A Conformal Mapping based Shape Signature for Object Recognition

M. Radhika Mani1          Dr. G.P. Saradhi Varma2          Dr. Potukuchi D.M.3          Dr. Ch. Satyanarayana4
Dept. of CSE                  Dept. of IT                           Dept. of Physics            Dept. of CSE
Pragati Engg. College     SRKR College of Engg.      JNTUKakinada             JNTUKakinada
Surampalem, A.P.           Bhimavaram, A.P.               Kakinada, A.P.             Kakinada A.P.
INDIA                             INDIA                                INDIA                          INDIA

radhika_madireddy@yahoo.com    gpsvarm@yahoo.com          potukuchidm@yahoo.com          chsatyanarayana@yahoo.com

Abstract: - A novel method for designing a signature based on the conformal mapping is proposed. Conformal mapping function preserves the local angles of the shape of the object. Schwarz Christoffel mapping transformation is effectively implemented. A conformal transformation of the upper half plane onto the interior of a simple polygon is carried out. The widely used in 2-D potential flow for the computation of the complex potential of the flow by a line source located in a two dimensional channel is familiarized. A novel Complex Potential Source (CPS) descriptor for the shape based object recognition is designed. CPS is mediated through the Fourier transformation during the description stage, while the feature vector is generated. The CPS descriptor uses Chebyshev distance measure for the shape toning stage. It is experimented over the versatile MPEG7 CE-1 Set B database. The results witness the efficiency of the proposed method than that reported for other descriptors of contemporary interests.

Key-Words: - Schwarz Christoffel transformation, Fourier transformation, Distance, Feature Vector, Shape Toning and Performance Measures.

1 Introduction

Shape based methods are widely used in the computerized image processing and pattern recognition process. Subsequently, reports of relative analysis for efficient shape based object recognition method [1] resumed importance. Shape based approaches involve different stages of processing viz., shape representation, shape description and shape toning. The shape representation stage identifies the number of effective representative points of an object. Two categories of shape representation methods viz., boundary based and region based methods [3] are reported in literature. Boundary based methods require less number of representative points than the compared with region based methods [2]. Moment Invariants [4], Zernike Moments [5, 6], Krawtchouk moments [9], Chebyshev moments [7, 8] constitute some of representative types of popular moments that are used for region based descriptors. However, the region based shape representation methods usually involve competing procedures like the medial axis transform [10,11,12], grid method [13], generic Fourier transform [14], convex hull [14], shock graph [17] and shape matrix [16] etc. To preserve the local angles, conformal mapping is prominently used in the real time applications [17, 18]. Conformal mapping techniques can be divided into two categories.

i) Methods which perform well on regions with smooth boundaries, but either fail or require elaborate modifications during their mapping of regions with corners;

ii) Schwarz-Christoffel transformation, which is specifically designed for mapping polygons with any number of corners, but which does not, at least in its original form, extend to mapping domains with curved sides.

Feizli and Mumford [19] represented the 2D shapes as the conformal fingerprint. This representation includes the Euclidean curvature encoding process. Any representation function described by the potential can be transformed by a conformal map. Elwin Bruno Christoffel and Hermann Amandus Schwarz independently discovered formulae for conformal maps extended over arbitrary polygons, which can be used in numerical computations to generate explicit Riemann maps. Schwarz Christoffel mapping [20] is a conformal transformation of the upper half plane on-to-the interior of a simple polygon. The popular application of the current transformation is the potential theory. Thus, it can be used to measure the complex potential flow with in a two dimensional channel [21] generated by the line source. Thus the measure of complex potential evolved by basing on
Schwarz Christoffel transformation provides crucial information about the shape representation of the object. As such, Christoffel transformation is argued to design the signature for shape based object recognition. The representation of shape is carried out with the help of a signature that depicts the object. For this type of description, Fourier transformation [22,23] is considered as an efficient method to generate the feature vector.

The paper is organized in 3-sections. Introduction to conformal map based shape description methods are reported in Section-I. The details of methodology implemented presently and the relevance are detailed in section-II. Results of implementing the novel algorithm designed by the proposed approach over the standard database and their analysis with relevant discussion are presented in Section-III.

2 Methodology

An innovative approach for shape based object recognition based on conformal map is proposed. The details of various steps of proposed method are discussed in section II-A, where as the performance indicators are presented in section II-B. Owing to the versatile and vivid nature of the objects entailing its variety of shapes, the Set B database is presently considered.

2.1 Design of the System

The proposed method consists of four steps given as follows

i. Contour based shape representation

ii. Complex Potential based Shape Signature based representation

iii. Fourier transformation based description

iv. Shape toning and ranking respectively.

Contour based representation is found to be efficient [24] than the region based representation. Hence, during the first step, the input object is represented through its contour. In order to optimize the representation points, evaluation of the Equivalent Arc Length [25] is used during its sampling process.

During the second step, a conformal map is used to represent the shape. For this, the Schwarz Christoffel mapping transformation based complex potential is computed. In this transformation, the channel is considered with the source [21] located in the mid of channel walls. If the source location is taken as the plane origin, then the corresponding potential flow field will be symmetrical about both x-axis and y-axis, while the first quadrant of the complex and orthogonal based field is sufficient to represent the field.

Fig.1 represents the first quadrant of the flow field in the z-plane, in which the source is located at z=0. In Figure-1, the region denoted as 0≤x, 0≤y≤l represents boundary of the polygon that is used for mapping. The shape points say A and B are chosen as to correspond to points -1 and 1. The interior angles viz., α and β corresponding to the shape points that depicted by its boundary in the z-plane are take the value as π/2.

The complex potential in the z plane is given by

\[ F(z) = \frac{m}{\pi} \log \left( \frac{\pi z}{2l} \right) \]

where m represents the source strength, z represents the shape representative points given by (x+iy) in the complex plane and l represents the width of the channel.

The Imaginary values of the F(Z) is a harmonic function and hence it can be used for designing the Complex Potential Source (CPS) signature. In the third step, the 1-D Fourier transform [22] is applied for the description. Further, the extracted features are testified as invariant to translation, rotation and scaling operations performed over the set of images. In the wake of the fact, that the CPS signature is obtained wrt the centroid, the obtained features proved to be invariant to the translation operation. Further, the resulting finite and stipulated magnitude values of the features would be validated for the rotation invariance. The scaling invariance is achieved by dividing the feature vector with the first feature value. In the third step, the feature vector is constructed, which describes the entire shape aspect of the object. To further improve the quality of CPS signature, three global descriptors (GD) are augmented to the CPS signature by its feature vector. The GD feature vector, viz., {S,C,A} contains the measures of solidity, circularity and aspect ratio.

In the fourth step, the shape toning process is executed. The distance with Chebyshev Distance (CD) is estimated to test and expose to the process of shape training (i.e. during the identical shape toning process).

\[ CD(TE, TR) = |TE, -TR|_{max} \]
where, TE represents the test shape feature vector, TR represents the trained shape feature vector and \( i \) represents the length of the feature vector.

Inclusion of GD is effectively carried out through the computation of relative distance measure given by:

\[
D(TE, TR) = CD(TE, TR) + \frac{D_3(TE, TR)}{3}
\]

\[
D_3(TE, TR) = \sum_{X} \frac{|X^{TE} - X^{TR}|}{\max(X^{TR})}
\]

where, \( ED(TE, TR) \) represents the ED between the test and trained shapes, \( D_X(TE, TR) \) represents the Global distance between the test and trained shapes, \( X \) represents the GD vector \( \{S, C, AR\} \), \( X^{TE} \) represents the GD feature of the test shape and \( X^{TR} \) represents the GD feature of the trained shape.

The specified data of distance measurement i.e. the so computed distances are further rearranged in ascending order and are assigned with specific ranks. In turn, the system is enabled to recognize top ranked images.

2.2 Performance Indicators

The performance of the proposed method is estimated by the computation of the confusion matrix, which contains [26] four elements viz., (i) False Negatives (FN) (ii) False Positives (FP) (iii) True Positives (TP) (iv) True Negatives (TN). From the confusion matrix, the performance measures viz., True Positive Recognition (TPR), Specificity (SPC), Positive Predicted Value (PPV), False Discovery Rate (FDR) and Accuracy (ACC) are computed by

\[
TPR = \frac{TP}{TP + FN}
\]

\[
SPC = \frac{TN}{FP + TN}
\]

\[
PPV = \frac{TP}{TP + FP}
\]

\[
FDR = \frac{FP}{FP + TP}
\]

\[
ACC = \frac{TP + TN}{TP + TN + FN + FP}
\]

3 Results and Discussions

Conformal mapping based shape representation and its description (detailed in section-II-A) is estimated for the input of MPEG-CE-Set B database. The results of proposed method and various other standard methods are presented in the following subsection-III-A. The proposed CPS+GD descriptor is validated with the relative performance measures and are given in the sub section III-B in the wake of the other reported descriptors.

3.1 Processing of CPS+GD descriptor

The proposed CPS signature is constructed with source width fixed to 1, flow velocity fixed to 1. With these inputs, the complex potential given by Eqn-1 is computed. The imaginary values of the complex potential act as the CPS signature. The proposed signature is constructed for various objects of Set B database and the CPS of three Teddy images (Teddy2, Teddy3 and Teddy11) are illustrated in Fig. 2. It is observed that the CPS of different objects of the same group are found to be similar.

Further, Chebyshev Distance (CD) measure is estimated between the target and test objects, while they are allocated with a ranking in accord to their distance. The top 20-ranked objects are only considered for computing the performance. Fig. 3 illustrates the recognition results of Carriage6 object from Set B database. The Fig. 3(i) represents the input object, 3(ii) represents the ZMD recognition result and 3(iii) represents the CPS+GD recognition result. From this, it is observed that the proposed CPS+GD descriptor reduces the dissimilar recognition result than the ZMD.

3.2 Validation of CPS+GD descriptor

To validate the proposed CPS+GD descriptor for shape based object recognition, the present paper
computes the confusion matrix. The confusion matrix contains FN, FP, TP, TN measures. The Table 1 shows the confusion matrix of the proposed CPS+GD descriptor. The present paper compares the confusion matrix of the proposed and four other standard descriptors viz., Zernike Moment Descriptor (ZMD), Angular Radial Transform Descriptor (ARTD), Moment Invariant Descriptor (MID) and Curvature Scale Space Descriptor (CSSD). The Table 1 gives the comparison of confusion matrix of proposed and standard descriptors. From the Table 1, the five performance measures are computed and shown in Table 2. In the Table 2, the TPR, SPC, PPV, FDR and ACC performance measures are computed for the CPS+GD, ZMD, ARTD, MID and CSSD descriptors. From the Table 2, it is found that the proposed CPS+GD yields improved performance and is validated.

Table 1 Confusion Matrix of Various descriptors

<table>
<thead>
<tr>
<th>Descriptor</th>
<th>FN</th>
<th>FP</th>
<th>TP</th>
<th>TN</th>
</tr>
</thead>
<tbody>
<tr>
<td>CSSD</td>
<td>0.309</td>
<td>0.205</td>
<td>0.691</td>
<td>0.814</td>
</tr>
<tr>
<td>MID</td>
<td>0.256</td>
<td>0.184</td>
<td>0.744</td>
<td>0.888</td>
</tr>
<tr>
<td>ARTD</td>
<td>0.252</td>
<td>0.175</td>
<td>0.748</td>
<td>0.891</td>
</tr>
<tr>
<td>ZMD</td>
<td>0.248</td>
<td>0.153</td>
<td>0.752</td>
<td>0.895</td>
</tr>
<tr>
<td>CPS+GD</td>
<td>0.205</td>
<td>0.135</td>
<td>0.795</td>
<td>0.894</td>
</tr>
</tbody>
</table>

Table 2 Performance Measure of various descriptors.

<table>
<thead>
<tr>
<th>Descriptor</th>
<th>PPV</th>
<th>TPR</th>
<th>SPC</th>
<th>FDR</th>
<th>ACC</th>
</tr>
</thead>
<tbody>
<tr>
<td>CSSD</td>
<td>0.771</td>
<td>0.691</td>
<td>0.799</td>
<td>0.229</td>
<td>0.745</td>
</tr>
<tr>
<td>MID</td>
<td>0.802</td>
<td>0.744</td>
<td>0.828</td>
<td>0.198</td>
<td>0.788</td>
</tr>
<tr>
<td>ARTD</td>
<td>0.810</td>
<td>0.748</td>
<td>0.836</td>
<td>0.190</td>
<td>0.793</td>
</tr>
<tr>
<td>ZMD</td>
<td>0.831</td>
<td>0.752</td>
<td>0.854</td>
<td>0.169</td>
<td>0.804</td>
</tr>
<tr>
<td>CPS+GD</td>
<td>0.855</td>
<td>0.795</td>
<td>0.869</td>
<td>0.145</td>
<td>0.832</td>
</tr>
</tbody>
</table>

The PR plot [22] of all the five descriptors is shown in Fig. 4. The CPS+GD descriptor clearly witnesses an improved precision prevalently at higher recalls from 60 to 90 than the other descriptors. At low recalls from (25 to 40) also, the proposed CPS+GD descriptor comes out with minor improvement of precision than that with the other descriptors.

4 Conclusion

- Conformal mapping based shape representation and its description represents a relative efficient method than the other standard descriptors.
- The complex plane based representation of the object would become efficient by the Schwarz Christoffel transformation.
- The involvement of efficiency of the complex potential flow in the complex plane would be efficient, if its performance is measured. Basing on the various elements of the confusion matrix measures, viz., through TPR, SPC, PPV, FDR and ACC.

References:
Conference on Signal and Image Processing, 2004, 80-84.


