Direction of Arrival Estimation Using Self-Organizing Map

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Abstract: - The direction of arrival (DOA) is a key issue of sensor network system. In this paper, a self-organizing map (SOM) neural network for DOA estimation is proposed, which is based on the time differences of arrival (TDOA) and a uniform linear sensor array in a 2-D plane. SOM is a network that makes a high-dimensional space mapped into a low (one or two)-dimensional space and keeps the same topological order of the primary events. We found that there are similar topological order between the TDOA vectors and the angles of arrival of the same signals, thus the scheme performs a two-level SOM network between them. It is shown that the network trained by the signals of near field can efficiently estimate the angles of arrival of the signals not only in near-field but also far-field.

Key-Words: - direction of arrival; self-organizing maps; time difference of arrival; topological order; uniform linear sensor array

1 Introduction
Target detection and localization are important problems in sonar, radar, radio emitter tracking and mobile communications, and estimation of direction is one of the basic issues. In the past few decades, variety of approaches have been proposed for solving the direction of arrival (DOA) of signal source, such as the multiple signal classification (MUSIC) algorithm [1], hyperbolic location [2], and the maximum likelihood (ML) method. However, MUSIC has a higher demand on the assumption of signals and noise, and the computation loads of hyperbolic location and ML are high, these limit their practical applicability. Recently, artificial neural networks have been applied successfully in estimation of DOA with high accuracy [3-8]. A neural network is proposed to approximate the relationship between the outputs of sensor array and the characters of signals to be estimated, through the network the DOA or location of unknown signal could be obtained. First Hopfield network was used to the signal processing community [3], then radial basis function (RBF) neural is known to be popular and high accuracy [4]. However, these two neural networks are limited to training data sets, the error of estimation will be large when the new signal is out of the range of training sets. On the other hand, target training are failed to achieve the requirement of real-time and blind.

SOM is a self-organizing system, the signals put in it will be mapped automatically onto a set of output response acquire the same topological order as the primary signals [10], and it is a self-organized network of unsupervised training. Recently, the application of self-organizing maps (SOM) appears in the field of DOA estimation. JunXu [9] proposed a self-organizing maps scheme for mobile location estimation, the network is set up between the strengths of signals and user's location, and it achieved efficient, robust, and easy to implementation.

Regardless of the various methods in form, the nature of the issue is to exploration the relationship between the time difference of arrival (TDOA) and the DOA (or location) of signals. In this paper, we proposed a two-level SOM network to approximate the relationship between TDOA and DOA when there is only single signal waveform, and a 2-D DOA problem with a uniform linear sensor array is considered. We found that under this assumption, the topological structure of the TDOA vectors is similar to the angles of arrival of same signals, but different to the location of the signals. Thus we set up a two-level SOM network between the two. The network was trained by simulation data in advance, and in practical application, just put the estimated TDOA into the trained network and the corresponding DOA could be estimated.
2 Background material

2.1 Data model

Assume that a uniform linear array of \( M + 1 \) sensors in the 2-D plane, and a sound source incident on the plane. Establish rectangular coordinate system as shown in figure 1, let the linear array placed along the x-axis, and the first left sensor is located at the origin, the array element spacing is \( r \), consider the distance from sound source \( S(x,y) \) to sensor \( i \) \((i = 0,1,\cdots,M)\) is \( d_i \), then

\[
d_i = \sqrt{(x - ir)^2 + y^2}, \quad (i = 0,1,\cdots,M).
\]

(1)

The distance difference of sensor \( i \) and \( i + 1 \) is

\[
d_{i+1} - d_i, \quad (i = 0,1,\cdots,M - 1).
\]

(2)

If \( c \) is the speed of sound wave propagation in the media, then TDOA between sensor \( i \) and \( i + 1 \) is

\[
t_{i+1} = t_i + c, \quad (i = 0,1,\cdots,M - 1).
\]

(3)

Let \( T = [t_{0,1},t_{1,2},\cdots,t_{M-1,M}]^T \in R^M \) be the TDOA vector, obviously it has the same topology order with

\[
d = [d_{0,1},d_{1,2},\cdots,d_{M-1,M}]^T \in R^M.
\]

(4)

Let \( \theta \) be the angle of arrival, it is the angle of connection of source and the array midpoint and the positive x-axis, obviously it is computed from

\[
\theta = \arctan \frac{y}{x - Mr/2}.
\]

(5)

The core issue of the DOA problem is to explore a mapping \( F: R^M \rightarrow R^r \) from the TDOA vector space to the angle of arrival space. In the next section, we will analysis the relationship of the two spaces and set up the maps.

2.2 DOA Estimation with SOM

The self-organizing map setup in this paper is a Kohonen self-organizing map, which is also called Kohonen feature map. It is a feed-forward neural network with unsupervised and competitive learning algorithm [11]. SOM is a system maps high-dimensional or complex data into a one- or two-dimensional data and keep the same topological order as original data in the meantime, thus the features of the input data will be visualizing. The other hand, SOM can be considered as an effective method for feature extraction and dimensionality reduction.

Kohonen SOM works as follows. When the input sample vectors are put into the network, the Euclidean distance between competition layer neurons weights and the input sample vectors are calculated to obtain the winning neuron which has the minimum distance. Then adjust the weights of the winning neuron and its neighboring neurons, to make them similar to the input sample. So that all neurons’ connected weights have a certain distribution by such training.

Take \( n \) points in the two-dimensional plane as the locations of \( n \) sound sources, each point corresponds to a distance difference vector \( d \). Consider that there is only a constant factor \( c \) difference between the DOA vector \( T \) and the distance difference vector \( d \); we take \( d \) to be an alternative input vector to \( T \) as training samples in this paper.

![Fig. 2. The architecture of the two-level SOM.](image-url)
map keep the same topological order with the distance difference vectors.

The second level of SOM is a 1-1 mapping process from the trained two-dimensional space to another two-dimensional space, and there are the same numbers of nodes on them. Each node of the second map represents an angle of arrival, which is obtained according to the signal’s location that activates the node. Assumed that node \( j \) in the first layer is activated by \( n_j \) sample vectors, where \( \sum_{j=1}^{N} n_j = n \), and the coordinates of the corresponding signals are \((x_i, y_i) (i = 1, 2, \ldots, n_j)\), the angle of arrival corresponding to \((x_i, y_i)\) is \( \theta = \arctan \frac{y_i}{x_i - M \rho/2} \). According to the node activation of the first level, the output of second level is constructed by the following rules:

1. If node \( j \) is activated by one training input vector, and the coordinates of which is \((x_i, y_i)\), then the angle of the corresponding signal will be the output, and
   \[ \hat{\theta}_j = \arctan \frac{y_i}{x_i - M \rho/2}. \] (6)

2. If node \( j \) is activated by more than one training input, that is \( n_j > 1 \), and the coordinates of them are \((x_{i_k}, y_{i_k}) (i = 1, 2, \ldots, n_j)\), then the average angles of the corresponding signals will be the output as this node stands for:
   \[ \hat{\theta}_j = \frac{1}{n_j} \sum_{i_k=1}^{n_j} \arctan \frac{y_{i_k}}{x_{i_k} - M \rho/2}. \] (7)

3. If node \( j \) has never activated by any training input, then the output will be considered as a null node. When it is activated by a new input vector, the output will be substituted with the value of the nearest node.

2.3 Analysis of reliability

The process of establishing the SOM network described above shows that the topological order of angle of arrival is the same as vector of TDOA’s. In another word, when the Euclidean distance between vectors of TDOA of two signals is small, then the Euclidean distance between angles of arrival will be small as well. This is the theoretical basis of our estimation of DOA, and we will analyze this nature as follows.

Let \((x, y)\) be the location of a sound source, and another sound source around it on point \((x + \Delta x, y + \Delta y)\), then the distance difference vectors of them are \( d \) and \( d_t = d + \Delta d \), and angles of arrival of them are \( \theta \) and \( \theta_t = \theta + \Delta \theta \) respectively. According to equation (1) and equation (2), the incremental can be obtained, that is

\[ \Delta d = [\Delta d_{1,1}, \Delta d_{1,2}, \ldots, \Delta d_{M-1,1}]^T, \] (8)

Where \( \Delta d_{i,j} = d_{i+1,j} - d_{i,j} \), and \( \Delta \theta = \theta_t - \theta \).

Obviously, function \( d_{i,j} \) is differentiable at point \((x, y) \in \mathbb{R}^2\), thus

\[ \Delta d_{i,j} \approx \frac{\partial d_{i,j}}{\partial x} \Delta x + \frac{\partial d_{i,j}}{\partial y} \Delta y \]

\[ = \left( \frac{1}{d_{i,j}} - \frac{1}{d_{i+1,j}} \right) (x \Delta x + y \Delta y) \]

\[ = A_i(x, y) (x \Delta x + y \Delta y), \] (9)

where \( A_i(x, y) = \frac{1}{d_{i,j}} - \frac{1}{d_{i+1,j}}, (i = 0, 1, \ldots, M - 1) \). Then

\[ \Delta d = [A_1(x, y), A_2(x, y), \ldots, A_M(x, y)]^T (x \Delta x + y \Delta y), \]

and the Euclidean distance between \( d \) and \( d_t \) is

\[ \|d_t - d\| = \left( \sum_{i=1}^{M} A_i^2(x, y) \right)^{1/2} \left[ (x \Delta x + y \Delta y) \right]. \] (10)

The same way, function \( \theta \) is differentiable at point \((x, y) \in \mathbb{R}^2 \), \((x \neq M \rho/2)\), then

\[ \Delta \theta \approx \frac{\partial \theta}{\partial x} \Delta x + \frac{\partial \theta}{\partial y} \Delta y \]

\[ = \frac{1}{(x - M \rho/2)^2 + y^2} \left[ -y \Delta x + (x - M \rho/2) \Delta y \right] \]

\[ = B(x, y) \left[ -y \Delta x + (x - b) \Delta y \right], \] (11)

where \( B(x, y) = \frac{1}{(x - M \rho/2)^2 + y^2} \) and \( b = M \rho/2 \). Then the Euclidean distance between \( \theta \) and \( \theta_t \) is

\[ \|\theta_t - \theta\| = |B(x, y)| \left| -y \Delta x + (x - b) \Delta y \right|. \] (12)

The ratio of the two distances is

\[ \frac{\|\theta_t - \theta\|}{\|d_t - d\|} = \left| \frac{B(x, y)}{\sum_{i=1}^{M} A_i^2(x, y)} \right| \left| -y \Delta x + (x - b) \Delta y \right|. \]

\[ = C(x, y) \cdot R(x, y, \Delta x, \Delta y), \] (13)

Where
\[ C(x, y) = \frac{|B(x, y)|}{\sqrt{\sum_{i=1}^{M} A_i^2(x, y)}} , \]

\[ R(x, y, \Delta x, \Delta y) = \frac{-y\Delta x + (x-b)\Delta y}{(x\Delta x + y\Delta y)} . \]

When point \((x, y)\) is fixed, \(C(x, y)\) is a constant, thus the ratio is decided to formula \(R(x, y, \Delta x, \Delta y)\). In the neighborhood of point \((x, y)\) out of \((0, 0)\), the value of ratio is bounded, and changes little, changes in the interval \([0,1]\), as shown in figure 3. This shows that there is consistency change trend between \(d\) and \(\theta\), when the Euclidean distance of \(d\) is small, the Euclidean distance of corresponding \(\theta\) is small too, and vice versa. That is to say when two signals’ TDOA vectors \(T\) are similar, the angles of arrival will be similar at the same time, they have a similar topological order and distribution.

3 Result and Discussion

In this section, simulations are carried out to verify the effectiveness of the SOM network. In the simulations, a uniform linear sensor array of four sensors \((M=3)\) is assumed along the x-axis, the array element spacing \(r=0.375\) m, and the array centered at \((0.5625, 0)\). Take \(50 \times 50\) uniform distribution points in region \([0,20] \times [0,20]\) as the location of \(50 \times 50\) signals, the distance difference vectors \(d\) of them are taken as sample vectors. For the SOM network, \(50 \times 50\) nodes are arranged on both first and second level.

Put the sample vectors into the network for training the network, as shown in figure 4, (a) (b) are distribution of sample vectors \(d\) and angles of arrival. It can be seen that (a) (b) have a similar topological order, their distribution density is consistent. The self-organized map of angles of the trained network is as shown in figure 5, each node in the hexagonal grid holds a model of angles, and the neighboring models are mutually similar, it has the same distribution with the corresponding \(d\) vectors.

For testing the network, we choose six groups points of different distances from the coordinate origin: 8 m, 16 m, 20 m, 30 m, 50 m, and 100 m, each group includes 21 points of different angles of arrival, calculate their distance difference vectors \(d\) of them are taken as sample vectors. For the SOM network, \(50 \times 50\) nodes are arranged on both first and second level.

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For testing the network, we choose six groups points of different distances from the coordinate origin: 8 m, 16 m, 20 m, 30 m, 50 m, and 100 m, each group includes 21 points of different angles of arrival, calculate their distance difference vectors, and then put them into the trained network to get the estimation of DOA. The absolute errors of test results are shown in figure 6. It can be seen from the figure, the network trained by near-field signals \([0,20] \times [0,20]\) works effectively in the estimation of the angle of arrival, not only in trained field...
signals (8, 16, 20), but also in far-field ones (30, 50, 100). As shown in Table 1, the average error of estimation results is about 0.1° to 0.3°.

Table 1. Results of angle estimation by SOM neural network

<table>
<thead>
<tr>
<th>Distance</th>
<th>8 m</th>
<th>16 m</th>
<th>20 m</th>
<th>30 m</th>
<th>50 m</th>
<th>100 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>0.201</td>
<td>0.102</td>
<td>0.123</td>
<td>0.112</td>
<td>0.118</td>
<td>0.169</td>
</tr>
<tr>
<td>Pr (err &lt; 0.3°)</td>
<td>0.852</td>
<td>0.952</td>
<td>0.905</td>
<td>1.00</td>
<td>0.852</td>
<td>0.905</td>
</tr>
<tr>
<td>Pr (err &lt; 0.2°)</td>
<td>0.810</td>
<td>0.877</td>
<td>0.810</td>
<td>0.810</td>
<td>0.905</td>
<td>0.852</td>
</tr>
</tbody>
</table>

To illustrate the efficacy and extensive adaptability of our method in estimation of DOA, a RBF neural network is taken in comparison. The RBF network is trained by the same data set as the SOM network, and the angles of arrival is as the target output. Then the two trained networks are tested with 20 simulation signals, the locations of test signals are distributed from 2 meters to 40 meters, which cover the training range and non-training range. It can be seen from figure 8 that the RBF neural network performs well in the training range but poor out of the training range, while the SOM’s estimation error changes very small with the fluctuation of distance.

In the processing, there is no need of calculating the covariance matrix and its eigenvectors, just the simulated location information of a group of the points at near-field is enough to set up and train the network. When a new signal is detected by the sensor array, put the estimated TDOA vector input the network, then the estimation of angle of arrival is obtained. In conclusion, the using of SOM network on estimation of DOA is worthy of being applied in practice.

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Conflicts of Interest
The authors declare no conflict of interest.

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