

On Cognitive Learning Methodologies for Cognitive Robotics

Yingxu Wang

International Institute of Cognitive Informatics and Cognitive Computing (ICIC)
 Laboratory for Computational Intelligence and Software Science
 Dept. of Electrical and Computer Engineering
 Schulich School of Engineering and Hotchkiss Brain Institute, University of Calgary
 2500 University Drive NW, Calgary, Alberta, Canada T2N 1N4
 Email: yingxu@ucalgary.ca

Abstract: A cognitive robot is an autonomous robot that is capable of inference, perception, and learning mimicking the cognitive mechanisms of the brain. The underpinning technologies for cognitive robots are the cognitive knowledge base (CKB) and the cognitive learning engine (CLE). This paper explores the cognitive foundations and denotational mathematical means of CLE and CKB for cognitive robotics. The conceptual architectures and cognitive functions of CKB and CLE are formally described. A content-addressed knowledge base methodology for CKB and a recursive learning algorithm for CLE are formally presented. The CLE and CKB theories and methodologies are not only designed to explain the mechanisms of human knowledge acquisition and learning, but also applied in the development of cognitive robots, cognitive computers, and knowledge-based systems as a key methodology.

Key-Words: Cognitive robotics, cognitive informatics, cognitive computing, abstract intelligence, cognitive systems, autonomous learning, computational intelligence, denotational mathematics, brain science, neuroinformatics, intelligence science, cognitive machine learning, cognitive knowledge base, and artificial intelligence.

1 Introduction

The hierarchy of human knowledge is categorized at the levels of *data*, *information*, *knowledge*, and *intelligence*. For instance, given an AND-gate with 1,000-input pins, it may be described very much differently at various levels of perceptions in the knowledge hierarchy. At the data level on the bottom, it represents a $2^{1,000}$ state space, known as ‘*big data*’ in recent terms, which appears to be a big issue in engineering. However, at the information level, it just represents 1,000 bit message that is equivalent to the numbers of inputs. Further, at the knowledge level, it expresses only two rules that if all inputs are one, the output is one; and if any input is zero, the output is zero. Ultimately, at the intelligence level, it is simply an instance of the logical model of an AND-gate with arbitrary inputs. This example reveals that human intelligence and wisdom are an extremely efficient and a fast convergent *induction* mechanism for knowledge and

wisdom elicitation and abstraction where data are merely factual materials and arbitrary instances in the almost infinite state space of the real world. Therefore, databases and knowledge bases are significantly different in both theories and manipulation mechanisms.

A Cognitive Knowledge Base (CKB) is a knowledge base that represents and manipulates knowledge as a dynamic concept network mimicking human knowledge processing. CKB is demanded in machine learning, knowledge-based systems, cognitive computers, and cognitive robots in general, as well as in the development of the Cognitive Learning Engine (CLE) for cognitive robots in particular. CKB is a central component for machine learning via autonomous knowledge acquisition and manipulation, because the general form of learning is a knowledge acquisition and manipulation process according to the latest studies in cognitive science, brain science, and neuroinformatics [3, 6, 17, 22, 24, 31, 34].

Conventional knowledge bases are studied in three categories known as the *linguistic knowledge bases* [5, 6, 7, 10, 13, 31], *expert knowledge bases* [1, 21, 34], and *ontology* [2, 9, 20, 21]. Typical linguistic knowledge bases are generic lexical databases such as WordNet and ConceptNet [7, 11]. The linguistic knowledge bases only provide general materials or dictionaries for applied knowledge bases of individuals and systems. Expert knowledge bases are elicitation of various domain knowledge represented by logical and fuzzy logical rules [1, 15, 32, 33, 35, 36]. However, human knowledge representation and retrieval are more complicated and semantics-centric beyond logical rules. Ontology deals with small-scale knowledge in a certain domain as a hierarchical network of a set of natural words and their semantic relations [2, 4, 8, 14, 16, 29]. Ontology represents knowledge in a static and application-specific model, which cannot be applied as a general knowledge base for machine learning and real-time knowledge manipulations.

According to studies in cognitive science and neurophysiology [9, 12, 25, 26, 30], the foundations of human knowledge and long-term memory can be represented by an Object-Attribute-Relation (OAR) model based on the synaptic structure of human memory. The OAR model represents the hierarchical and dynamic neural clusters of knowledge retained in memory, which leads to the development of the logical model of cognitive knowledge bases.

Definition 1. The *OAR model of knowledge* as retained in long-term memory (LTM) is a triple, i.e.:

$$OAR \triangleq (O, A, R) \quad (1)$$

where O is a finite set of objects identified by unique symbolic names, A is a finite set of attributes for characterizing each object, and R is a set of relations between objects and attributes.

The OAR model can be illustrated as shown in Fig. 1 for formally modelling the structure of human knowledge and its representation in LTM and CKB.

This paper presents a novel cognitive learning engine (CLE) for cognitive robots powered by the cognitive knowledge base (CKB). The structure model of CKB is described in Section 2, which encompasses the formal concept model for itemized knowledge representation and the dynamic concept network model for the entire knowledge base composition. Knowledge manipulations in CKB are embodied by a set of knowledge acquisition and retrieval operations on the structural models of CKB.

On the basis of CKB, the CLE architecture and methodologies are formally described in Section 4 for implementing the autonomous learning of cognitive robots.

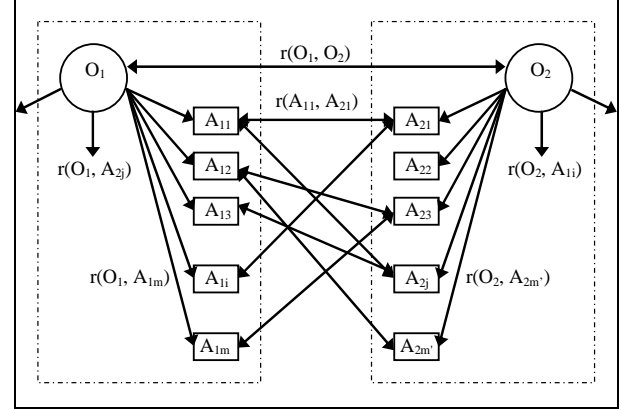


Figure 1. The OAR model of human knowledge representation

2 The Cognitive Knowledge Base for Robot Learning

It is recognized that the generic form of machine learning is a knowledge acquisition and manipulation process mimicking the brain. Therefore, knowledge representation as a dynamic concept network is centric in the design and implementation of the intelligent knowledge base for the cognitive learning engine in a cognitive robot.

The logical structure of CKB is modeled as a dynamic network of acquired knowledge of concepts and themes as shown in Fig. 2. The formal concept in CKB represents a set of itemized knowledge in the form of the OAR model according to concept algebra [18]. The theme and view represent a set of composite knowledge on a subject with multiple associate concepts in CKB. Knowledge acquisition, fusion, and retrieval in CKB are controlled by the knowledge manipulation engine.

2.1 The formal Concept Model of Knowledge

Definition 2. Let O denote a finite nonempty set of *objects*, and A be a finite nonempty set of *attributes*, the *semantic discourse* \mathcal{U}_c of knowledge bases is a triple, i.e.:

$$\begin{aligned} \mathcal{U}_c &= (\mathcal{O}, \mathcal{A}, \mathcal{R}) \\ &= \mathcal{R} : \mathcal{O} \rightarrow \mathcal{O} / \mathcal{O} \rightarrow \mathcal{A} / \mathcal{A} \rightarrow \mathcal{O} / \mathcal{A} \rightarrow \mathcal{A} \end{aligned} \quad (2)$$

where R is a set of relations between O and A .

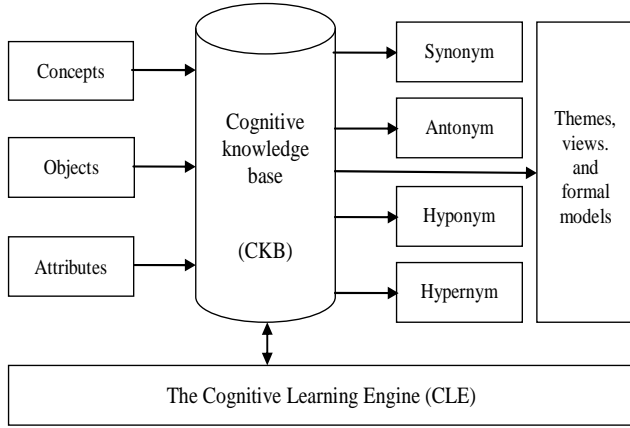


Figure 2. Architectural model of the cognitive knowledge base

On the basis of the semantic discourse of knowledge, an abstract or general concept can be formally modeled as follows.

Definition 3. A *formal concept*, C , as the basic unit of knowledge and the unique semantic model of linguistics in the discourse of the universal concept environment \mathcal{U}_c is a 5-tuple, i.e.:

$$C \triangleq (A, O, R^c, R^i, R^o) \quad (3)$$

where A is a finite nonempty set of *attributes (intension)* of C , $A \subseteq \mathcal{P}\mathcal{A} \sqsubset \mathcal{U}_c$ where \mathcal{P} represents a power set and \sqsubset denotes that a set is a substructure of a given hyperstructure; O is a finite nonempty set of *objects (extension)* of C , $O \subseteq \mathcal{P}\mathcal{O} \sqsubset \mathcal{U}_c$; $R^c \subseteq O \times A$ is a finite nonempty set of internal relations, $R^c \subseteq \mathcal{P}\mathcal{R} \sqsubset \mathcal{U}_c$; $R^i \subseteq A' \times A$, $A' \subseteq C' \wedge A' \not\subseteq C$, is a finite nonempty set of input relations where C' is a set of external concepts, $C' \subseteq \mathcal{P}\mathcal{C} \sqsubset \mathcal{U}_c \wedge C' \neq C$. For convenience, $R^i = A' \times A$ will be simply denoted as $R^i \subseteq C' \times C$; and $R^o \subseteq C \times C'$ is a finite nonempty set of output relations.

The formal concept is the minimum meaningful unit of knowledge, while words in natural languages are not because of their overloaded ambiguity. The structure model (SM) of concepts, Concept|SM , describes an itemized knowledge in CKB as shown in Fig. 3. Concept|SM represents a formal concept as a 5-tuple in Real-Time Process Algebra (RTPA) [19] where each field in the tuple is defined by a type and

a constraint. In the concept model, ConceptID|S denotes the name of the concept as a string upto 100 characters; $A|U$ a set of attributes that denotes the intention of the concept; $O|U$ a set of objects that instantiates the extension of the concept; $R|U$ a set of internal relations between the objects and attributes; and $RE|U$ a set of external relations between the concept and other concepts in the knowledge base.

```

Concept|SM  $\triangleq$ 
{ <ConceptID: S | 1 ≤ ConceptID|S ≤ 100>, // ID of the concept
  <A: U | A|U = {A1|SM, A2|SM, ..., An|SM}>, // Attributes
  <O: U | O|U = {O1|SM, O2|SM, ..., Om|SM}>, // Objects
  <R: U | R|U = O|U × A|U>, // Internal relations
  <RE: U | RO|U = C|SM × C'|SM> // External relations
}
    
```

Figure 3. The structure model of formal concepts

In the CKB models, a set of typical type suffixes of RTPA is adopted for denoting the semantic categories of entities such as SM (structure models), PM (process models), N (natural numbers), R (real numbers), L (Boolean variables with two constants T – true or F – false), S (strings), and U (sets). An entity in CKB is defined as $\text{EntityID:TypeSuffix}$ and invoked as $\text{EntityID|TypeSuffix}$, e.g., $A|U$ is a set of attribute A in the set type.

2.2 The Structure of Cognitive Knowledge Base for Cognitive Robots

Definition 4. A *generic knowledge* K in \mathcal{U}_c is an n -ary relation R_k among a set of n concepts, C , i.e.:

$$K = R_k : C \rightarrow \prod_{i=1}^n C_i \quad (4)$$

where X denotes a Cartesian product and $R_k \in \mathcal{R} \sqsubset \mathcal{U}_c$.

According to Definition 4, the *entire knowledge* \mathfrak{K} of a person or a cognitive system is a Cartesian product of all formal concepts acquired in the form of a concept network.

Theorem 1. The *entire knowledge* \mathfrak{K} in Θ is a Cartesian product among all formal concepts C in the CKB, i.e.:

$$\mathfrak{K} \triangleq DCN = R_k : \prod_{i=1}^n C_i \rightarrow \prod_{j=1}^n C_j, i \neq j \quad (5)$$

where \mathfrak{K} is embodied as a *dynamic concept network (DCN)*.

Proof: Theorem 1 can be directly proven based on Definitions 1 through 4 with neurophysiological support as shown in Fig. 1. Because the relations between concepts are transitive, the generic topology of knowledge is a DCN, which is continuous updating when new knowledge in the form of formal concepts is acquired. ■

The advantages of the DCN representation of knowledge are natural (as described in the OAR model in Fig. 1), dynamic, and evolvable. It is dynamic because the knowledge base can be updated flexibly during knowledge acquisition and learning without destructing the existing nodes and edges. It is evolvable because the knowledge base may adaptively grow without changing the existing structure and topology of DCN.

The formal structure of CKB, CKB|SM, is modeled as a set of interconnected concepts Concept|SM as shown in Fig. 4 in RTPA. The current numbers of concepts in CKB|SM is registered in #Concepts|N. The itemized knowledge of Concept|SM as modeled in Fig. 3 is extended by a concept index as a serial number and a time stamp that indicates when the concept is created. The logical model of CKB is designed according to the nature of human knowledge as described in Theorem 1. The DCN can be embodied as a digraph where a node is a formal concept, and an edge is one of the built-in relations such as attribute, object, synonym, antonym, hyponym, and hypernym as formally

described in Fig. 4. The formal model of CKB|SM indicates that a CKB in cognitive computing and cognitive linguistics is not merely a relational database. Instead, it is a complex network of itemized knowledge denoted by a set of formal concepts and their relational themes and views. The Cartesian product between all concepts in CKB indicates the semantic complexity of a knowledge base.

According to concept algebra [18] and semantic algebra [27], a formal concept is represented in a hierarchical semantic context related to the synonym and antonym concepts at the same conceptual level, to the hypernym concepts at the higher level, and to hyponyms as well as attributes and objects at the lower level. This hierarchical semantic framework represents the knowledge base acquired by a person or a cognitive robot during learning.

2.3 Manipulations of Knowledge Acquisition in CKB for Robot Learning

Knowledge acquisition in CKB as a result of leaning by humans and cognitive systems can be formally manipulated by the cognitive processes of concept memorization and knowledge fusion. The former enters an acquired concept into CKB based on the formal structure Concept|SM as formally defined in Fig. 3. The latter analyzes the relationship between a newly established concept and all existing concepts in CKB|SM according to Theorem 1.

$$\begin{aligned}
 \text{CKB|SM} \triangleq & \{ \langle \#Concepts: N \mid 1 < \#Concept|N \leq \text{SizeOfCKB|N} \rangle, \\
 & \langle \begin{matrix} \#Concepts|N \\ \mathbf{R} \\ i|N=1 \end{matrix} \text{Concept}(i|N)|\text{SM} :: \\
 & (\langle \text{Concept|SM} \rangle, \\
 & \langle \text{Index: } N \mid \text{Index|N} := i|N \rangle, \\
 & \langle \text{TimeStamp: YYYY:MM:DD:hh:mm:ss} \rangle \\
 & \langle \begin{matrix} \#Concepts|N & \#Concepts|N \\ \mathbf{R} & \mathbf{R} \\ i|N=1 & j|N=1 \end{matrix} \text{Concept}(i|N)|\text{SM} \times \text{Concept}(j|N \mid i|N \neq j|N)|\text{SM} :: \\
 & \quad | \langle \text{Synonym: } U \mid \text{Synonym|U} \mid \text{C}(i|N)|\text{SM.A|U} = \text{C}(j|N)|\text{SM.A|U} \rangle, \\
 & \quad | \langle \text{Antonym: } U \mid \text{Antonym|U} \mid \text{C}(i|N)|\text{SM.A|U} = \text{C}(j|N)|\text{SM.}\bar{A}|U \rangle, \\
 & \quad | \langle \text{Hyponym: } U \mid \text{Hyponym|U} = \text{C}(i|N)|\text{SM.A|U} \subset \text{C}(i'|N)|\text{SM.A|U} \rangle, \\
 & \quad | \langle \text{Hypernym: } U \mid \text{Hypernym|U} = \text{C}(i|N)|\text{SM.A|U} \supset \text{C}(i'|N)|\text{SM.A|U} \rangle, \\
 & \quad | \langle \text{Independent: } U \mid \text{Independent|U} = \{ \text{C}(i|N)|\text{SM.A|U} \cap \text{C}(i'|N)|\text{SM.A|U} \} = \emptyset \rangle \\
 & \rangle) \\
 & \rangle \\
 & \}
 \end{aligned}$$

Figure 4. The structure model of the entire knowledge base

The first step of knowledge acquisition is to retain a newly acquired itemized knowledge in CKB. The second step of knowledge acquisition is to connect the newly acquired itemized knowledge to existing knowledge in CKB by mapping the newly acquired concept into the entire knowledge base via comparative analyses. This process is called knowledge fusion that analyzes potential relations between the new concept and the n existing ones by 1-to- n pairwise mapping throughout CKB. The mathematical model of knowledge fusion is formally described in Eq. 4. It reveals the mechanisms of knowledge memorization and fusion where acquired knowledge buffered in short-term memory is selectively moved into long-term memory [17, 21, 23, 28]. It is also the cognitive mechanism of human subconscious learning and knowledge comprehension.

2.4 Manipulations of Knowledge Retrieval in CKB for Robot Learning

As the result of machine learning, knowledge retrieval from CKB can be implemented at three levels such as those of concept retrieval, subject retrieval, and entire knowledge base retrieval. The bottom level retrievals for concepts from CKB can be further classified as those of formal concepts, antonyms, hyponyms (attributes and objects), and hypernyms.

An important function of CKB is its content-addressed mechanism for knowledge retrieval and manipulations enabled by the structure models and cognitive search algorithms.

Concept retrieval seeks a targeted itemized knowledge by content-addressed matching, which adaptively allocates and fetches a target concept in CKB without the requirement for a memory address as that in databases. The ConceptRetrieval|PM process is designed as a set of iterative matching searches on all concepts learnt and acquired in CKB|SM. Then, synonym, antonym, hypernym, and hyponym retrievals from CKB can be implemented in the similar way.

On the basis of the retrieval algorithms of individual concept as well as antonyms, hyponyms, and hypernyms, the retrieval of a subject/theme and the entire knowledge base is logically reduced to an iterative retrieval of all concepts encompassed in them, respectively. As a result, a subject or the entire CKB is embodied by an interconnected concept network,

denoted by the Dynamic Concept Network (DCN), where a node is a concept and an edge is an external relation between a pair of concepts established by the algorithm of knowledge fusion.

3 The Cognitive Learning Engine of Cognitive Robots

On the basis of CKB as formally described in the preceding section, a general Cognitive Learning Engine (CLE) is designed and implemented for autonomous machine learning of natural language contents and online documents mimicking the cognitive process of human beings.

3.1 The Architecture of CLE

CLE is an autonomous machine learning system to learn text-based knowledge in natural languages and symbolic notations. The architecture of CLE is described by the functional model as shown in Fig. 5. CLE encompasses two subsystems known as the learning kernel and CKB. The former enables autonomous machine learning based on a recursive learning algorithm, which will be formally described in Section 3.2. The latter supports the manipulation of existing and newly acquired knowledge via conceptual and logic knowledge representation.

3.2 Implementation of CLE

CLE implements autonomous machine learning by a recursive learning algorithm. As analysed in Section 2, concept is the basic unit of learning that reserves the complete and stable semantics in natural languages. Therefore, at the most fundamental level, concept learning is implemented as the kernel of CLE. Based on the cognitive algorithm of concept learning, high-level learning mechanisms such as sentence, subject (paragraph), and general (essay) learning are implemented recursively as shown in Fig. 6.

There are seven relational operators, five reproductive operators, four compositional operators, on formal concepts according to concept algebra [18]. On the basis of concept algebra, human knowledge and semantics in natural languages can be rigorously modeled as formal concepts.

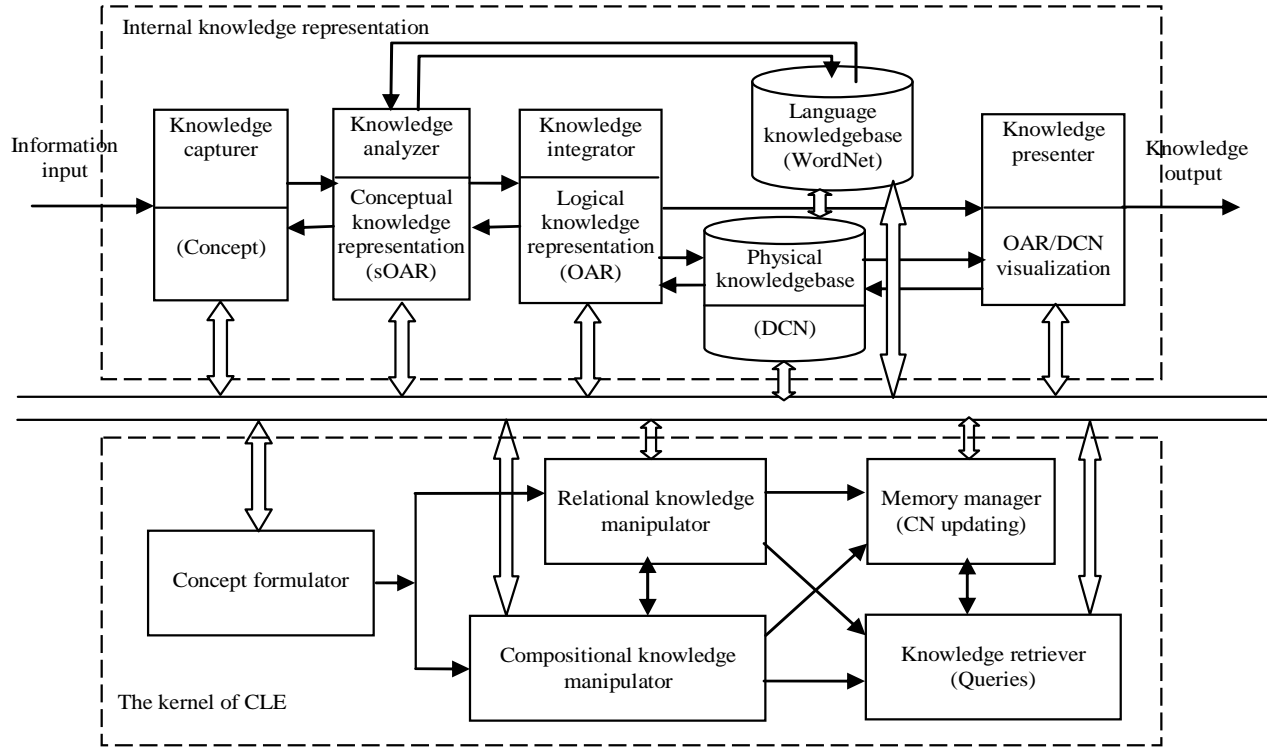


Figure 5. Architecture of the cognitive learning engine (CLE)

The learning of fundamental concepts can be reduced to the algebraic operations on formal concepts in CKB. For instance, the *composition* of two concepts C_1 and C_2 yields a superconcept C by the intersection of both sets of attributes, the union of both sets of objects, the updating of internal relations, and the incremental unions of the input/output relations, respectively. Given two formal concepts $C_1(\text{pen})$ and $C_2(\text{pencil})$, the composite semantics of a superconcept $C_3(\text{stationery})$, can be rigorously derived as formally described in Eq. 6.

The example of concept composition as given in Eq. 6 yields a superconcept $C_3(\text{stationery})$ as illustrated in Fig. 7 where the superconcept is composed by the given subconcepts $C_1(\text{pen})$ and $C_2(\text{pencil})$.

Similarly, the *aggregation* of a superconcept C_1 from a given concept C_0 is an inductive generalization of C_0 by a broader intension with fewer specific attributes A' and extended coverage of objects O' . Given the subsets of attributes and objects $A' = \{\text{ink}\}$ and $O' = \{\text{pencil}\}$ to be removed or extended, respectively. The aggregation of the superconcept $C_{41}(\text{stationery}_{41})$ based on $C_1(\text{pen})$ can be obtained as shown in Eq. 7. The example of

concept aggregation yields a superconcept relation with increasingly broader concept, i.e., $C_1(\text{pen}_1) \Rightarrow C_{41}(\text{stationery}_{41})$ as illustrated in Fig. 8.

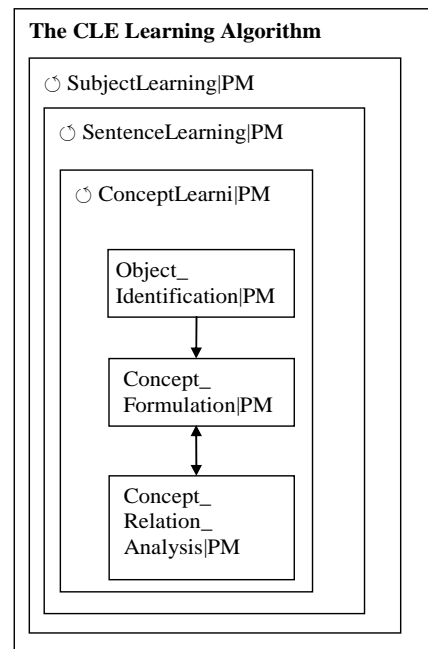


Figure 6. The recursive learning mechanism of CLE

$$\begin{aligned}
 &C_3(\text{stationery: } A_3, O_3, R_3^c, R_3^i, R_3^o) \triangleq C_1(\text{pen: } A_1, O_1, R_1^c, R_1^i, R_1^o) \uplus C_2(\text{pencil: } A_2, O_2, R_2^c, R_2^i, R_2^o) \\
 &= C_3 \left\{ \begin{aligned}
 &A_3 = \{a_{31}, a_{32}, a_{33}, a_{34}, a_{35}\} = \{\text{instrument, writing, nib, paper, office}\} \\
 &O_3 = \{o_{31}, o_{32}, o_{33}, o_{34}, o_{35}\} \\
 &\quad = \{\text{ballpoint, fountain, brush, normal_pencil, automatic_pencil}\} \\
 &R_3^c = O_3 \times A_3 = \{[(o_{31}, a_{31}), (o_{31}, a_{32}), (o_{31}, a_{33}), (o_{31}, a_{34}), (o_{31}, a_{35}), (o_{31}, a_{36})], \\
 &\quad [(o_{32}, a_{31}), (o_{32}, a_{32}), (o_{32}, a_{33}), (o_{32}, a_{34}), (o_{32}, a_{35}), (o_{32}, a_{36})], \dots, \\
 &\quad [(o_{35}, a_{31}), (o_{35}, a_{32}), (o_{35}, a_{33}), (o_{35}, a_{34}), (o_{35}, a_{35}), (o_{35}, a_{36})]\} \\
 &R_3^i = R_1^i \cup R_2^i \cup \{(C_1, C_3), (C_2, C_3)\} \\
 &R_3^o = R_1^o \cup R_2^o \cup \{(C_3, C_1), (C_3, C_2)\}
 \end{aligned} \right. \tag{6}
 \end{aligned}$$

$$\begin{aligned}
 &C_1(\text{pen: } A_1, O_1, R_1^c, R_1^i, R_1^o) \Rightarrow C_{41}(\text{stationery}_{41}: A_{41}, O_{41}, R_{41}^c, R_{41}^i, R_{41}^o) \\
 &= C_{41} \left\{ \begin{aligned}
 &A_{41} = \{a_{414}, a_{412}, a_{413}, a_{414}, a_{415}\} = \{\text{instrument, writing, nib, paper, office}\} \\
 &O_{41} = \{o_{411}, o_{412}, o_{413}, o_{414}\} = \{\text{ballpoint, fountain, brush, pencil}\} \\
 &R_{41}^c = O_{41} \times A_{41} = \{[(o_{411}, a_{411}), (o_{411}, a_{412}), (o_{411}, a_{413}), (o_{411}, a_{414}), (o_{411}, a_{415})], \\
 &\quad [(o_{412}, a_{411}), (o_{412}, a_{412}), (o_{412}, a_{413}), (o_{412}, a_{414}), (o_{411}, a_{415})], \\
 &\quad [(o_{413}, a_{411}), (o_{413}, a_{412}), (o_{413}, a_{413}), (o_{413}, a_{414}), (o_{411}, a_{415})], \\
 &\quad [(o_{414}, a_{411}), (o_{414}, a_{412}), (o_{414}, a_{413}), (o_{414}, a_{414}), (o_{411}, a_{415})]\} \\
 &R_{41}^i = R_1^i \cup (C_1, C_{41}) \\
 &R_{41}^o = R_1^o \cup (C_{41}, C_1)
 \end{aligned} \right. \tag{7}
 \end{aligned}$$

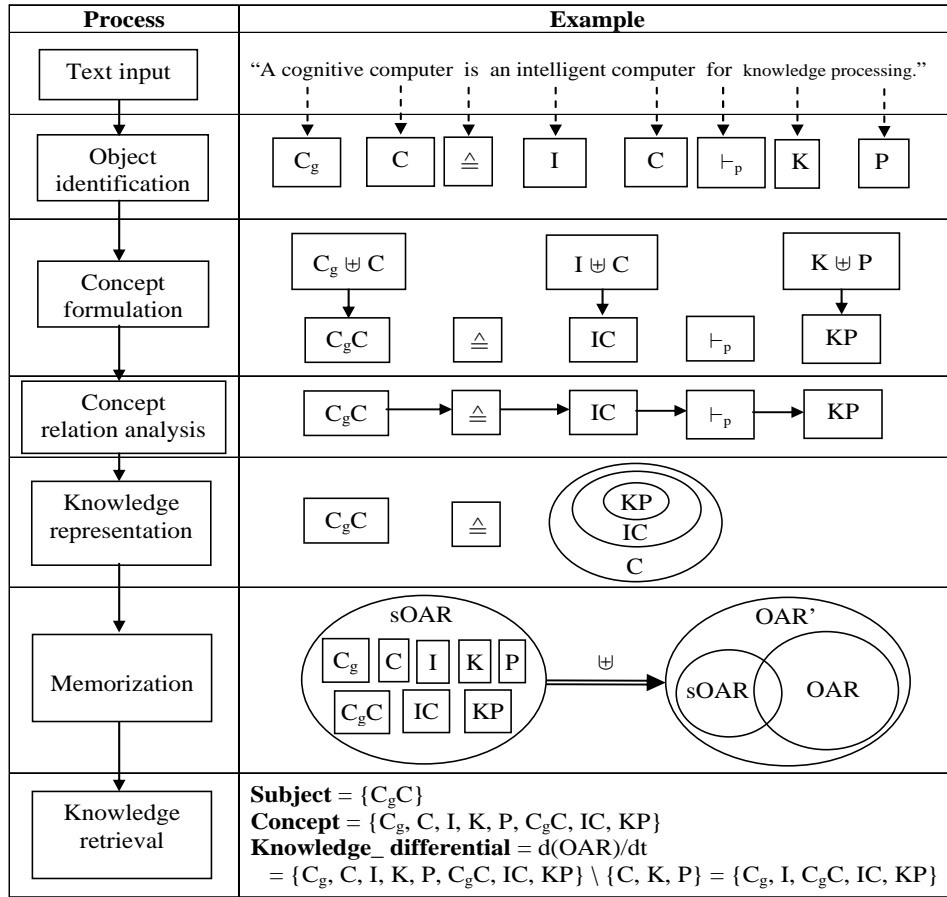


Figure 9. Cognitive robot learning implemented by CLE and CKB

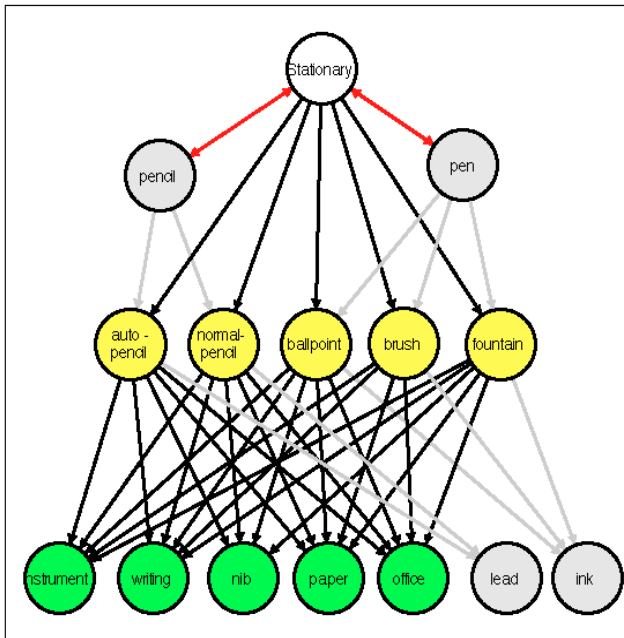


Figure 7. Robot learning by concept composition

With the support of CKB for knowledge representation, acquisition, and retrieval, the cognitive learning system, CLE, mimics human cognitive processes of learning and knowledge acquisition as illustrated in Fig. 9 where the denotational mathematical operators are adopted from concept algebra and semantic algebra [18, 27].

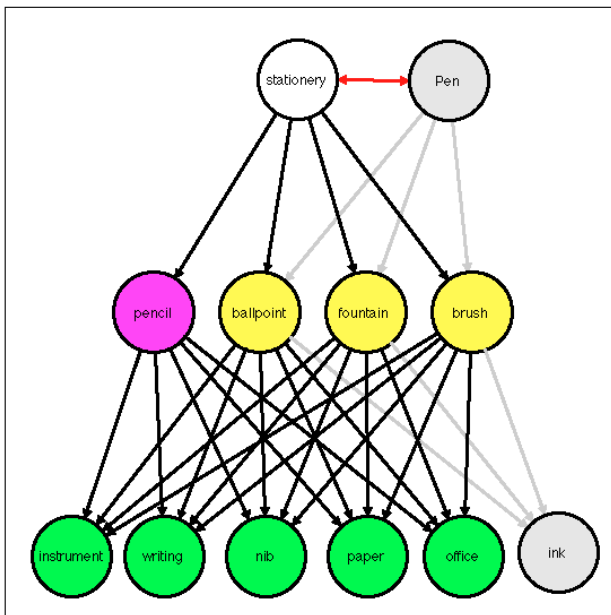


Figure 8. Robot learning by concept aggregation

4 Conclusion

This paper has presented a novel cognitive learning engine (CLE) for cognitive robotics powered by the cognitive knowledge base (CKB). CKB and its theoretical foundations on cognitive science and neuroinformatics are formally presented in this paper, which explains how humans acquire, memorize, and manipulate knowledge in learning. It also demonstrates how cognitive machines and systems may learn from the human cognitive processes of knowledge manipulations and learning on the basis of the denotational mathematical models and algorithms. The CKB system has been characterized by its content-addressed knowledge access, which is different from those of conventional databases.

CLE as an autonomous robot learning system has been developed based CKB for text-based knowledge acquisition in natural languages and symbolic notations. CLE has implemented the recursive learning technology at concept, subject, and essay levels from the bottom up. An interest finding in this work has been the shareability and transformability of learnt knowledge among cognitive machines when the same structure of cognitive knowledge base is adopted. The CLE and CKB systems have not only been adopted to simulate the mechanisms of human knowledge acquisition, comprehension, and learning, but also been applied in the development of cognitive robots, cognitive learning systems, cognitive computers, and knowledge-based systems as a key methodology and novel technology.

Acknowledgments

This work is supported in part by a discovery fund granted by the Natural Sciences and Engineering Research Council of Canada (NSERC). The author would like to thank the anonymous reviewers for their valuable comments on the previous version of this paper.

References

- [1] Bender, E.A. (1996). *Mathematical Methods in Artificial Intelligence*. Los Alamitos, CA: IEEE CS Press.
- [2] Brewster C., K. O'Hara et al. (2004). Knowledge Representation with Ontologies: The Present and Future. *IEEE Intelligent Systems*, 19 (1), 72-81.
- [3] Chang, C.-H., Kayed, M., Girgis, M. R., & Shaalan, K. (2006). A Survey of Web Information Extraction System. *IEEE Transactions on Knowledge and Data Engineering*, 18(10), 1411-1428.

- [4] Cocchiarella, N. (1996). Conceptual Realism as a Formal Ontology. In R. Poli & P. Simons (Eds.), *Formal Ontology*. London: Kluwer Academic, 27–60.
- [5] Crystal, D. (1987). *The Cambridge Encyclopedia of Language*. NY: Cambridge University Press.
- [6] Debenham, J.K. (1989). *Knowledge Systems Design*. NY: Prentice Hall.
- [7] Fellbaum, C. (1998). *WordNet: An Electronic Lexical Database*. Cambridge, MA: MIT Press.
- [8] Gruber, T. (1993). A Translation Approach to Portable Ontology Specifications. *Knowledge Acquisition*, 5(2), 199-220.
- [9] Leone, N., G. Pfeifer et al. (2006). The DLV System for Knowledge Representation and Reasoning. *ACM Transactions on Computational Logic*, 7(3), 499-562.
- [10] Liddy, E.D. (2001). Natural Language Processing. In *Encyclopedia of Library and Information Science*, (2nd ed.). NY: Marcel Decker.
- [11] Liu, H. and P. Singh (2004). ConceptNet - A Practical Commonsense Reasoning Toolkit. *BT Technology Journal*, 22(4), 211–225.
- [12] Pojman, L.P. (2003). *The Theory of Knowledge: Classical and Contemporary Readings*. Belmont, CA: Wadsworth/Thomson Learning.
- [13] Pullman, S. (1997). *Computational Linguistics*. Cambridge, UK: Cambridge University Press.
- [14] Sanchez, D. (2010). A Methodology to Learn Ontological Attributes from the Web. *Data & Knowledge Engineering*, 69(6), 573-597.
- [15] Surmann, H. (2000). Learning a Fuzzy Rule Based Knowledge Representation. *Proc. ICSC Symposium on Neural Computation*. Berlin, 349 - 355.
- [16] Tiberino, A., D. Embley, et al. (2005). Towards Ontology Generation from Tables. *Internet and Information Systems*, 8(3), 261–285.
- [17] Wang, Y. (2003). On Cognitive Informatics. *Brain and Mind*, 4(3), 151-167.
- [18] Wang, Y. (2008a). On Concept Algebra: A Denotational Mathematical Structure for Knowledge and Software Modeling. *International Journal of Cognitive Informatics and Natural Intelligence*, 2(2), 1-19.
- [19] Wang, Y. (2008b). RTPA: A Denotational Mathematics for Manipulating Intelligent and Computational Behaviors. *International Journal of Cognitive Informatics and Natural Intelligence*, 2(2), 44-62.
- [20] Wang, Y. (2009a). On Cognitive Computing. *International Journal of Software Science and Computational Intelligence*, 1(3), 1-15.
- [21] Wang, Y. (2009b). Formal Description of the Cognitive Process of Memorization. *Transactions of Computational Science*, Springer, 5, 81-98.
- [22] Wang, Y. (2010a). Cognitive Robots: A Reference Model towards Intelligent Authentication. *IEEE Robotics and Automation*, 17(4), 54-62.
- [23] Wang, Y. (2010b). On Formal and Cognitive Semantics for Semantic Computing. *International Journal of Semantic Computing*, 4(2), 203–237.
- [24] Wang, Y. (2012a). On the Denotational Mathematics Foundations for the Next Generation of Computers: Cognitive Computers for Knowledge Processing. *Journal of Advanced Mathematics and Applications*, 1(1), 118-129.
- [25] Wang, Y. (2012b). On Abstract Intelligence and Brain Informatics: Mapping Cognitive Functions of the Brain onto its Neural Structures. *International Journal of Cognitive Informatics and Natural Intelligence*, 6(4), 54-80.
- [26] Wang, Y. (2013a). Neuroinformatics Models of Human Memory: Mapping the Cognitive Functions of Memory onto Neurophysiological Structures of the Brain. *International Journal of Cognitive Informatics and Natural Intelligence*, 7(1), 98-122.
- [27] Wang, Y. (2013b). On Semantic Algebra: A Denotational Mathematics for Cognitive Linguistics, Machine Learning, and Cognitive Computing. *Journal of Advanced Mathematics and Applications*, 2(2), 38-56.
- [28] Wang, Y., & Wang, Y. (2006). Cognitive Informatics Models of the Brain. *IEEE Transactions on Systems, Man, and Cybernetics (Part C)*, 36(2), 203-207.
- [29] Wang, Y., Tian, Y., & Hu, K. (2011). Semantic Manipulations and Formal Ontology for Machine Learning Based on Concept Algebra. *International Journal of Cognitive Informatics and Natural Intelligence*, 5(3), 1-29.
- [30] Wang, Y. and G. Fariello (2012). On Neuroinformatics: Mathematical Models of Neuroscience and Neurocomputing. *Journal of Advanced Mathematics and Applications*, 1(2), 206-217.
- [31] Wang, Y. and R.C. Berwick (2012). Towards a Formal Framework of Cognitive Linguistics. *Journal of Advanced Mathematics and Applications*, 1(2), 250-263.
- [32] Wang, Y. and R.C. Berwick (2013). Formal Relational Rules of English Syntax for Cognitive Linguistics, Machine Learning, and Cognitive Computing. *Journal of Advanced Mathematics and Applications*, 2(2), in press.
- [33] Wang, Y. (2014). Fuzzy Causal Inferences based on Fuzzy Semantics of Fuzzy Concepts in Cognitive Computing. *Transactions on Computers*, 13, 430-441.
- [34] Wilson, R.A., & Keil, F.C. (2001). *The MIT Encyclopedia of the Cognitive Sciences*. Cambridge, MA: MIT Press.
- [35] Zadeh, L.A. (1965). Fuzzy Sets. *Information and Control*, 8, 338-353.
- [36] Zadeh, L.A. (2004). Precisiated Natural Language (PNL). *AI Magazine*, 25(3), 74-91.