Operation optimisation of supermarket refrigerated display case

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Abstract: The main goal of this paper is to optimise the supermarket display case operation. Optimisation was performed through the analysis of the developed hybrid model that describes the temperature dynamics in a refrigerator system and as well all the crucial discrete events that influence the refrigerator system performance. Model parameters were estimated based on the measured data from a supermarket and our experiences. The model was realised with the simulation software AnyLogic and was used to analyse how the defrosting strategy, the frequency of food loading and the frequency of door openings influence the total operational cost and the product quality.

Key–Words: Supermarket refrigeration systems, Hybrid modelling, AnyLogic

1 Introduction

Energy efficiency is becoming one of the key success factors for manufacturing and retail companies. Advances in operational energy savings tend to increase the competitiveness of the company and reduce their dependence on changeable energy prices. Main tendency in the filed is to implement automatic data gathering equipment and support the companies with the support systems that allow users to analyse historical data to reach operation optimisation [1].

Energy represents a major part of the total costs of retail companies and therefore energy efficiency is their strategic goal. The biggest consumer in the shopping centers are typically refrigeration systems as they can consume up to 40% of the total electrical consumption [2]. Bigger refrigeration systems are already equipped with the automatic data acquisition systems, but the main question remains: how to employ already gathered data in order to find additional optimisation opportunities.

The paper presents the framework where refrigeration data is employed in order to help the user to validate the importance of different optimisation approaches. The main goal is to support the refrigeration system managers to rate the potential energy savings and the effort needed to implement the optimisation approach. For the analysis purposes a hybrid model of the refrigeration cabinet was developed from the store data. The model tends to reflect the conditions in the shopping center and allows us to perform the operation optimisation analysis.

In the next section the dynamical and discrete part of the refrigeration system model is introduced. In section 3, the optimisation framework and optimisation results are presented. Finally, the conclusion and future work is presented in Section 4.

2 Modelling

The most common refrigerators and freezers have four major parts in their refrigeration system (Fig. 1): compressor, condenser, expansion valve and evaporator. In the evaporator section, a refrigerant is vaporised to absorb heat added into the refrigerator due to heat transfer across the refrigerators walls and convection through the door, seals and during door opening. The evaporator temperature is maintained at, or near, the refrigerant boiling temperature. In the next stage, an electric motor runs a piston compressor and the refrigerant is pressurised. This raises the temperature of the refrigerant and the resulting superheated, high-pressure gas (it is still a gas at that point) is then condensed to a liquid in an air-cooled condenser. Supermarket refrigeration systems usually apply common compressor and condenser coils for several refrigerators. From the condenser, the liquid refrigerant flows through an expansion valve, in which its pressure and temperature are reduced and these conditions are maintained in the evaporator. The whole process operates continuously by transferring heat from the evaporator section (inside the refrigerator) to the con-

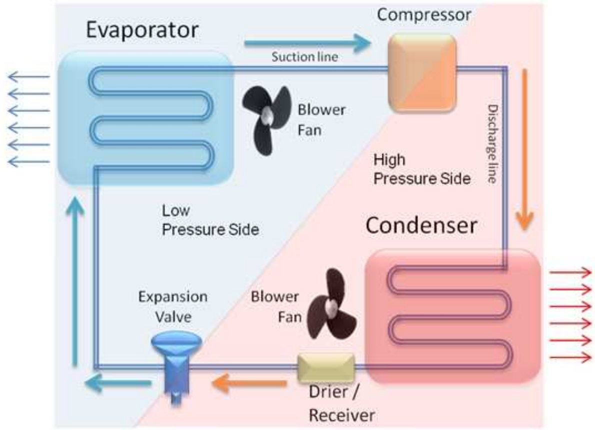


Figure 1: Cooling system of the refrigerator/freezer

denser section (outside the refrigerator) by pumping refrigerant continuously through the system described above.

The costs of the refrigeration systems in a supermarket are in general affected by two main factors: the technological overall efficiency of the appliances and the losses due to interaction with the system (buyers and shopping centre's personnel). For a single refrigerator, the most influential variables which determine the energy costs are the following: defrosting of the evaporator's coil, filling in the refrigerators with new products, degradation of the performance due to buyer's interaction and products mass fluctuations, etc.

To study a refrigerator display case, one has to develop dynamic model of the refrigerator with additional discrete events. For modelling we used simulation software AnyLogic [3], which supports various simulation methodologies, i.e., System Dynamics, Discrete Events and Agent Based Modelling.

2.1 Dynamical part of the refrigeration model

A simplified model is developed to describe the dynamics of the display case. The dynamical model consists of three main parts: the display case model, evaporation model and ice formation model. A low temperature display case, located in one of the local shopping centres, is chosen as a case study. The model is calculated from the specifications and parameters identified from the field measurements, in order to comply with the dynamics of the considered display case.

The following variable nomenclature is used: T denotes the temperatures, $\dot{Q}$ represents heat flows, UA is overall heat transfer coefficient; and index variables are f (food), c (core), w (wall), a (air), e (evaporator), r (refrigerant) s (store) and h (defrost heater).

2.1.1 Display case model

Thermodynamical equations are derived to describe the dynamics of the refrigeration display case. A detailed heat flow diagram of the considered model is shown in Fig. 2. Model structure is based on the work presented in [4], where additional model simplification is assumed ($m_a = 0$).

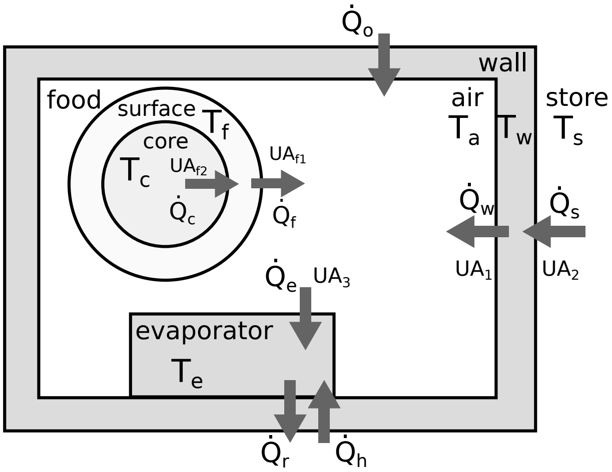


Figure 2: Thermodynamical part of the model

Linear heat transfer dynamics can be represented with a state-space representation. States x consist of the main display case temperatures (T_f, T_c, T_w, T_e), output vector y is selected according to the available measured data (T_f, T_c, T_a, T_e) and input vector u consists of store temperature, defrost heater heat flow and evaporator heat flow $(T_s, \dot{Q}_h, \dot{Q}_r)$. The state-space equations of the model are defined as:

$$\dot{x} = Ax + Bu \tag{1}$$

$$y = Cx \tag{2}$$

where matrices A, B, C are:

$$A = \begin{bmatrix} \frac{-UA_{f1}-UA_{f2}}{m_f c_{Pf}} + \frac{UA_{f1}^2}{m_f c_{Pf} UA_{tot}} & \frac{UAf_2}{m_f c_{Pf}} & \frac{UA_{f1}UA_1}{m_f c_{Pf} UA_{tot}} & \frac{UA_{f1}UA_3}{m_f c_{Pf} UA_{tot}} \\ \frac{UA_{f2}}{m_c c_{Pc}} & \frac{-UA_{f2}}{m_c c_{Pc}} & 0 & 0 \\ \frac{UA_1 UA_{f1}}{m_w c_{Pw} UA_{tot}} & 0 & \frac{-UA_1-UA_2}{m_w c_{Pw}} + \frac{UA_1^2}{m_w c_{Pw} UA_{tot}} & \frac{UA_1 UA_3}{m_w c_{Pw} UA_{tot}} \\ \frac{UA_3 UA_{f1}}{m_e c_{Pe} UA_{tot}} & 0 & \frac{UA_3 UA_1}{m_e c_{Pe} UA_{tot}} & \frac{-UA_3}{m_e c_{Pe}} + \frac{UA_3^2}{m_e c_{Pe} UA_{tot}} \end{bmatrix} \tag{3}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{UA_2}{m_w c_{Pw}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{1}{m_e c_{Pe}} & \frac{-1}{m_e c_{Pe}} \end{bmatrix} \tag{4}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{UA_{f1}}{UA_{tot}} & 0 & \frac{UA_1}{UA_{tot}} & \frac{UA_3}{UA_{tot}} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

and $UA_{tot} = (UA_{f1} + UA_1 + UA_3)$ is used to simplify the notation.

On-field measurements of seven defrost periods are used to identify free model parameters.

2.1.2 Evaporation model

Evaporator has nonlinear dynamics that arise from the thermophysical properties of the refrigerant. The opening degree (OD) of the expansion valve is controlled to reach the desired refrigeration temperature. The refrigerant mass flow can be defined with the following equation [5]:

$$\dot{m}_r = OD \cdot k_v \cdot \sqrt{\rho_r(p_c - p_{suc})} \tag{6}$$

where k_v is the expansion valve coefficient obtained from valve specifications, ρ_r denotes density of the refrigerant, p_c and p_{suc} are pressures in the suction manifold (after the evaporator) and in the display case conduit (before the expansion valve), respectively. Heat flow due to the evaporation of the refrigerant can be estimated as:

$$\dot{Q}_r = \Delta h_{lg} \cdot \dot{m}_r \tag{7}$$

where the heat flow through the expansion valve is further limited with the maximum heat transfer flow. The nonlinear expressions for refrigerant density $\rho_r = f(p_{suc}, T_r)$ and its change of enthalpy $\Delta h_{lg} = f(T_r, p_{suc}, p_c)$ are linearised from the refrigerant R404A characteristic chart obtained from CoolPack Software [6].

The current power consumption of the compressor is evaluated with the work needed to change the enthalpy of the refrigerant that flows through the evaporator of the considered display case ($\dot{m}_r$):

$$P_{ref} = \frac{1}{\eta_{mes}} \dot{m}_r (h_{is,c} - h_{i,c}), \tag{8}$$

where η_{mes} is overall mechanical, electrical and isentropic efficiency and $h_{is,c}$ and $h_{i,c}$ are the isentropic enthalpy of the refrigerant at the output of the compressor and input enthalpy of the refrigerant at the input of the compressor, respectively. Again, refrigerant characteristics from CoolPack [6] were linearised.

2.1.3 Ice formation model

As evaporator is continuously exposed to the moist air, frosting of the evaporator's coils occurs. The fan air flow through the evaporator is decreased with frost growth, which is the main cause for decreased evaporator cooling capacity and consequently the overall refrigeration efficiency [7]. A simple model was assumed to describe the relation among the ice and evaporator's cooling capacity. Mass of the ice (m_{fr}) formed on the coils of the evaporator depends on the air flow from the store ($\dot{m}_{a,ent}$) and the difference among the specific air humidity in the store (x_s) and in the refrigerator (x_a):

$$\dot{m}_{fr} = \dot{m}_{a,ent}(x_s - x_a) \tag{9}$$

Air flow consists of the two parts, constant part $\dot{m}_{a,0}$ and part $\dot{m}_{a,door}$ that is triggered by the discrete door opening events (δ):

$$\dot{m}_{a,ent} = \dot{m}_{a,0} + \dot{m}_{a,door}\delta \tag{10}$$

Maximal air flow rate $\dot{V}_0$ is being slowly degraded as the ice is formed. This degradation rate is simplified with the following relation:

$$\frac{d\dot{V}}{dt} = -k\dot{m}_{fr} \tag{11}$$

where the air flow degradation rate is much higher when the critical frost mass ($m_{fr,critical}$) is reached:

$$k = \begin{cases} k1 & \text{if } m_{fr} <= m_{fr,critical} \\ k2 & \text{if } m_{fr} > m_{fr,critical} \end{cases} \tag{12}$$

Finally, the degradation of the cooling capacity due to air flow degradation can be defined as the ratio of air flow of the frosted evaporator and evaporator without frost:

$$\eta_v = \frac{\dot{V}(t)}{\dot{V}_0}. \tag{13}$$

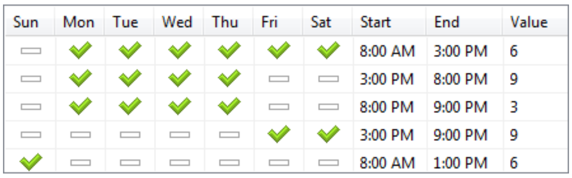
Parameters of the ice formation model were calculated according to available measurements.

2.2 Refrigerator system - Discrete event part

While the refrigerator system is in operation, various situations occurs that are of discrete nature. In the following we present a discrete part of the refrigeration system model, where some parameters were taken from the observed store and the others were estimated based on our experience.

2.2.1 Customers behaviour

Customers are arriving and taking products from the refrigerator according to the time and day of the week. Since we do not have historical data from the analysed store, we have selected these parameters according to preliminary measurements and our experience. In the weekdays, we consider that customers are arriving with exponentially distributed interarrival time with mean $\lambda = 1/6\ h^{-1}$, i.e., with a rate of 6 customers per hour. Costumers arrival rate is higher in peak hours (3 PM to 8 PM) and on Sundays ($\lambda = 1/9\ h^{-1}$), and lower in the weekday evenings (8 PM to 9 PM), $\lambda = 1/3\ h^{-1}$. The schedule of customers arrival rates, depending on time in a week is given in Fig. 3.



Sun	Mon	Tue	Wed	Thu	Fri	Sat	Start	End	Value
☐	✓	✓	✓	✓	✓	✓	8:00 AM	3:00 PM	6
☐	✓	✓	✓	✓	☐	☐	3:00 PM	8:00 PM	9
☐	✓	✓	✓	✓	☐	☐	8:00 PM	9:00 PM	3
☐	☐	☐	☐	☐	✓	✓	3:00 PM	9:00 PM	9
✓	☐	☐	☐	☐	☐	☐	8:00 AM	1:00 PM	6

Figure 3: Schedule of customers arrivals, defined with rate per hour.

When customer takes a product, the refrigerator's door is open for an exponentially distributed time of 10 seconds ($\lambda = 0.1\ s^{-1}$) and the quantity of product items within the refrigerator is decreased. For every taken product, the core and surface masses are decreased respectively by $0.5kg$ (core weight of a single product) and $0.057kg$ (surface weight of a single product).

Block *source1* generates the entities (customers) with a rates defined by schedule from Fig. 3. The *queue1* represents the waiting queue of customers waiting to open the door and take the product (we assume that only one customer can open the door at the same time). The *RefOpTake* block represents the action of opening the door and defines how long the door is open by each customer. It implements also the mass decrease. Last block (*sink1*) disposes the entities.

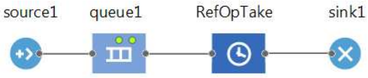


Figure 4: DE model of door openings and total mass decrease.

Door openings influence also on the heat transfer of the refrigerator. This is included with the following term that is added to the model introduced in the previous section.

$$\dot{Q}_{door} = \dot{m}_{in}(h_s - h_r)\delta. \tag{14}$$

Here $\dot{m}_{in}$ is an air mass flow that enters the refrigerator, h_s is an enthalpy incoming air and h_r is an enthalpy of the refrigerated air. δ is a door opening variable that considers the status of the door according to the discrete event model describing the customers' arrivals, as shown in Fig. 4.

$$\delta = \begin{cases} 1 & if\ the\ door\ is\ open \\ 0 & if\ the\ door\ is\ closed \end{cases} \tag{15}$$

2.2.2 Filling in the products

To define how full is the refrigerator, we introduce a variable $Fullness$ ($Fullness = 1$ means full and $Fullness = 0$ means empty). When $Fullness$ is below a certain threshold, the refrigerator is refilled with new product item entities. This threshold is defined randomly for each refrigerator loading through a triangular distribution $T(min, max, mode)$ with mode $Thr_m = 0.4$, i.e., $Thr = T(Thr_m - 0.2, Thr_m + 0.2, Thr_m)$.

Also time for refilling is presumed to be variable, however this should depend on how many products have to be refilled. This was implemented with a triangular distribution, which depends on Thr parameter: $T(2, 2 + 6(1 - Thr), 8)$mins.

We assume that the temperature of incoming entities is not always the same. We consider it to be distributed in a range defined with $T(-20, -16, -19)$°C.

Also during the refilling time the refrigerator's door is open. The influence on the heat transfer is considered similarly as previously shown by Eq.14.

2.2.3 Defrosting

In order to increase the overall refrigerator efficiency, the evaporator has to be defrosted periodically. A common practice in commercial refrigeration systems is to define a defrosting schedule that defrosts the evaporator's coils. Coils are defrosted with the use of the heater that actively removes the formed frost. In our case defrost heaters are discretely turned on during defrosting periods and are considered to operate at constant power (i.e. $\dot{Q}_h = 3100W$).

Discrete defrosting events are included into the model with the use of statechart depicted in Fig. 5. When the initial condition is triggered by the scheduler, the refrigeration system is turned off and the system goes in the defrosting state (*DefrostOnOff*). When the defrosting is finished (after 22.5 min), the heater is turned off and the system moves to the state that delays the activation of the refrigeration system for 4.2 minutes (*SystemOFF1*).

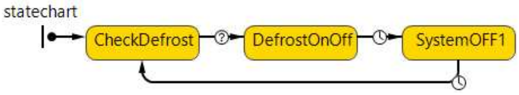


Figure 5: Statechart connecting dynamical part of the refrigeration model and discrete defrosting events

3 Optimisation

The optimisation goal is to determine how the defrosting strategy, the frequency of food loading and the frequency of door openings influence the total operational cost and the product quality.

From the derived model it is expected that frost formation decreases the evaporator's cooling capacity (eq. 13) which is reflected in higher operating cost. But also defrosting is very energy demanding as it uses energy for the heater and also heats the display case that needs to be cooled down again after defrosting. Therefore, too frequent defrosting is expected to increase the overall energy consumption and significantly affect the food quality. Cabinet refilling strategy and customer door opening habits also have an important influence on the operating costs of the cabinet. Too frequent door openings increase the energy losses, increase the frost formation and decrease the cabinet temperature. All of this is reflected in the higher operational costs and lower product items quality.

The main question of the optimisation is to determine to what extent these strategies affect on the system optimality and what operation settings should be applied to optimise its performance.

3.1 Optimisation objectives

The key objective is to minimise the total energy cost, calculated as the following integral over the simulated time T:

$$C_{tot} = \int_0^T (P_{ref} + \dot{Q}_h) \cdot c_e dt \qquad (16)$$

where P_{ref} is the refrigeration power applied (see eq. 8), $\dot{Q}_h$ is the defrost heater power and c_e is time-dependent energy fare, i.e., 0.04391€ per kWh from 22:00 to 06:00 every weekday and 0.07929€ per kWh every weekday from 06:00 to 22:00 and during the weekends.

When optimising cost, we have to satisfy the requirements about the food quality. Food quality decay can be determined by many environmental factors, but of all these, temperature is the most influential. As described in [8], this indicator can be calculated as follows:

$$q_{food,loos} = \frac{e^{\frac{T_c - T_{ref}}{z}} \cdot 100}{D_{T,ref}} \qquad (17)$$

where z is the product temperature sensitivity indicator and $D_{T,ref}$ is time for loss of quality at T_{ref}. Reference temperature for frozen products is normally $T_{ref} = -18°\text{C}$. In our case we use: $z = 20°\text{C}$ and $D_{T,ref} = 125$ days to describe the property of the refrigerated items. The current food quality index $Q_{food,loos}$ can be evaluated as an integral of the food decay ratio, where uniform quality of the product items in the refrigerator is observed. Additional assumption is made that newly added items have 100% quality and the overall food quality is averaged with the older items in the refrigerator.

3.2 Optimisation results

In order to achieve optimal operation of the refrigerator, various defrost schedule scenarios were analysed, i.e., (i) one day defrosting and (ii) two day defrosting. Schedule was systematically varied, and for each case 30-day simulation run was performed. Moreover, 50 repetitions per case were conducted to account for the model randomness.

In the first step, the scenario of one defrosting per one day (scheduled at the same time of a day) was analysed. Timing of the defrost is systematically varied over one day with 0.5 hour steps and the distribution of the monthly quality decay and operating costs are compared for the alternative runs. Mean and two sigma data variability of the analysis results are shown in Fig. 6. As expected it is more favourable to defrost during low energy rate, but this yields slightly higher quality decay of the product items (which is not critical). From the results it can be concluded that early morning defrosting scenario can be found as the most optimal. Using of scenario with two defrosts in one day results in higher average costs and quality decay.

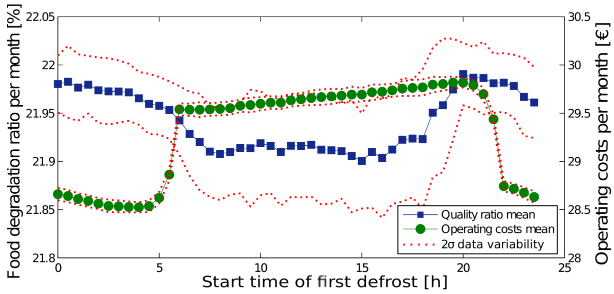


Figure 6: One-day defrost scenario analysis – one defrost per day

In the next step, a two-day defrosting schedule was tested where the schedule is defined for two days in advance and is being repeated thereafter. Defrost schedule for the first day was fixed at the previously found optimal setting (i.e. 4 AM) and the defrost schedule for second day was systematically varied. The results in the Fig. 7 suggests that there is no need to use two-day defrosting schedule as an early morning scenario is found again as optimal. Moreover, results also indicate that less frequent defrosting schedules would not be appropriate because after 24 hours with no defrosting the ice formed on the evaporator's coils already significantly influence the performance of the refrigerator. This leads to higher operating costs, however the comparable food quality decay indicates that the temperature control has not yet been affected.

In the next stage we analysed how often the display cases should be refilled with fresh product items. The distribution mean of the minimal product threshold in the refrigerator was systematically varied over the predefined range. Fig. 8 depicts how this affects the food quality degradation and the overall costs. It can be concluded that less frequent loading of the refrigerator results in lower operating costs (less energy loss due to food loading) and also that near empty refrigerator has bigger problems at keeping the food at the desired temperature which causes slightly higher quality degradation.

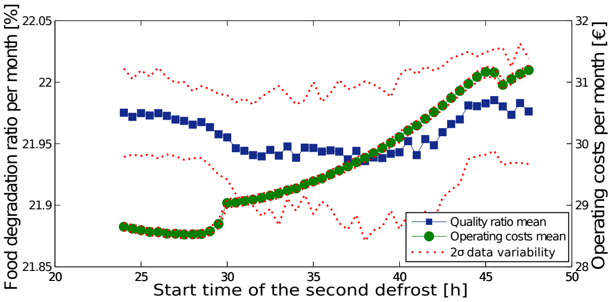


Figure 7: Two-day defrost scenario analysis (first defrost is scheduled at 4 AM of the first day) – two defrosts per two days

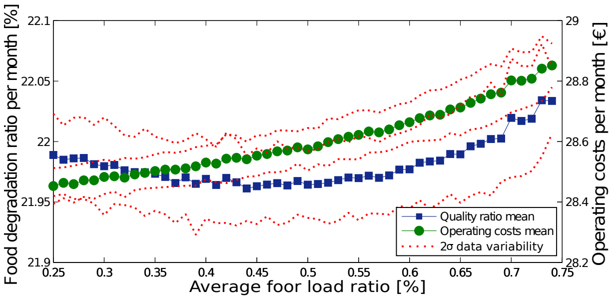


Figure 8: Analysis of the product refilling threshold parameter

It is also interesting to know if customers have a significant influence on the refrigeration systems and if they should also be additionally motivated to reduce the door opening time. For this, the door opening distribution was varied (mode of Thr distribution). Results are show in the Fig. 9 and indicate almost linear dependency among the door opening time and the operating costs. Therefore it can be concluded, that to some extent it is worth to motivate and help the customers to shorten their reach-in time.

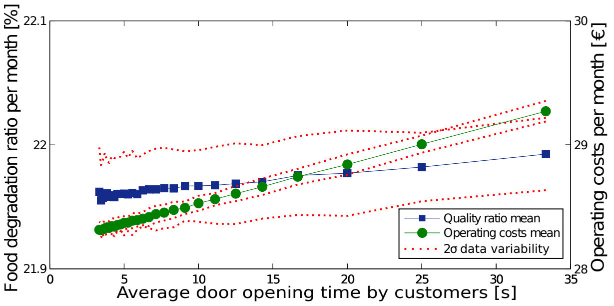


Figure 9: Significance of the customer door openings

4 Conclusion

A framework for analysing and optimising the display case operation has been presented in this paper. Optimisation is based on a detailed model that describes the temperature dynamics in a refrigerator system and all the crucial discrete events, that influence the refrigerator performance. Main benefit of such an approach is to test and directly rank the importance of different operation optimisation strategies. Analyse has shown how defrosting schedule can be optimised. Moreover, food loading strategy and door opening customs were analysed through evaluation of their influence on the operating costs and monthly degradation rate of the products. Some parameters of the model (e.g., customer arrivals, ice formation model) had to be estimated solely on our experiences and would need to be identified more precisely in the future in order to reflect the nature of the system more precisely.

Presented cost reductions could be seen as relatively small, but it is important to recognise the true dimension of the problem. Such an approach will be of a much higher importance, when the operation optimisation is further scaled to the level of a complete supermarket or even to the level of company (i.e., several supermarkets). Our future work will be focused in this direction, where the operation optimisation of several parallel display cases will be taken into account.

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