An Improved Fuzzy Fractal Dimension for Texture Analysis

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Abstract: In the real life, a lot of phenomena cannot be described by the traditional geometry; fractals are one of these objects. It serves to give a suitable description for these objects. Texture analysis plays an important role in image processing. Fractal dimension is utilized in texture segmentation and classification and prove to be an interesting parameter to characterize image roughness and extract image feature. In some complex and irregular scenes, it becomes not effective for feature extracting and classification. Therefore, a more general approach known as fuzzy fractal dimension can be used to model such types of scenes effectively. A new fuzzy fractal dimension method is proposed in this paper, it is verified by the experiment on a set of nature texture images to show its efficiency and accuracy, a satisfactory result is found. It also offers promising performance when it is tested among some types of noises to show a good robustness to them.

Key–Words: Fractal dimension, Fuzzy fractal dimension, Texture, Box counting method

1 Introduction

This work is motivated due to the great demands for new techniques to handle the complex and irregular scenes. Many areas of image description, recognition, and segmentation are based on fractal dimension that play a key role in these problem. Texture analysis is an important issue in many applications. It composes of sub-patterns that have some characteristic such as; slope, color, size, brightness, etc., which give rise to recognize the roughness, regularity, randomness, smoothness etc. It is believed to be a prosperous source of visual information.

The Hausdorff- Besicovitch measure in (1) is one of the earliest definitions of fractal dimension which is considered as a basic for the definition of the fractal set [1].

\[ H^s(E) = \lim_{\delta \to 0} \inf \sum_U |U_i|^{s} \]  

Since this type of dimension is computationally hard, a new approach known as Box counting dimension to calculate the fractal dimension is emerged as the best estimator for self-similar images. It takes its popularity due to the ease of its numerical computation. In 1982, Mandelbrot [2, 3] was the first who describes an approach to find the fractal dimension while he tried to find the length of the coastlines. After that, many approaches had been proposed to estimate the FD. Since most real textures are in fact semi fractals; it cannot be precisely characterized by fractal dimension. Many researchers have argued to extend Mandelbrot’s method to 2D, 3D and nD based on different approaches; such as, fractal Brownian motion (fBm), wavelet transform, contourlet transform. S. Peleg [4] extended Mandelbrot’s method the 2D images where the image can be represented as a hilly terrain surface; the height from the ground represents the gray value of the image. Therefore, all points which have a distance \(2\varepsilon\) from both sides of the surface formed a blanket divided by \(2\varepsilon\). For a different \(\varepsilon\) the area of the blanket is estimated in (2) and the area of the fractal surface behaves according to the following expression:

\[ A(\varepsilon) = F \varepsilon^{2-D} \]  

where \(D\) is the fractal dimension. The estimation of fractal dimension from power spectral density of fBm is introduced by Pentland [5], who considered the image intensity as fBm. J.Gangepain and R.Carmes [6], proposed a method named the reticular cell counting approach using the Box counting method. In their method, they partitioned the 3D space into boxes of size \(L \times L \times L\) for a given scale \(L\) and by calculating the number of the sub cubes \(N_L\) in (3) that at least one sample from the intensity surface, \(D\) represent the fractal dimension.

\[ N_L = \frac{1}{r^D} = \left[ \frac{L_{\max}}{L} \right]^D \]
In 1989, Keller et al. [7] used the Box counting dimension to distinguish between different texture. They showed that, it is not enough to be used for texture analysis or to classify natural texture. N. Sarkar and B. B. Chaudhuri [8] designed one of the popular methods in box counting theorem it is named differential box counting algorithm (DBC) used to estimate the dimension for gray level images (3D). The appearance of wavelet transform have revolutionized image processing field. Flandrin [9] uses multiscale property with self-similarity of fBm to estimate the fractal dimension, then Liu et al. [10], estimated fBm of noisy images by using variance of wavelet coefficients. Do [11] introduced contourlet transform to estimate fractal dimension, he showed that it give better representation of edges in an image. Yazdi and Mahyari [12] proposed a method based on box covering method may cause loss of information for different texture that provides information for different texture. Their approach serves as an important fuzzy fractal dimension based on escape time dimension model of complex networks with fuzzy set. Zhang et. al. [21] proposed a fuzzy fractal dimension is performed better than fractal dimension for complex image objects based on their textures. It has good effects in the characterization of a particular texture [13, 14]. Huntsberger [15] generalized the classical fractal dimension to what you know as the fuzzy fractal dimension by incorporating the fuzzy sets instead of pixel covering method, after Huntsberger many fuzzy fractal dimensions have been proposed based on box counting method [16, 17, 18]. O. Y. Bas and A. M. Erkmen [19] proposed new method to find the FFD by using the length and the width of the entry image. Their method yields more precious dimension than the traditional method, and also takes less time. O. Castillo and P. Melin [20] describe a new method for fractal dimension estimation of a geometrical object using fuzzy logic techniques. W. Pedrycz and A. Bargiela [16] proposed an algorithmic framework necessary to carry out all computing of the FFD. Their experimental results show that this approach produces more consistent models for the fuzzy set. Zhang et. al.[21] proposed a fuzzy fractal dimension model of complex networks with fuzzy set. They showed that their approach is efficient and less time consuming. Alsaidi et. al. in [22] proposed new local fuzzy fractal dimension based on escape time dimension in [0,1], their approach serves as an important characteristic that provides information for different gray scale images.

However, the FFD proposed by T.L.Huntsberger and others has a problem in robustness against noise; in this work we developed a method to solve this problem, the reminder of this work is structured as follows. Section 2 summarized the basic definition of the FFD. Section 3 presents the proposed method with its algorithm; Section 4 discusses the experiments results. Finally, Section 5 exposes the conclusions.

2 Basic Definition of The FFD Approach

The main idea of the FFD is that, for any image of 256 gray scale levels, each pixel \( p \) is calculated as \( p/256 \) and produces the fuzzy set. The calculation method of the FFD for any image is as follows [23, 24, 25]:

1. For a discrete scale \( d \), divide the region of the image into small \( n \)-dimension boxes of width \( d \).

2. For any \( n \)-dimension box \( C \), the characteristic value is defined as follows

\[
F(C) = \forall d_j \in C \ f(d_j)
\]

where \( f(d_j) \) is the membership function for \( d_j \), the membership function value of a pixel is defined in dividing pixels gray-scale value by 256. The operator \( \forall \) is defined as follows:

\[
a \forall b = max(a, b) \text{ where } a, b \in [0, 1]
\]

3. For the discrete scale \( d \), the characteristic value of the image on scale \( d \) is defined as:

\[
N_d(C) = \sum_{i=1}^{N} F(C_i^d)
\]

where \( N \) is the number of boxes that cover the image surface, therefore the FFD is defined as:

\[
FFD = \log N_d(C)
\]

3 The Proposed Method

In this section a new method to estimate fuzzy fractal dimension is proposed and the experimental is performed on the matrices of some fractal images. The method is explained as in the following:

Let \( L \) be the matrix of size \( M \times N \) of fuzzy numbers as follows:

\[
L = \begin{pmatrix}
0_{1,1} & 0_{1,2} & \cdots & 0_{1,N-1} & 0_{1,N} \\
0_{2,1} & 0_{2,2} & \cdots & 0_{2,N-1} & 0_{2,N} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0_{M-1,1} & 0_{M-1,2} & \cdots & 0_{M-1,N-1} & 0_{M-1,N} \\
0_{M,1} & 0_{M,2} & \cdots & 0_{M,N-1} & 0_{M,N}
\end{pmatrix}
\]
By dividing the columns of the matrix into small columns consist of 4 rows, when the number of column is not possible to divide by 4 then the last package represents the remaining rows and as follows:

\[
\begin{align*}
c_1 &= \{a_{1,1}, a_{2,1}, a_{3,1}, a_{4,1}\} \\
c_2 &= \{a_{1,2}, a_{2,2}, a_{3,2}, a_{4,2}\} \\
&\vdots \\
c_d &= \{a_{1,N}, a_{2,N}, a_{3,N}, a_{4,N}\} \\
&\vdots \\
c_{d+1} &= \{a_{5,1}, \cdots, a_{M,1}\} \\
&\vdots \\
c_D &= \{a_{5,N}, \cdots, a_{M,N}\}
\end{align*}
\]

Then the fuzzy fractal dimension can be calculated by the following formula and Algorithm1:

\[
FFD = \log \left( \max(c_1) + \max(c_2) + \cdots + \max(c_D) \right)
\]

**Algorithm 1:**

Read image %input the image
\[F = M \mod 4; S = 0; Mt = M - F;\]
For \(I = 1:N\) % split the image to columns of 4 rows
For \(J = 1:Mt:4\)
For \(K = 1:4\)
\[B(K) = A(I, K + J - 1);\] % read the columns elements
End K
\[S = S + \max(B);\] % take the maximum value in each column
End J
\[B = 0;\]
For \(K = Mt + 1:Mt + F\) % move to another column
\[B(K) = A(I, K);\]
End K
\[S = S + \max(B);\]
End I
FFD = \log S % find the dimension

4 Experimental results

In this section, the proposed method is compared with Huntsberger method [15] by introducing the Gaussian noise. Six natural texture images with the same size and resolution as shown in Figure 1 have been tested.

<table>
<thead>
<tr>
<th>Images</th>
<th>Before noise</th>
<th>After adding Gaussian noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>FF D by T.Huntsberger</td>
<td>FF D by the proposed method</td>
<td>Execution error</td>
</tr>
<tr>
<td>F1</td>
<td>0.54562</td>
<td>0.50032</td>
</tr>
<tr>
<td>F2</td>
<td>0.53077</td>
<td>0.55002</td>
</tr>
<tr>
<td>F3</td>
<td>0.54743</td>
<td>0.59035</td>
</tr>
<tr>
<td>F4</td>
<td>0.52711</td>
<td>0.56022</td>
</tr>
<tr>
<td>F5</td>
<td>0.52924</td>
<td>0.59119</td>
</tr>
<tr>
<td>F6</td>
<td>0.59906</td>
<td>0.59462</td>
</tr>
</tbody>
</table>

Figure 1: Set of six natural images
Figure 2 shows that the fitted slopes of the proposed method remain close to each other more than the fitted slopes of the Huntsberger method and as follows: The results are shown in Table 2. We conclude that FFD estimated by Huntsberger have approximate value to the proposed method, but the proposed method is more robust to noise than Huntsberger method. We can also notice from the error that the proposed method is performed better, especially, for F3 the accuracy of the FFD after adding the noise is around 99.862 in comparing with the same one before noise.

5 Conclusions

This paper has focused on an important mathematical feature called fuzzy fractal dimension and its application in texture analysis. Our main contribution is through proposing a new method that has some improvements over the classical methods. The classical FFD such as Huntsberger method there is a noise robustness problem, so using these methods for FFD estimation will resulted in an inaccurate value especially when some noise is adding to the image. A new method is proposed to handle this problem, the experiment is performed on a set of six images, and satisfactory results have been obtained, some useful features are pointed out. The experiment showed that the proposed method is more efficient and robust to noise than the Huntsberger method.

References:


