-4.72	-4.66	-4.66	-4.66	-4.66
	20	-4.656	-4.654	-4.656	-4.43	-4.66	-4.66	-4.66	-4.66
	21	-4.656	-4.662	-4.643	-4.4	-4.66	-4.66	-4.66	-4.66
	22	-7.367	-7.378	-7.385	-7.28	-7.37	-7.37	-7.37	-7.37
	23	-7.367	-7.355	-7.397	-7.3	-7.37	-7.37	-7.37	-7.37
	24	-7.367	-7.365	-7.361	-6.86	-7.37	-7.37	-7.37	-7.37
	25	-7.367	-7.358	-7.333	-6.66	-7.37	-7.37	-7.37	-7.37
Energy (kNm)		-3.7645	-3.7645	-3.7645	-3.7628	-3.7645	-3.7645	-3.7645	-3.7645

Table 6: Analysis results of 25-bar space truss system by different methods (Loading 2)

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		Loading 2							
		FEM Nonlinear	HS [18]	PSO [19]	TLBO (5 Class)	TLBO (10 Class)	TLBO (20 Class)	TLBO (30 Class)	TLBO (40 Class)
Joint Displacements (mm)	u(1)	1261.101	1257.533	1260.63	1349.25	1261.07	1261.08	1261.08	1261.08
	v(1)	-528.823	-527.509	-529.80	-623.94	-528.69	-528.69	-528.69	-528.69
	w(1)	-456.667	-455.548	-455.35	-443.66	-456.73	-456.73	-456.73	-456.73
	u(2)	-1261.101	-1264.620	-1261.15	-1158.66	-1261.08	-1261.08	-1261.08	-1261.08
	v(2)	528.823	529.739	529.20	486.27	528.68	528.69	528.69	528.69
	w(2)	-456.667	-457.905	-455.38	-412.38	-456.73	-456.73	-456.73	-456.73
	u(3)	-48.610	-48.858	-47.50	6.71	-48.72	-48.72	-48.72	-48.72
	v(3)	-288.313	-288.344	-287.03	-225.92	-288.42	-288.42	-288.42	-288.42
	w(3)	-362.743	-362.120	-361.33	-318.17	-362.84	-362.84	-362.84	-362.84
	u(4)	-202.750	-203.107	-203.19	-197.64	-202.7	-202.7	-202.7	-202.7
	v(4)	-296.792	-296.619	-295.57	-238.79	-296.9	-296.9	-296.9	-296.9
	w(4)	-68.466	-67.535	-68.84	-122.79	-68.42	-68.42	-68.42	-68.42
	u(5)	48.610	48.774	47.10	21.05	48.72	48.72	48.72	48.72
	v(5)	288.313	288.714	286.57	256.54	288.42	288.42	288.42	288.42
	w(5)	-362.743	-363.583	-361.26	-323.53	-362.84	-362.84	-362.84	-362.84
	u(6)	202.750	202.232	202.95	237.26	202.7	202.7	202.7	202.7
	v(6)	296.792	297.259	295.18	253.34	296.9	296.9	296.9	296.9
	w(6)	-68.466	-69.186	-68.67	-48.22	-68.42	-68.42	-68.42	-68.42
Member Forces (kN)	1	692.496	692.590	691.627	661.26	692.55	692.54	692.54	692.54
	2	-334.607	-334.408	-334.677	-350.17	-334.57	-334.57	-334.57	-334.57
	3	72.764	73.046	73.231	77.19	72.73	72.73	72.73	72.73
	4	72.764	72.343	73.245	94.45	72.73	72.73	72.73	72.73
	5	-334.607	-334.711	-334.761	-330.29	-334.57	-334.57	-334.57	-334.57
	6	274.675	275.294	275.049	259.96	274.62	274.62	274.62	274.62
	7	-189.233	-189.137	-189.519	-195.99	-189.22	-189.22	-189.22	-189.22
	8	-189.233	-189.279	-189.291	-193.64	-189.22	-189.22	-189.22	-189.22
	9	274.676	273.927	275.003	308.52	274.62	274.62	274.62	274.62
	10	-132.732	-132.535	-132.998	-147.03	-132.7	-132.7	-132.7	-132.7
	11	-132.732	-132.868	-132.948	-140.24	-132.7	-132.7	-132.7	-132.7
	12	35.767	35.618	35.618	32.64	35.78	35.77	35.77	35.77
	13	35.767	35.798	35.781	27.26	35.77	35.77	35.77	35.77
	14	-52.346	-52.294	-51.883	-34.7	-52.39	-52.38	-52.38	-52.38
	15	-125.277	-125.339	-125.189	-126.36	-125.26	-125.26	-125.26	-125.26
	16	-125.277	-125.143	-125.368	-129.63	-125.26	-125.26	-125.26	-125.26
	17	-52.346	-52.509	-51.825	-40.14	-52.39	-52.38	-52.38	-52.38
	18	116.025	116.226	115.587	83.17	116.06	116.06	116.06	116.06
	19	-174.822	-174.651	-174.273	-152.31	-174.86	-174.86	-174.86	-174.86
	20	-174.822	-175.121	-174.142	-159.59	-174.86	-174.86	-174.86	-174.86
	21	116.025	115.946	115.450	110.99	116.06	116.06	116.06	116.06
	22	-46.168	-46.776	-45.834	-15.55	-46.19	-46.19	-46.19	-46.19
	23	-55.264	-54.925	-55.429	-74.01	-55.23	-55.23	-55.23	-55.23
	24	-46.168	-45.581	-45.918	-59.13	-46.19	-46.19	-46.19	-46.19
	25	-55.264	-55.421	-55.700	-57.95	-55.23	-55.23	-55.23	-55.23
Energy (kNm)		-1444.6	-1444.6	-1444.6	-1441.8	-1444.6	-1444.6	-1444.6	-1444.6

Table 7: Analysis results of 25-bar space truss system by different methods (Loading 3)

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		Loading 3							
		FEM Nonlinear	HS [18]	PSO [19]	TLBO (5 Class)	TLBO (10 Class)	TLBO (20 Class)	TLBO (30 Class)	TLBO (40 Class)
Joint Displacements (mm)	u(1)	2522.974	2522.412	2521.86	2451.74	2522.99	2522.99	2522.99	2522.99
	v(1)	1840.856	1840.283	1839.88	1812.02	1840.89	1840.89	1840.89	1840.89
	w(1)	-3083.146	-3078.900	-3076.40	-2824.06	-3083.3	-3083.3	-3083.3	-3083.3
	u(2)	2441.255	2437.300	2435.57	2233.23	2441.46	2441.46	2441.46	2441.46
	v(2)	2523.148	2522.971	2522.31	2460.02	2523.12	2523.12	2523.12	2523.12
	w(2)	-1441.611	-1435.857	-1433.56	-1115.23	-1441.89	-1441.89	-1441.89	-1441.89
	u(3)	-394.610	-393.940	-393.83	-386.49	-394.64	-394.64	-394.64	-394.64
	v(3)	-371.344	-371.211	-371.22	-353.21	-371.34	-371.34	-371.35	-371.35
	w(3)	-905.218	-904.675	-903.69	-815.5	-905.22	-905.22	-905.22	-905.22
	u(4)	579.039	579.074	579.02	569.49	578.99	578.99	578.99	578.99
	v(4)	199.088	198.927	198.35	180.89	199.1	199.1	199.1	199.1
	w(4)	44.423	44.972	45.45	24.06	44.45	44.45	44.45	44.45
	u(5)	1015.303	1013.812	1012.54	935.38	1015.39	1015.39	1015.39	1015.39
	v(5)	1038.580	1037.329	1036.29	963.18	1038.66	1038.66	1038.66	1038.66
	w(5)	-356.179	-353.069	-351.08	-201.44	-356.41	-356.41	-356.41	-356.41
	u(6)	408.348	407.347	406.51	343.3	408.39	408.39	408.39	408.39
	v(6)	803.160	802.637	802.21	746.51	803.16	803.16	803.16	803.16
	w(6)	-36.688	-36.622	-36.68	5.18	-36.65	-36.65	-36.65	-36.65
Member Forces (kN)	1	106.578	107.640	107.691	189.92	106.51	106.51	106.51	106.51
	2	274.288	273.603	273.432	237.83	274.33	274.33	274.33	274.33
	3	-310.686	-310.372	-310.660	-300.22	-310.71	-310.71	-310.71	-310.71
	4	246.394	247.023	247.501	267.68	246.34	246.34	246.34	246.34
	5	-70.015	-70.532	-70.574	-82.87	-69.99	-69.99	-69.99	-69.99
	6	-10.677	-10.172	-10.110	32.36	-10.73	-10.73	-10.73	-10.73
	7	359.449	359.301	359.219	353.19	359.46	359.46	359.46	359.46
	8	187.700	187.014	186.599	141.07	187.73	187.73	187.73	187.73
	9	472.059	471.844	471.760	452.99	472.07	472.07	472.07	472.07
	10	-111.426	-110.993	-111.052	-130.15	-111.45	-111.45	-111.45	-111.45
	11	-180.523	-180.275	-179.696	-174.31	-180.56	-180.56	-180.56	-180.56
	12	-26.811	-26.908	-26.821	-58.96	-26.81	-26.81	-26.81	-26.81
	13	-106.798	-106.789	-106.819	-109.62	-106.81	-106.81	-106.81	-106.81
	14	-260.940	-260.641	-260.427	-245.64	-260.95	-260.95	-260.95	-260.95
	15	-237.647	-237.250	-236.904	-199.95	-237.65	-237.65	-237.65	-237.65
	16	238.741	238.850	238.851	228.27	238.73	238.73	238.73	238.73
	17	-178.359	-177.597	-177.046	-140.97	-178.41	-178.41	-178.41	-178.41
	18	-125.978	-125.783	-125.457	-124.18	-125.97	-125.97	-125.97	-125.97
	19	-248.093	-248.091	-247.944	-227.46	-248.09	-248.09	-248.09	-248.09
	20	-190.717	-190.078	-189.732	-155.44	-190.76	-190.76	-190.76	-190.76
	21	332.665	332.335	332.074	311.07	332.68	332.68	332.68	332.68
	22	-52.285	-52.549	-52.744	-47.88	-52.25	-52.25	-52.25	-52.25
	23	-106.579	-106.707	-106.436	-87.42	-106.57	-106.57	-106.57	-106.57
	24	-50.960	-50.752	-50.697	-65.05	-50.94	-50.94	-50.94	-50.94
	25	542.082	542.045	541.876	535.83	542.07	542.07	542.07	542.07
Energy (kNm)		-2860.5	-2860.5	-2860.5	-2843.97	-2860.5	-2860.5	-2860.5	-2860.5

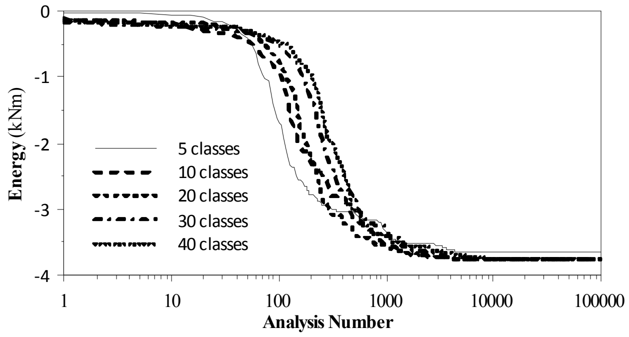

Fig. 6. Convergence speed of optimum results for different population sizes for Loading 1 (example 3)

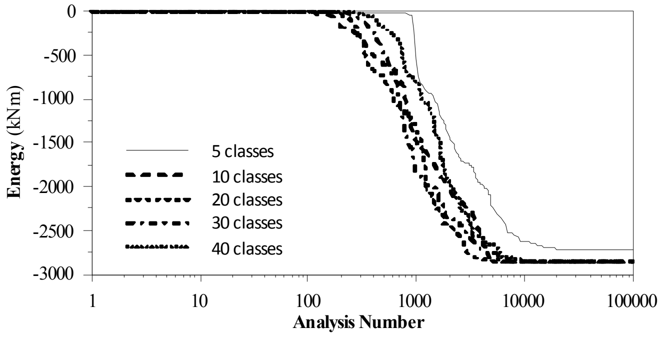

Fig. 8. Convergence speed of optimum results for different population sizes for Loading 3 (example 3)

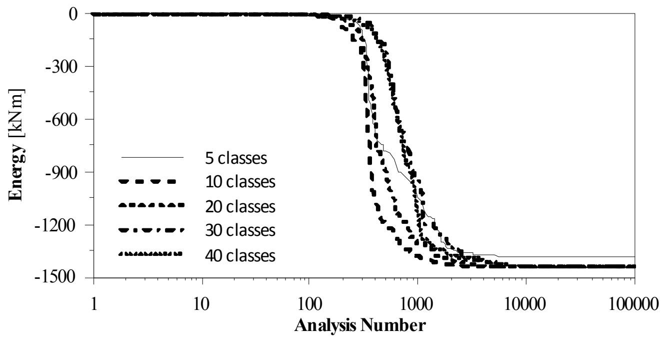

Fig. 7. Convergence speed of optimum results for different population sizes for Loading 2 (example 3)

4 Conclusion

In this paper, it is tried to observe the performance of TLBO algorithm which is a recently developed metaheuristic method on some structural analysis problems. Applications are performed through TPO/MA formulation on truss structures which have 2 (example 1), 4 (example 2) and 18 (example 3) design variables.

The results obtained have shown that TLBO algorithm is successful for finding minimum potential energies of systems.

The results have also shown that population size 5 can be considered as insufficient for obtaining reliable results, and that population size 10 is the most suitable one as far as computation time is concerned.

As a conclusion, one can say that TLBO algorithm is an effective, robust and powerful method for this kind of problem.

Table 8: Statistical evaluation of 100 independent analyses by TLBO algorithm (Example 3)

	Particle	Total Potential Energy [kNm]			
		Max	Min	Aver.	St. Dev.
Loading 1	5	-3.7628	-1.9350	-3.5941	0.23
	10	-3.7645	-3.7645	-3.7645	0
	20	-3.7645	-3.7645	-3.7645	0
	30	-3.7645	-3.7645	-3.7645	0
	40	-3.7645	-3.7645	-3.7645	0
Loading 2	5	-1441.77	-1264.98	-1413.39	29.03
	10	-1444.57	-1444.57	-1444.57	0
	20	-1444.57	-1444.57	-1444.57	0
	30	-1444.57	-1444.57	-1444.57	0
	40	-1444.57	-1444.57	-1444.57	0
Loading 3	5	-2843.97	-690.42	-2284.47	420
	10	-2860.49	-2860.49	-2860.49	0
	20	-2860.49	-2860.49	-2860.49	0
	30	-2860.49	-2860.49	-2860.49	0
	40	-2860.49	-2860.49	-2860.49	0

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