Design of Streamline Dies for Drawing Driven by Fracture

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Abstract: - This paper presents an efficient analytical method for design of streamline dies driven by fracture. The method is based on Bernoulli’s theorem relating pressure and velocity along any streamline extended to ideal flows in plasticity. The Cockroft-Latham criterion is adopted to predict the initiation of ductile fracture. In order to apply the method developed, it is not necessary to know the solution to the boundary value problem of plasticity. The final result is a simple relation between geometric parameters of the process and the constitutive parameter involved in the fracture criterion. Since the latter is supposed to be known for a given material, the relation determines a safe domain for drawing without fracture.

Key-Words: - drawing, streamline die, fracture, ideal flow, Bernoulli’s theorem, perfectly plastic material

1 Introduction
Ideal flows in plasticity are widely used as the basis for inverse methods for the preliminary design of metalworking processes (see, for example, [1-5]). A comprehensive review on the ideal flow theory and its applications has been provided in [6]. Fracture is one of the most important modes of failure in metalworking processes. However, no fracture criterion is involved in the ideal flow theory. In the present paper, design solutions of stationary processes based on the ideal flow theory are supplemented with design driven by fracture.

Empirical ductile fracture criteria are often used to predict fracture in metal forming. In particular, such criteria are included in modern commercial finite element packages. Reviews of empirical ductile fracture criteria are provided, for example, in [7 – 9]. In the present paper, the fracture criterion proposed in [10] is adopted. Note that a modified version of this criterion has been introduced in [11]. However, the original and modified criteria coincide for rigid perfectly/plastic materials. Therefore, both criteria are referred to as the Cockroft and Latham criterion in the present paper. This criterion has been used and/or evaluated for several metals in [9, 12 - 19]. In particular, it has been indicated in [12, 13, 15] that the Cockroft and Latham criterion is the best amongst the various existing criteria under the conditions investigated in these papers.

Various approximate methods have been adopted to predict the initiation of fracture in drawing. In particular, the upper bound theorem has been used in [20, 21] and the solution for plastic flow through an infinite conical channel given in [22] has been adopted in [23] for drawing through conical and wedge-shape dies. Numerical solutions have been given in [17, 24]. The present approach is based on the extended Bernoulli theorem proven in [25] assuming that the shape of the die has been found using the ideal flow theory. A remarkable property of this approach is that there is no need to know the solution to the plasticity problem to apply the Cockroft and Latham criterion. The final expression is extremely simple and can be directly used for preliminary design of drawing driven by fracture.

2 Ideal Flows and Fracture Criterion
Ideal flows constitute a wide class of solutions in the theory of rigid perfectly/plastic solids. A comprehensive review on ideal flows has been provided in [6]. They are of particular interest in Tresca’s solids, i.e. solids satisfying Tresca’s yield criterion and its associated flow rule. Let \( \sigma_1, \sigma_2 \) and \( \sigma_3 \) be the principal stress. It is possible to assume with no loss of generality that \( \sigma_1 > \sigma_2 \) and \( \sigma_1 > \sigma_3 \). In what follows, it is sufficient to consider
the edge regime of Tresca’s yield criterion determined by
\[ \sigma_1 - \sigma_2 = 2k, \quad \sigma_1 - \sigma_3 = 2k. \] (1)
Here \( k \) is the shear yield stress, a material constant. In this case, the plastic work rate is given by
\[ \frac{dW}{dt} = 2k \xi_1. \] (2)
Here \( t \) is the time and \( \xi_1 \) is the principal strain rate of maximum magnitude. Note that the yield criterion (1) and the associated flow rule ensure the inequality \( \xi_1 > 0 \). It is evident that the trajectories of \( \sigma_1 \) and \( \xi_1 \) coincide. A remarkable property of steady ideal flows is that the streamlines in such flows are everywhere coincident with trajectories of the principal stress \( \sigma_1 \) [6]. Equation (2) for steady flows becomes
\[ \frac{\partial W}{\partial s} = 2k \xi_1. \] (3)
Here \( \partial / \partial s \) is the space derivative taken along streamlines and \( u \) is the magnitude of the velocity vector. By definition,
\[ \xi_1 = \frac{\partial u}{\partial s}. \] (4)
Substituting (4) into (3) yields
\[ \frac{\partial W}{\partial s} = 2k \frac{\partial u}{\partial s}. \] (5)
The ductile fracture criterion proposed in [10] is
\[ \int_0^t \sigma_1 \xi_{eq} dt = kC. \] (6)
Here \( C \) is a constitutive parameter, \( \xi_{eq} \) is the equivalent strain rate and \( t_f \) is the value of \( t \) at the initiation of fracture. The definition for the equivalent strain rate is usually associated with the plastic work rate [26]. Therefore, it follows from (2) that in the case under consideration it is natural to assume that \( \xi_{eq} = \xi_1 \). Then, using (4) equation (6) is transformed to
\[ \int_0^s \frac{\sigma_1 \xi_{eq}}{u} ds = kC. \] (7)
It has been taken into account here that steady flows are under consideration. It has been also assumed that \( s = 0 \) at the entrance to the plastic zone. Moreover, \( s = s_f \) at the exit from the plastic zone if the initiation of fracture occurs at the exit.

3 Fracture in Drawing
A schematic diagram of the drawing process is shown in Fig.1. There are two rigid zones and one plastic zone. The motion of the rigid zones is a translation along the same direction. The speed of rigid zones 1 and 2 are \( U_1 \) and \( U_2 \), respectively. Introduce an orthonormal basis \( e_1,e_2,e_3 \) of eigenvectors of the stress tensor. The vector \( e_1 \) corresponds to \( \sigma_1 \) and is tangent to streamlines. The velocity vector \( u \) is represented as
\[ u = u_1 e_1 + u_2 e_2 + u_3 e_3 \] (8)
with \( u_i > 0 \). It is evident that \( u_i = u \). It has been shown in [25] that in the case of quasi-static stationary flow of incompressible material without body forces the following relation along any streamline \( L \) coinciding with a trajectory of the stress \( \sigma_1 \) is valid
\[ \frac{\partial \sigma_1}{\partial s} - \frac{\partial W}{\partial s} + \left[ (\sigma_2 - \sigma_1) \text{div} (u e_2) + (\sigma_3 - \sigma_1) \text{div} (u e_3) \right] = 0. \] (9)
It has been mentioned before that all streamlines coincide with trajectories of the principal stress \( \sigma_1 \) in steady ideal flows. Therefore, \( u_2 = 0 \) and \( u_3 = 0 \) everywhere and equation (9) simplifies to
\[ \frac{\partial \sigma_1}{\partial s} = \frac{\partial W}{\partial s} \] (10)
Using (5) this equation is transformed to
\[ \frac{\partial \sigma_1}{\partial s} - 2k \frac{\partial u}{u \partial s} = 0. \] (11)
No active force is applied to rigid zone 1 (Fig. 1). Therefore, \( \sigma_i = 0 \) for \( u = U_i \). Integrating equation (11) with the use of this boundary condition leads to

\[ \sigma_i = 2k \ln \left( \frac{u}{U_i} \right). \]  

(12)

Substituting this solution into (7) gives

\[ 2 \int \ln \left( \frac{u}{U_i} \right) \frac{\partial u}{\partial x} ds = C. \]  

(13)

Assume that the initiation of fracture occurs at the exit from the plastic zone (Fig. 1) where \( u = U_2 \). Then, integrating (13) yields

\[ \ln \left( \frac{U_2}{U_1} \right) = C. \]  

(14)

It follows from the equation of incompressibility that

\[ U_1 S_1 = U_2 S_2. \]  

(15)

Here \( S_1 \) and \( S_2 \) are the cross-sectional areas of rigid zones 1 and 2, respectively. Substituting (15) into (14) leads to

\[ \ln \left( \frac{S_2}{S_1} \right) = C. \]  

(16)

Here \( S_2 / S_1 \) is understood as the minimum possible ratio for drawing without fracture. Since \( C \) is supposed to be known for any given material, equation (16) determines a safe range of \( S_2 / S_1 \) for drawing with no fracture. The function \( \ln x \) is monotonically decreasing in the range \( x < 1 \). Therefore, it is evident from (16) that the minimum possible ratio of \( S_2 / S_1 \) for drawing with no fracture decreases as the value of \( C \) increases. This is in agreement with physical expectations that the maximum possible reduction with no fracture increases as the ductility of material increases.

4 Conclusions

It has been shown that the design of drawing based on the ideal flow theory can be supplemented with the Cockcroft-Latham ductile fracture criterion by means of equation (16). This simple equation determines the maximum possible drawing ratio without fracture. The simplicity of equation (16) makes it suitable for quick design of the process. This preliminary design can also be used as an initial guess for sophisticated design solutions based on numerical methods.

In general, equation (16) is valid for any three-dimensional flow. Moreover, one does not need to know the solution to the boundary value problem of plasticity to apply this equation. However, it is important to know that the solution exists. In the case of strip, wire and tube drawing the ideal flow solutions have been given in [1 -3]. Therefore, equation (16) can be used for such processes without any restriction. However, three-dimensional ideal flows may not exist. Necessary conditions for the existence of such ideal flows have been derived in [27].

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References:


