CAPACITY OF COMMUNICATION CHANNEL AS A QUALITY GUARANTEE OF DIGITAL REMOTE CONTROL OF CONTINUOUS TECHNICAL PLANT

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Abstract: The article shows that the restriction on the capacity of communication channel provided by the channel environment can be a major limitation on the achievable quality of digital remote control of continuous technical plant. The provisions of article are illustrated by examples.

Keywords: Capacity, Communication Channel, Remote Control, Continuous System, Environment, Bandwidth, Transfer Function, Barred Code

1. INTRODUCTION

We consider a continuous-type system «Single-Input and Single-Output» (SISO), which is analytically constructed by any of the currently available methods (Kwakernaak et al 1972; Ushakov et al, 2011) so that it describes the transfer function (TF) "input-output" (IO) parameterized by characteristic frequency ω_0.

$$\Phi(s,\omega_0)=\frac{v_n\omega_0^n}{s^n+\sum_{i=1}^{n}v_i\omega_0^i s^{(n-i)}} \tag{1}$$

In the transfer function (1), the coefficients $v_i\left(i=\overrightarrow{1,n}\right)$ set (Kwakernaak et al 1972; Ushakov et al, 2011) distribution pattern of the roots of the polynomial $D(s,\omega_0)=s^n+\sum_{i=1}^{n}v_i\omega_0^i s^{(n-i)}$, and the characteristic frequency ω_0 - the size of the region localization of the roots in the complex plane. Parameter ω_0 of the transfer function (1) defines the basic parameters (Kwakernaak et al 1972; Ushakov et al, 2011) of the continuous system described by it as in transition as well as in the steady state. This is the length of the transition process, the quality factor of the speed, bandwidth at different levels of amplitude frequency characteristics of relations "input-output" and "input-error". Invariant to the parameter value ω_0 there are parameters of system such as the value of the overshoot of the transient response, the number of its vibrations during the transition process and the stability margins of the system. These parameters are determined by the coefficients $v_i\left(i=\overrightarrow{1,n}\right)$ that define the character of the region of localization of the roots of the denominator polynomial TF (1).

We set the goal to evaluate how the conditions of feasibility of the desired value of the characteristic frequency ω_0 will change for "immersion" of the transfer function (1) in the task of organization of the digital remote «online» control of continuous technical plant (CTP) in the channel environment, connecting through the straight communication channel (CC) the apparatus of digital control signal (FADCS) with CTP and CTP with FADCS on the reverse of the CC.

2. SYSTEMIC FACTORS OF CHANNEL ENVIRONMENT IN THE PROBLEM OF DIGITAL REMOTE «ONLINE» CONTROL

The presence of channel environment in the problem of digital remote «online» control Protocol PPP («point-to-point protocol») (Ushakov et al, 2013), which mathematically is a scalar, ie, consistent, and not a vector, ie parallel, at the transmission of signals, and the receiver interference caused by physical means in its entirety, forces to consider the following systemic factors:

1. Implementation of the four-phase transformation "parallel - serial" at the transmission and "serial - parallel" at the reception in the forward and reverse channels of communication terminal hardware with the time costs, equal to the length of the converted code in each CC;

2. Use barred codes at the transmission, which leads to an increase in the format of the code by introducing its members check digit;

3. Implementation of the detected distortion correction noise-free codes transmitted with time costs, determined by methods of the organization of correction;

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4. Implementation of the exchange of information in the "duplex" or "half-duplex" forms depending on the specific properties used by the serial interface (telemechanical protocol), accompanied by additional time costs;

5. Realization the procedure of scrambling - descrambling transmitted - received codeword digital control signal for synchronous operation of the generators of the transmitting and receiving sides, which requires additional time costs;

6. Usage of the dynamic monitored devices at generating of control signal even in case of immediate measurable components of the state vector due to scalar nature of the channel environment.

The first systemic factor increases the dimension of the discrete model CTP in two, so that the development of digital control has to deal with an aggregate of discrete plants (ADO) (delay on the cycle of transmission of digital control in direct CC - discrete model with order n CTP - delay the transmission cycle of the measurement signal released in reverse CC) dimension $n_A = n + 2$. In the case of modal control device (Ushakov et al, 2011; Julius, 1971; Iserman, 1984; Grigoriev et al, 1983), it is necessary to choose the modal model (MM) and dimension n_A.

Systemic factors "2.-5." increase the interval of discreteness to various extent with which information is exchanged on the forward and reverse channels of digital remote control as compared with the duration of bits used in telemechanical protocol (TMP). Accounting of systemic factors leads to the following representation of aggregated interval of discreteness:

$$\Delta t = \Delta t_b (n_d + m + 1) \quad (2)$$

in the case of "duplex" without scrambling;

$$\Delta t = 2\Delta t_b (n_d + m + 1) \quad (3)$$

in the case of "half-duplex" without scrambling;

$$\Delta t = \Delta t_b (n_d + m + 3) \quad (4)$$

in the case of "duplex" with scrambling;

$$\Delta t = 2\Delta t_b (n_d + m + 3) \quad (5)$$

in the case of "half-duplex" with scrambling.

In the expressions (2) - (5): n_d – number of digits of hardware formation and transformation of the digital and analog signals (typically from the range: 8; 12; 16; 24; 32); m – number of check digits of the used barred code (BC), which in the task of remote «online» control protocol PPP must provide error correction mode transmission BC signals in the forward and reverse channels, and not detection, which requires the number m meet the condition:

$$m = \arg\left\{ N_{ns} = 2^m - 1 \ge N_f = \sum_{i=1}^{s} C^i_{n_d+m} \ \& \ P_{tf} = \right. \\ \left. = \sum_{i=s+1}^{n_d+m} \left(C^i_{n_d+m} \right) p^i (1-p)^{n_d+m-i} \le P_{atf} \right\} \quad (6)$$

In the expression (6): N_{ns} – the number of non-zero syndromes; N_f – the number of options distortions BC multiplicity to s; $C^i_{n_d+m}$ – the number of combinations of $(n_d + m)$ on i; P_{tf} – the probability of passing BC through barred environment; p - the probability of distortion BC bit in the CC; P_{atf} – allowable probability of receiving false commands (depending on the category of the remote control from the series 10^{-7};10^{-10};10^{-14}).

Interval of discreteness Δt defined by the formula (2) in the case of TMP with duplex character of the exchange of information without scrambling the code packages, with "1" in parentheses takes into account the cost of duration of one bit at correcting distortions BC. Interval of discreteness Δt defined by the formula (3) in the case of TMP a half-duplex character of the exchange of information without the scrambling code packages, where "1" in brackets also considers the cost of one bit length of the correction of distortion BC. Interval of discreteness Δt defined by the formula (4) in the case of TMP with duplex character of the exchange of information and scrambling-descrambling code packages which is taken into account together with the cost of duration of one bit at correcting distortions BC number "3". Interval of discreteness Δt defined by the formula (5) in the case of TMP with half-duplex character of the exchange of information with the scrambling-descrambling code packages which is taken into account together with the cost of a duration of one bit at correcting distortions BC number "3".

The variant of the interval forming in the case of correction of the BC using quasi-syndromes is possible (Rosenthal et al, 2001; Bleyhut, 1986; Liholetova et al, 2012; Liholetova et al, 2014), which demands for the realization the additional division cycle of accepted distorted BC, carried out at a pace "of channel time", which will replace the number "1" in all formulas on "$n_d + m$", thereby increasing Δt almost two times compared with (2) - (5).

Thus, independent of the speed (capacity) provided by TMP, factor of serial character transmission information in its physical environment for remote continuous technical plant SISO type increases in two the discrete model, becoming equal to $n_A = n + 2$. If CTP is the plant of "multidimensional input-output multi-dimensional" (MIMO) type with r-inputs and r-outputs, the overall dimension of the discrete model of the plant MIMO type increases by $2r$. Moreover in r-times compared to (2) - (5) increases the duration of interval of discreteness Δt with consistent TMP service of r -separate channels of CTP MIMO-type.

3. MAIN RESULT

For demonstration of main results, we consider continuous system SISO-type with transfer function of the form (1), a subscript «A», and in which n is replaced by n_A, so that we get

$$\Phi_A(s,\omega_0) = \frac{v_{n_A}\omega_0^{n_A}}{s^{n_A} + \sum_{i=1}^{n_A} v_i \omega_0^i s^{(n_A - i)}} = \frac{v_{n_A}\omega_0^{n_A}}{D(s,n_A,\omega_0)} \qquad (7)$$

The transfer function (7) is the desired continuous analogue of the digital remote control of continuous technical plant, the discrete model representation of which is modified taking into account the factors of channel environment. Now, we need to solve the key problem of implementation of the transfer function (7) which is to evaluate the maximum value of the characteristic frequency ω_0, achievable at the interval of discreteness Δt of information exchange during the digital remote control, implemented in a form of list of options (2) - (5). To solve this problem, let's address to the table of dynamic performance of continuous systems SISO-type transfer functions of (7) type, confining one selves to the cases of roots of polynomials $D(s,n_A,\omega_0)$ distributions by Butterworth and Newton and aggregate dimension values $n_A = 4, n_A = 5$, thereby assuming that the original CTP has the dimension of a number of $n = 2, n = 3$.

Dynamic parameters of a continuous system SISO-type (Ushakov et al, 2011) with the transfer function of the form (7) with the distribution of the roots of the polynomial $D(s,n_A,\omega_0)$ Butterworth are shown in Table 1. The dynamic parameters of a continuous system SISO-type transfer function of the form (7) with the distribution of the roots of the polynomial $D(s,n_A,\omega_0)$ Newton (binomial) (Ushakov et al, 2011) are shown in Table 2.

Table 1.

Order n	$D(s,\omega_0)$ with Butterworth distribution	σ,%	$t_\sigma\omega_0$	$t_{\Pi}\omega$	$\frac{D}{\omega_0}$	$\Delta\varphi^\circ$	$\frac{\omega_c}{\omega_0}\perp$	Bandwidth $\Delta\omega/\omega_0$			
								|1-M|≤0.05	M≥0.707	M≥0.05	δ≤0.05
4	$s^4+2.6\omega_0 s^3+3.4\omega_0^2 s^2+ +2.6\omega_0^3 s+\omega_0^4$	11	5.55	7	0.385	59.8	0.3934	0.701	1	2.11	0.02
5	$s^5+3.24\omega_0 s^4+5.24\omega_0^2 s^3+ +5.24\omega_0^3 s^2+3.24\omega_0^4 s+\omega_0^5$	13	6.3	8	0.31	60.0	0.319	0.774	1	1.782	0.017

Table 2.

Order n	$D(s,\omega_0)$ with Newton binomial distribution	σ,%	$t_\sigma\omega_0$	$t_{\Pi}\omega$	$\frac{D}{\omega_0}$	$\Delta\varphi^\circ$	$\frac{\omega_c}{\omega_0}\perp$	Bandwidth $\Delta\omega/\omega_0$			
								|1-M|≤0.05	M≥0.707	M≥0.05	δ≤0.05
4	$s^4+4\omega_0 s^3+6\omega_0^2 s^2+ +4\omega_0^3 s+\omega_0^4$	0	–	7.8	0.25	68.6	0.248	0.144	0.44	2.0	0.013
5	$s^5+5\omega_0 s^4+10\omega_0^2 s^3+ +10\omega_0^3 s^2+5\omega_0^4 s+\omega_0^5$	0	–	9	0.2	66.9	0.2	0.128	0.4	1.54	0.011

To establish the connection of the characteristic frequency ω_0 with interval of discreteness it is necessary to use two system positions. The first position is associated with the analytical dependence of the bandwidth $(\Delta\omega)_s$ of the system with the transfer function (7) at the level of five percent of its value amplitude frequency characteristic relationship "input-output" from the characteristic frequency ω_0 in the form

$$(\Delta\omega(\omega_0))_s = \omega = \max_{\omega} \arg\{|\Phi_A(j\omega,\omega_0)| = M(\omega,\omega_0) \geq 0.05\} = = \gamma_{(*)n} \cdot \omega_0. \qquad (8)$$

In expression (8), the coefficient of proportionality $\gamma_{(*)n}$, which element $(*)$ of double index may receive meaning $(*) = B$ and $(*) = N$, which indicates on the polynomial $D(s,n_A,\omega_0)$ owning respectively Butterworth and Newton distribution of its roots. The second element n of double index takes the meaning $n = 4$ and $n = 5$ dimension of the model (7). Thus, tables 1 and 2 allow us to write the coefficients of proportionality: $\gamma_{B4} = 2.11$, $\gamma_{B5} = 1.782$, $\gamma_{N4} = 2.0$, $\gamma_{N5} = 1.54$.

The second system position, based on the theorem of Shannon-Kotelnikov (Julius, 1971; Iserman, 1984; Drozdov et al, 1983; Shannon, 1948), analytically links bandwidth $(\Delta\omega)_c$ channel environment with an interval of discreteness Δt of information exchange in the digital remote control relation

$$(\Delta\omega)_c = \pi/\Delta t. \qquad (9)$$

Obviously, the dynamic parameters of the system (7) will be implemented in a digital remote control of continuous technical plant, if the following correlation is performed:

$$(\Delta\omega)_c = \pi/\Delta t \geq (\Delta\omega(\omega_0))_s = \gamma_{(*)n} \cdot \omega_0. \qquad (10)$$

If (10) is solved with respect to the characteristic frequency ω_0, then it will get

$$\omega_0 \leq \pi/(\Delta t \cdot \gamma_{(*)n}). \qquad (11)$$

The maximum value of the frequency characteristic ω_0 realized in the provided channel environment is determined by the expression

$$\omega_0 = \pi/(\Delta t \cdot \gamma_{(*)n}). \qquad (12)$$

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If relation (12) is placed into the analytical representation of quality parameters of the continuous dynamical systems, given in Tables 1 and 2 in the form of the functions of the characteristic frequency ω_0, we obtain the limiting values of the table systems SISO-type with transfer function "input-output" (7) as a function of interval of discreteness Δt of information exchange in the digital remote control.

Table 3.

Order n	$D(s,\omega_0)$ With Butterworth distribution	σ,%	$\frac{t_\sigma}{\Delta t}$	$\frac{t_{\Pi}}{\Delta t}$	$D\Delta t$	$\Delta\varphi^\circ$	$\frac{\omega_c}{\omega_0}$ ⊥	Bandwidth $\Delta\omega \cdot \Delta t$			
								$\lvert 1-M\rvert \le 0.05$	$M \ge 0.707$	$M \ge 0.05$	$\delta \le 0.05$
4	$s^4+2.6\omega_0 s^3+3.4\omega_0^2 s^2+$ $+2.6\omega_0^3 s+\omega_0^4$ $\omega_0=1.4889(\Delta t)^{-1}$	11	3.73	4.7	0.573	59.8	0.3934	1.044	1.49	3.142	0.03
5	$s^5+3.24\omega_0 s^4+5.24\omega_0^2 s$ $+5.24\omega_0^3 s^2+3.24\omega_0^4 s+$ $\omega_0=1.763(\Delta t)^{-1}$	13	3.57	4.54	0.547	60.0	0.319	1.365	1.763	3.142	0.03

Table 4.

Order n	$D(s,\omega_0)$ with Newton binomial distribution	σ,%	$\frac{t_\sigma}{\Delta t}$	$\frac{t_{\Pi}}{\Delta t}$	$D\Delta t$	$\Delta\varphi^\circ$	$\frac{\omega_c}{\omega_0}$ ⊥	Bandwidth $\Delta\omega \cdot \Delta t$			
								$\lvert 1-M\rvert \le 0.05$	$M \ge 0.707$	$M \ge 0.05$	$\delta \le 0.05$
4	$s^4+4\omega_0 s^3+6\omega_0^2 s^2+$ $+4\omega_0^3 s+\omega_0^4$ $\omega_0=1.5708(\Delta t)^{-1}$	0	–	4.97	0.393	68.6	0.248	0.226	0.691	3.142	0.021
5	$s^5+5\omega_0 s^4+10\omega_0^2 s^3+$ $+10\omega_0^3 s^2+5\omega_0^4 s+\omega_0^5$ $\omega_0=2.04(\Delta t)^{-1}$	0	–	4.42	0.408	66.9	0.2	0.261	0.816	3.142	0.022

Dynamic parameters of a continuous system SISO-type with transfer function of the form (7) with Butterworth distribution roots of the polynomial $D(s,n_A,\omega_0=\pi/(\gamma_{Bn}\Delta t))$ are shown in Table 3. The dynamic parameters of a continuous system SISO-type with transfer function of the form (7) with a Newton binomial distribution roots of the polynomial $D(s,n_A,\omega_0=\pi/(\gamma_{Bn}\Delta t))$ are shown in the table 4. Dynamic parameters of a continuous system SISO-type with transfer function of the form (7) are shown in Tables 3 and 4 parameterized by interval of discreteness Δt of information exchange interval during a digital remote control.

4. ILLUSTRATIVE EXAMPLES

Example 1. Evaluate achievable dynamic parameters of system digital remote control (SDRC) of continuous technical plant SISO-type with following system and channel conditions:

1. CTP SISO-type dimension $n=2$;

2. Multi-phase transformation "parallel - serial" at the transmission and "serial - parallel" at the reception in the forward and reverse CC by terminal hardware with the time costs, equal to the length of the converted code in each of the BC generates aggregated discrete model CTP dimension $n_A=n+2=4$;

3. Hardware Environment terminal nodes SDRC performance, formation and transformation of the control signal to the number of digits $n_d=8$;

4. Granted telemechanical protocol (TMP) (http://ctsspb.ru) has transmission speed (capacity) $c=1200bps$ and is characterized by the bit duration $\Delta t_b=8.333\cdot 10^{-4}c$;

5. Barred environment is such in CC that the allowable probability of receiving false command $P_{atf}=10^{-7}$ due to (6) leads to the format of the BC (15,8) with the number of check digits $m=7$;

6. Distortion correction of BC is carried out at one bit;

7. Granted TMP allows to organize a "duplex" exchange of information in the digital remote control CTP;

8. Granted TMP does not use the procedure "scrambling-descrambling" transmitted-received codeword signal exchange.

9. In order to ensure lack of overshoot, to give the system of digital remote control Newton binomial distribution of the roots of the characteristic polynomial.

Solution of Example 1.

1. The calculation of aggregate interval of discreteness Δt for conditions described above with the aid of (2) gives it to

$$\Delta t=\Delta t_b(n_d+m+1)=8333\cdot 10^{-4}\cdot(8+7+1)=0.01333c$$

2. On the basis of Table 4 dynamic parameters SDRC CTP SISO-type fourth-order will have assessing the maximum attainable values:

- overshoot $\sigma=0$%;

- The duration of the transition process $t_{\Pi}\ge 4.97\Delta t=0.0663c$;

- quality factor speed $D\le 0.393/(\Delta t)=29.48c^{-1}$;

- Characteristic frequency $\omega_0\le 1.5708(\Delta t)^{-1}=117.84c^{-1}$.

Example 2. To evaluate achievable dynamic parameters of system of digital remote control (SDRC) of continuous technical plant SISO-type with following system and channel conditions:

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1. CTP SISO-type dimension $n = 2$;

2. Multi-phase transformation "parallel - serial" at the transmission and "serial - parallel" at the reception in the forward and reverse CC by terminal hardware with the time costs, equal to the length of the converted code in each of the BC generates aggregated discrete model CTP dimension $n_A = n + 2 = 4$;

3. Hardware Environment terminal nodes SDRC performance, formation and transformation of the control signal to the number of digits $n_d = 12$;

4. Granted telemechanical protocol (TMP) (http://ctsspb.ru) has transmission speed (capacity) $c = 600bps$ and is characterized by the bit duration $\Delta t_b = 16.666 \cdot 10^{-4} c$;

5. Barred environment is such in CC that the allowable probability of receiving false command $P_{atf} = 10^{-7}$ due to (6) leads to the format of the BC (23,12) with the number of check digits $m = 11$;

6. Distortion correction of BC is carried out at one bit;

7. Granted TMP allows to organize a "duplex" exchange of information in the digital remote control CTP;

8. Granted TMP does not use the procedure "scrambling-descrambling" transmitted-received codeword signal exchange.

9. In order to ensure lack of overshoot, to give the system of digital remote control Newton binomial distribution of the roots of the characteristic polynomial.

Solution of Example 2.

1. The calculation of aggregate interval of discreteness Δt for conditions described above with the aid of (5) gives it to

$$\Delta t = 2\Delta t_b (n_d + m + 3) = 16.666 \cdot 10^{-4} \cdot (12 + 11 + 3) = 0.0433c$$

2. On the basis of Table 4 dynamic parameters SDRC CTP SISO-type fourth-order will have assessing the maximum attainable values:

- overshoot $\sigma = 0\%$;

- The duration of the transition process $t_{\Pi} \geq 4.97\Delta t = 0.2154c$;

- quality factor speed $D \leq 0.393/(\Delta t) = 9.0762c^{-1}$;

- Characteristic frequency $\omega_0 \leq 1.5708(\Delta t)^{-1} = 36.2771c^{-1}$.

6. CONCLUSIONS

It is shown that the capacity of provided telemechanical protocols, which are now quite widely-used form of implementation of the interface functions of the channel environment in the task of the digital remote control of continuous technical plant SISO-type at large distances, can be a serious constraint on the possibility of achieving the required performance of SDRC. This situation may manifold redoubled at the construction of STSDU by technical plants MIMO-type.

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