Natural Convection Heat Transfer Enhancement from An Isothermal Array Of Circular Cylinders Under Horizontal Vibrations

A. SHAMS TALEGHANI¹, S. ABDOLAHI¹, A. FALAHIAN²
1-Aerospace Research Institute, Tehran, IRAN
2- Department of Mechanical Engineering, Dezful Branch, Islamic Azad university, Dezful, IRAN
Taleghani@ari.ac.ir

Abstract: - Vibration in heat exchangers sometimes has positive effect in heat transfer enhancement. In this article a characteristics based method in Arbitrary Lagrangian Eulerian framework for 2D unsteady flow was used. The governing equations are Navier–Stokes for incompressible flow and energy equation. Vibrations applied to an array of isothermal cylinders in a cavity that is a simple model for simulation of natural convection in a heat exchanger. The effects of frequency of oscillations, cylinder spacing, amplitudes of vibrations and Rayleigh number have been investigated. The results indicate that, the increment of distance of two cylinders by cylinder diameter ratio makes the increment of the averaged Nusselt number only at middle and top cylinders and doesn’t have a substantial change in the bottom cylinder. Moreover, increment of the amplitudes of vibrations at a fixed frequency, enhances the averaged Nusselt number due to augmentation of velocity of oscillations.

Key-Words: - Natural Convection, Finite Element, ALE method, Characteristic Based Split, Oscillating Cylinder

1 Introduction
Free convection heat transfer from a vertical array of horizontal cylinders has been the subject of many investigations because of its numerous engineering and industrial applications. Typical examples are space heating, heating of high-viscosity oils for pumping ease, heating or cooling of fluids in heat exchangers. Natural convection from an array of cylinders is quite different in comparison with a single cylinder because of the interaction of the flow fields around adjacent cylinders in effect of buoyant plume generated by each cylinder. Because of this interaction the devices in the system are frequently under dynamic situation and caused the device being unavoidably subjected to vibrational motion. So, investigating the mechanism of natural convection on the vibrating heat surface induced by the operation of the system is an urgent need for the design of precise and effective device. Previously, a great number of experimental and numerical works has been carried out to study the effect of vibration on the natural convection. Gururatana and Li[1] used vibrating pin fin for heat transfer enhancement of small scale heat sinks. They applied the vibration frequency between 50 to 1,000 Hz. Their numerical results showed heat transfer augmentation that it increased with frequency until certain value then it drops rapidly. Yao et. al. [2] experimentally investigated the heat transfer enhancement of water-water heat exchanger in shell-and-tube type assisted by power ultrasonic and they evaluated impact of some parameters like as water flow rate, inlet water temperature in the tube, and the ultrasonic power. They found that the water flow rate and ultrasonic power levels would produce great influence on the enhancement by power ultrasound which decreased with the increasing water velocity in the tube and the decreasing acoustic power. Eid and Gomma[3] investigate the possibility of the enhancement of heat transfer rate from heat sink having thin planner fins by normal vibration. The effect of both vibration frequency and displacement amplitude on the enhancement of heat transfer rate was clarified by them. They report 85% enhancement of heat transfer rate by normal vibration. The effect of vibrations and gravity on the service ability of the heat pipes was carried out by Prisniakova et al. [4]. An overview of vibration analysis procedures and recommended design guidelines was presented by Pettigrewa and Taylorb [5]. Cheng et. al.[6] proposed a novel approach to enhance the heat transfer by using the flow-induced vibration of a new designed heat transfer device. They studied the effects of vibrations on the heat transfer numerically and experimentally, and the correlation of the shell-side convective heat transfer coefficient was obtained. It was found that the new designed heat exchanger can significantly increase the convective heat transfer coefficient and decrease the fouling resistance.
In present work, the effects of horizontal sinusoidal vibrations applied to an isothermal array of cylinders situated in a cavity have been investigated on Nusselt number augmentation. The influence of most important parameters to heat transfer performance, inclusive frequency of oscillations, cylinder spacing, amplitudes of vibrations and Rayleigh number were studied numerically. Due to oscillation in cylinders, flow and thermal fields would be unsteady. The governing equations were continuum, Navier-Stokes and energy. The equations were solved by finite element method based on Characteristic Based Split (CBS)[7-8]. Because of the cylinders movement, an Arbitrary Lagrangian Eulerian (ALE) approach was used. In this work, the computational mesh is independent of the material motion. Therefore the ALE viewpoint is suitable for our simulation. For more description of ALE method, you can see [9-11].

2 Governing Equations

The array of iso-thermal circular cylinders in a cavity is shown in Fig. 1. The diameter of cylinders is d and the center-to-center distance between each cylinder is s. The temperature of cylinders is constant and equal to \( T_h \) and the ambient temperature is assumed \( T_\infty \). As the time \( t > 0 \), the cylinders are subjected to sinusoidal motion, which is in x direction and normal to gravity direction. The amplitude and frequency of cylinders are \( L \) and \( f \), respectively. The flow is incompressible, physical properties are constant and viscous dissipation and pressure work are negligible. The buoyancy effects on momentum transfer are taken into account through the Boussinesq approximation. The flow field and thermal field due to cylinders oscillations and interaction of fluid around adjacent cylinders are unsteady and because of the boundary conditions in the cylinders are classified into a class of moving boundary problem. The ALE method is utilized to investigate this problem.

Non-dimensional variables are defined as follows based on the d, \( T_h \), and \( T_\infty \) [12]:

\[
x^* = \frac{x}{d}, \quad y^* = \frac{y}{d}, \quad L^* = \frac{l}{d}, \quad t^* = \frac{at\sqrt{Ra}}{d^2}
\]

\[
f^* = \frac{f d^2}{\alpha \sqrt{Ra}}, \quad u^* = \frac{ud}{\alpha \sqrt{Ra}}, \quad v^* = \frac{vd}{\alpha \sqrt{Ra}}
\]

\[
\hat{u}^* = \frac{\hat{u} d}{\alpha \sqrt{Ra}}, \quad p^* = \frac{pd^2}{\rho(\alpha \sqrt{Ra})^2}, \quad T^* = \frac{T - T_\infty}{T_h - T_\infty}
\]

\[
Gr = \frac{g B d^3 (T_h - T_\infty)}{\nu^2}, \quad Pr = \frac{\nu}{\alpha}
\]

B. \( \nu \), \( \alpha \), \( Ra \), \( t \), \( \rho \), \( p \), \( u \), \( v \) and \( \hat{u} \) are volumetric thermal expansion coefficient, kinematic viscosity, thermal diffusivity, Rayleigh number, time, density, pressure, x direction velocity, y direction velocity and mesh velocity in x direction, respectively.

None-dimensional governing equations are described by the following expressions. The stars have been omitted for simplicity.

Rayleigh number:

\[
Ra = Gr \cdot Pr = \frac{g B d^3 (T_h - T_\infty)}{\nu \alpha}
\] (1)

Continuity:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\] (2)

X-momentum:

\[
\frac{\partial u}{\partial t} + (u - \hat{u}) \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + Pr/\sqrt{Ra} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
\] (3)

Y-momentum:

\[
\frac{\partial v}{\partial t} + (u - \hat{u}) \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + Pr/\sqrt{Ra} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + Pr \cdot T
\] (4)

Energy Eq.:

\[
\frac{\partial T}{\partial t} + (u - \hat{u}) \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = 1/\sqrt{Ra} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\] (5)

The none-dimensional oscillation velocity \( U_c \) is calculated from the following equation:

\[
U_c = U_m \cos(2\pi ft)
\] (6)
The dimensionless maximum vibration velocity $U_m$ is assumed to be $U_m = 2\pi f l$ as [13]. The boundary conditions are as follows as mentioned in [13]:

a) At each cylinder surface
\[ u = U_m, \quad v = 0, \quad T = 1 \]  

b) At BC boundary
\[ \frac{\partial u}{\partial x} = 0, \quad v = 0 \]
If $u \leq 0$, $T = 0$
If $u > 0$, $\frac{\partial T}{\partial x} = 0$

\[ \frac{\partial u}{\partial x} = 0, \quad v = 0 \]
If $u \geq 0$, $T = 0$
If $u < 0$, $\frac{\partial T}{\partial x} = 0$

c) At DA boundary
\[ \frac{\partial u}{\partial x} = 0, \quad v = 0 \]
If $u \geq 0$, $T = 0$
If $u < 0$, $\frac{\partial T}{\partial x} = 0$

d) At CD boundary
\[ u = 0, \quad \frac{\partial v}{\partial y} = 0 \]
If $v \leq 0$, $T = 0$
If $v > 0$, $\frac{\partial T}{\partial y} = 0$

e) At AB boundary
\[ u = 0, \quad \frac{\partial v}{\partial y} = 0 \]
If $v \geq 0$, $T = 0$
If $v < 0$, $\frac{\partial T}{\partial y} = 0$

3 Numerical method
The governing equations are solved by CBS finite element method. The CBS algorithm for the solution of the Navier–Stokes and energy equation equations can be summarized by the following steps [14]:

1. Solution of the momentum equation without the pressure term
2. Calculation of the pressure using the Poisson equation.
3. Correction of velocities.
4. Calculation of energy equation or any other scalar equation.

For better understanding of the CBS method see the book of Lewis et. al. [8]

4 Results and discussion
In the present modeling, fluid is air and the Prandtl number is $Pr=0.71$. The effects of important parameters in natural convection included Rayleigh number, distance of two cylinders by cylinder diameter ratio and oscillation frequency have been investigated. We have used the unstructured grid with about 600,000 nodes for all the runs. This amount has been determined by the grid independency for the averaged Nusselt number. Similarly, we have used $\Delta t = 0.01$ as the time step. First of all, the averaged Nusselt number for a single cylinder in a cavity according to Fig. 2 is compared to references [15] and [16], for the code validation. This verification has been presented in table 1. According to this table, the maximum error of present work in comparison to other researchers is about 3.2%.
Table 1: Comparison of the present averaged Nusselt number with other. References for an isothermal cylinder inside a square enclosure

<table>
<thead>
<tr>
<th>$Ra$</th>
<th>Present Work</th>
<th>[15]</th>
<th>[16]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^4$</td>
<td>3.302</td>
<td>3.414</td>
<td>3.331</td>
</tr>
<tr>
<td>$10^5$</td>
<td>5.295</td>
<td>5.138</td>
<td>5.080</td>
</tr>
</tbody>
</table>

In the first step, to verification our results, we calculate the average Nusselt numbers for a single cylinder in a cavity (it has been shown in Fig.2) in different Rayleigh numbers and compare them with the benchmarks values given in [15, 16]. The comparisons are listed in the Table1. As is shown in the Table 1, a good consistency of the present results to [15, 16] can be seen. The differences are due to the computational domain and a small grid size was taken in present work.

The temperature and velocity contours around the cylinders at different times of a cycle have been shown in Fig. 3 and 4. These contours have been presented at $Ra = 10^4$, s/d$=4$, f=0.1Hz and L=1. The conditions of 1 to 4 are related to different time of a cycle which at conditions 1 and 3 the average Nusselt number are maximum in a cycle. The points of 1 and 3 are related to maximum amount of velocity in positive and negative direction, respectively. Due to these contours you can see the maximum temperature around the top cylinder is more than bottom cylinder. It’s related to buoyancy effect due to temperature difference between the cylinders and surrounding. The hot air move up and accelerate and consequently the velocity increase far from the top cylinder. The region of maximum velocity is moved in effect of cylinder oscillations and can be seen in Fig. 4.

Fig. 5 illustrates the Nusselt number versus the variation of Rayleigh number for three cylinders with sinusoidal vibration in horizontal direction. The conditions for vibration are s/d$=4$, f=0.1 and L=1. As can be observed from the figure, by increasing the Rayleigh up to $Ra = 10^5$, the Nusselt number increases, after which it seems the trend of increment is decreased. The Nusselt numbers for three cylinders have small difference to each other, but the top cylinder has lowest Nusselt number in comparison to the other cylinders.
Fig. 3: Temperature contours in Kelvin around the cylinders under horizontal vibrations in a cycle (1 is related to maximum velocity in x direction, 2 is minimum velocity at the end of positive motion, 3 is maximum velocity in the opposite direction of x, and 4 is minimum velocity at the end of negative motion)

The time averaged Nusselt number evolution with the frequency of vibration at three cylinders are given in Fig. 6 for s/d=4 , Rₚ = 10⁴ and L=1. It should be noted that the variations of the Nusselt number have small dependency to frequency of vibrations. As can be seen in Fig. 6, the variation range of the Nusselt number in this Rayleigh number is negligible and increment of frequency of oscillations does not always lead to increment of average Nusselt number.

In order to better understand how much increment in Nusselt number can be produced by displacement of the cylinders locations, the effect of variations of S/d on the time averaged Nusselt number are shown in Fig. 7. In this figure, Nusselt numbers are compared for three cylinders at f=0.1Hz , Rₚ = 10⁴ and L=1. It can be seen that Nusselt number is approximately a linear function of the S/d in the considered range. The increment of S/d has little effect on the bottom cylinder and Nusselt number increase softly. But increment of S/d lead to better and more increment of Nusselt number at the other cylinders. Increment of S/d makes more space for heat exchange and consequently leads to considerable increase in Nusselt number.
Fig. 4: Velocity contours in m/s around the cylinders under horizontal vibrations in a cycle (1 is related to maximum velocity in x direction, 2 is minimum velocity at the end of positive motion, 3 is maximum velocity in the opposite direction of x, and 4 is minimum velocity at the end of negative motion)

The time averaged Nusselt number variations versus the change in amplitudes of vibrations for all the cylinders at $R_a = 10^4$, $f=0.3Hz$ and $s/d=4$ is illustrated in Fig. 8. It should be noted that Nusselt number is approximately a linear function of the amplitudes of vibrations in the considered range of values. The enhancement of the amplitudes of vibrations at a fixed frequency leads to augmentation of oscillation velocity. Consequently, natural convection is increased. The increment of velocity of the cylinders is equal to relative Rayleigh number enhancement assuming the cylinders don’t have any motion, so the Nusselt number is increased.

Fig. 9 shows time series of Nusselt number for three cylinders at different $s/d$ ratios. These plots have been presented at $R_a = 10^4$, $f=0.3Hz$ and $s/d=4$ conditions. The increment of distance between the cylinders lead to significant enhancement of the Nusselt number at mid and top cylinders, whereas doesn’t have considerable influence on bottom cylinder. As regards the flow direction is to the top, the interactions and influences of mid and top cylinders over the bottom cylinder is negligible. Therefore the increment of space between the cylinders has little influence on heat transfer of the cylinders.
Fig. 8: The effect of the variation of amplitudes of vibrations on the time averaged Nusselt number for three oscillating cylinders at $R_a = 10^4$, $f=0.3\text{Hz}$ and $s/d=4$

- a) Top cylinder
- b) Mid cylinder
- c) Bottom cylinder

Fig. 9: The variation of the Nusselt number during time for different $s/d$ ratios

5 Conclusion

Natural convection heat transfer from an array of circular cylinders in presence of horizontal vibrations has been studied numerically in this paper. The presented results concerning the averaged Nusselt number show the oscillations in horizontal direction usually increase the averaged Nusselt number. The heat transfer rate is strongly increased by increment of Rayleigh number. The increment of S/d makes the increment of the averaged Nusselt number only at middle and top cylinders and doesn’t have a substantial change in the bottom cylinder. Increment of frequency of oscillations does not always lead to increment of average Nusselt number and a considerable variation cannot be seen in the evaluated condition.

References:


