An Improvement of Coupling Coefficient for Weakly Coupled Multi Fiber Coupler

DEDI IRAWAN1,2, SAKTIOTO2, IWANTONO2, ERMAN TAER2, JUANDI2

1 Department of Industrial Engineering, FST, State Islamic University of Sultan Syarif Kasim Riau, Pekanbaru 28282 INDONESIA
2 Department of Physics, FMIPA, University of Riau, Pekanbaru 28282 INDONESIA
dedi.dawan@yahoo.com

Abstract: The directional fiber couplers consist of multi fibers joint have been successfully fabricated by heating and pulling the coupling region. Good performance of the coupling coefficient and the coupling ratio was obtained by setting the initial parameter of coupler machine such as the coupling coefficient, pulling speed, pulling length, and heating temperature. The Coupling coefficient of 3-dB fused directional fiber coupler was determined based on perturbation method. A good agreement between theoretical purposed and empirical coupling coefficient from experiment was reported. The coupling coefficient between the waveguides is significantly induced by the distance of separation between them and geometrical coupling region. It is exponentially decreases with increasing separation between fiber axis. However by controlling the coupling coefficient, the desired coupling ratio can be easily attained.

Keywords: Directional fiber coupler; Coupling coefficient; Bessel and Henkel function and Power Exchange.

1. Introduction

Directional fiber coupler is key element in optical communication [1]. Fusion and elongation method as scathed in figure 1, is used to fabricate two single mode fibers. The coupling region is heated until 1350 °C by injecting 1 Bar H2 gas. At the same time, both sides of twisted fibers are pulled by two motorized pulling stages. Heating and pulling process will be stopped automatically when the preset coupling ratio reached. The performance of fiber coupler such as other passive devices is examined in term of low loss, high power transmission and uniformity [2]. Since the required coupling ratio is slightly difficult to be maintained during fusion, coupling coefficient is one important parameter that can be control. It is much depends on separation of fiber axis, fiber’s refractive index and wavelength operation in optical networking. The coupling coefficient was determined from the solution of power mismatch between the fibers. It is given theoretically as follows [3].

\[
\kappa = \left( \frac{2\Delta}{\pi} \right)^{1/2} \frac{U^2}{a} \frac{K_0(wd/a)}{K_1(w)}
\]

(1)

In recent years, the empirical equation of the coupling coefficient is expressed by Khare (2002). It is estimated from the experimental measurement.

\[
\kappa = \frac{\pi \sqrt{\delta}}{2a} \left[ \frac{A+B+C\delta}{\delta} \right]^{1/2}
\]

(2)

where

\[
A = 5.2789 – (3.663V)^2 + (0.3841V^2)
\]

\[
B = -0.7769 – (1.2252V) + (0.0152V^2)
\]

\[
C = 0.0175 – (0.0064V) + (0.0009V^2)
\]

In this paper, the coupling coefficient of directional fiber coupler will be optimized by determining it mathematically using Basel function. Then it will be compare to the previous coupling ratio reported reference [3] and [4].

2. Fabrication of Fused Fiber Coupler

The fabrication of directional fiber coupler is carried out by placing three single mode fibers on the stages. Corning fiber (SMF-28e®), are connected to the laser source (1310 nm) and displayed to the photo detector. The two fibers are twisted and held by a vacuum system in both stages. All components are recorded and by a data acquisition card installed to the computer system. The initial step is to set parameters such as coupling ratio, maximum pulling length, x-y-z position of torch flame, and flowing of H2 gas. 1 mW laser launched to the one of input ports is detected by photo detector and kept for calibration. Fusion and pulling process are started. During torch flame heating the coupling region, the fibers are elongated by pulling stages with suitable pulling speed. Heating and pulling process will be
automatically stopped when the pre-set coupling ratio is reached.

Fig. 1. Experimental set up to fabricate fiber coupler

3. Geometrical Coupling Region of Fused Multi Directional Fiber Coupler

The transmission of electric field in fused directional fiber coupler is significantly affected by the geometrical properties of fusion region. This chapter will be analyzed the geometrical properties and its effect to the performance of fiber coupler. It is focused on the coupling region. The effect of fabrication process to the profile of diameter cross section and distance between fiber axis will be estimated. Then, the refractive index profile of coupling region also analyzed both modeling and experimental.

A directional fiber coupler has been successfully fabricated by heating the coupling region. At the same time, both sides of the coupling region are pulled together 800-4500 µm with pulling speed 100 µm/s. Of course, heating and pulling process stretch the coupling region and causes the geometrical changes. The significant geometrical change can be seen at coupling region cross section. After fusion, the profile of coupling region diameter is characterized to three parts as given in Fig. 2.

Since the diameter, $D$ of coupling region varies along the coupling region $z$, shown in Figure 2. The taper region can be determined as given.

$$ (V \cdot D) = \frac{D}{\Delta l} $$  \hspace{1cm} (3)

$$ \frac{dD}{dz} = \frac{D}{\Delta l} \Rightarrow \frac{dD}{D} = \frac{1}{\Delta l} dz $$  \hspace{1cm} (4)

$$ \int_{l_0}^{l_1} \frac{dD}{D} = \frac{1}{\Delta l} \int_{0}^{l_1} dz $$  \hspace{1cm} (5)

$$ \ln \frac{D}{D_0} = \frac{1}{\Delta l} l $$  \hspace{1cm} (6)

$$ D(l) = D_0 \exp \left(\frac{l}{\Delta l}\right) $$  \hspace{1cm} (7)

Now the diameter profile of coupling region of multi directional fiber coupler can be summarized as follow.

$$ D_{cl}(l) = D(l) + l + D(l_o) $$

$$ D_{t}(l) = [D_o \exp \left(\frac{l}{\Delta l}\right)] + l + [D_o \exp \left(\frac{l}{\Delta l}\right)] $$  \hspace{1cm} (8)

Fig. 3. Profile of coupling region diameter for Multi directional fiber coupler

4. Derivative of Coupling Coefficient Based on Bessel and Henkel Function

Since the fibers having identical propagation constants are electromagnetically isolated, they will transfer their power each other due to coupling coefficient between them. The coupling coefficient can be expressed as follow [4]

$$ \kappa = \frac{6\epsilon \omega}{4} \left(n_i^2 - n_o^2 \right) e_z e_z^* dS $$  \hspace{1cm} (9)

The electric fields in both fibers are varies exponentially, and it was summarized by [6,7].

Fig. 2 Geometrical coupling region, $l$ is the critical region, $l_0$ is left and right tapered region, and $\Delta l$ is the fusion zone.
where $C$ is constant coefficient that related to the normalization of electric field $C = \left(1/\sqrt{N} \right)$, and $J_n, J_l$ are the first kind of Bessel function. $K_0$ and $K_1$ are the second kind of modified Bessel function. 

And $U_i = u_i a_i = \kappa_i a_i \sqrt{n_i^2 - \beta_n^2}$ . Where $\beta_n = \beta_1 / \kappa_0$ and $\beta_0 = \beta_0 / \kappa_0$ are the expression denoted as propagation constant of $LP_0$ and $LP_1$, respectively. The wave number $k$ is inversely proportional with wavelength, $k_0 = 2\pi / \lambda$.

By substituting (10) into (9) yields.

$$\frac{1}{\sqrt{N_1 N_2}} \int_0^{2\pi} \int_0^{2\pi} \frac{J_n(u_i r_i)}{J_l(U_i)} J_n(w_i r_i) \cos \varphi_i dr_i d\varphi_i$$

To solve (11), coordinate $(r_i, \varphi_i)$ as field in fiber 1 has to be transformed in term of coordinate $(r, \varphi)$. It can be done by using transform method given by reference [6].

$$K_{m(n)}[\cos m\varphi_i \sin m\varphi_i] = \sum_{p=-\infty}^{\infty} (-1)^p K_{m+p}(ad) I_p(ar_i) \cos p\varphi_i \sin p\varphi_i$$

In this case, the calculation of coupling coefficient is done on the fundamental fiber couplers, so $m = 1$ and $a = (W_i / a_i)$. By substituting in to (11), it can be written as follow.

$$\sum_{p=-\infty}^{\infty} (-1)^p \frac{K_{m+p}(w_d)}{J_n(U_i) J_l(W_i)} I_p(u_i r_i) I_p(w_i r_i) \int_0^{2\pi} \cos p\varphi_i d\varphi_i$$

From part 3 of (13) it can be seen that,

$$\sum_{p=-\infty}^{\infty} \int_0^{2\pi} \cos p\varphi_i d\varphi_i = 0 \quad \text{if} \quad p = \pm 1, \pm 2, \ldots, \pm \infty$$

To solve part 2 of (13) which has different kind of Bessel Function, can be done by using Bessel relation as follow [6],

$$\int_0^z I_n(az) I_n(bz) dz = \frac{z}{a^2 + b^2} \left[ b I_0(az) I_1(bz) + a J_1(az) J_0(bz) \right]$$

Now, it yields.

$$\int_0^z J_n(u_i r_i) J_n(w_i r_i) r_i dr_i = \frac{a_i}{U_i^2 + W_i^2} \left[ W_n(U_i) I_n(w_i a_i / a_i) + U_n(J_i) J_n(w_i a_i / a_i) \right]$$

Then, by substituting Equation (14) in to Equation (13), and using boundary given by (14), the coupling coefficient between two fibers can be written as following Equation.

$$\kappa_{12} = \left[ \frac{2(n_i^2 - n_j^2)}{n_i n_j} \right]^{1/2} \frac{K_n(w_d)}{K_0(W_j) K_0(W_i) K_0(W_1)}$$

$$U_i U_j \left[ \frac{1}{a_i K_1 W_i^2} \left[ W_i K_n(U_i) I_n(w_i a_i / a_i) + W_j K_n(U_j) I_n(w_j a_j / a_j) \right] \right]$$

![Fig 4. Coupling coefficient versus separation between fiber axis](image-url)

It can be seen in Fig. 4 that the purposed model of coupling coefficient is more accurate that previous one again empirical. When the fiber separation is longer, the purposed coupling coefficient more decreases following the experimental one compared to the previous coupling model.

Fig. 3 shows the profile of coupling coefficient for various percentage of refractive index difference ($\Delta n$). It exhibits that three types of coupling coefficient having difference $\Delta n$ are exponentially decreased as function of separation distance. The higher number of $\Delta n$ yields the lower coupling coefficient between fibers. This means that higher difference between refractive index of fibers’ core and cladding contributes lower power to transfer electric field through the junction among the fibers. Since the profile of refractive index of core and cladding slightly change after fusion, the relative difference of refractive indices is also change, and that is not as a single mode fiber any more.
The distance of separation between fiber’s axis significantly changes due to the increment of fusion degree. This leads to relate the effect of degree of fusion to the coupling coefficient which it can be mathematically written as follows,

\[ \kappa = \kappa \left[ d(f) \right] \] (18)

where \( d(f) = D_0 \left[ 1 - f(2 - \sqrt{2}) \right] \) is the separation distance between fiber axis, and the degree of fusion, \( f \) varies from 0 to 1.

Since the coupling coefficient is significantly dependent on the separation between fibers, of course, it is also as function of degree of fusion. Fig. 6 shows that the coupling coefficient is exponentially increased with the increment of the degree of fusion. This exhibits that the coupling coefficient can be easily obtained since the degree of coupled fiber can be controlled during fusion.

5. Results and Discussion: Power Exchange Between Fibers

If the fiber coupler consists of more than two fibers such as \( M \) input and \( N \) output ports, it can be mathematically expressed by a matrix of \( MXN \) as the following equation.

\[
\begin{bmatrix}
A_1(z) \\
A_2(z) \\
A_3(z) \\
\vdots \\
A_N(z)
\end{bmatrix}
= -j
\begin{bmatrix}
\kappa_{12} & \kappa_{13} & \cdots & \kappa_{1N} \\
\kappa_{21} & \kappa_{23} & \cdots & \kappa_{2N} \\
\kappa_{31} & \kappa_{32} & \cdots & \kappa_{3N} \\
\vdots & \vdots & \ddots & \vdots \\
\kappa_{N1} & \kappa_{N2} & \cdots & \kappa_{NN}
\end{bmatrix}
\begin{bmatrix}
A_1(0) \\
A_2(0) \\
A_3(0) \\
\vdots \\
A_N(0)
\end{bmatrix}
\]

If the amplitudes of input and output powers denoted by \( A_N(0) \) and \( A_{N}(z) \) respectively, and by assuming the interaction of fields only occurs between the nearest fibers only, Equation (19) can be written in term of differential equations as follow.

\[
\frac{dA_N(z)}{dz} = -j\beta_N A_N(z) - j\kappa_{N(N-1)} A_N(z - 1) (20a)
\]

\[
\frac{dA_{N}(z)}{dz} = -j\beta_{N} A_{N}(z) - j\kappa_{N(N-1)} A_{N}(z) (20b)
\]

It can be simplified as the following equation.

\[
\begin{bmatrix}
A_1(z) \\
A_2(z) \\
A_3(z) \\
\vdots \\
A_N(z)
\end{bmatrix}
= -j
\begin{bmatrix}
\kappa_{12} & 0 & \cdots & 0 \\
0 & \beta_3 & \kappa_{34} & \cdots & \kappa_{3N} \\
0 & 0 & \kappa_{45} & \cdots & \kappa_{4N} \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & 0 & \kappa_{N(N-1)}
\end{bmatrix}
\begin{bmatrix}
A_1(0) \\
A_2(0) \\
A_3(0) \\
\vdots \\
A_N(0)
\end{bmatrix}
\]

A solution of the inner matrix of Equation (21) is determined by the eigenvalue and the eigenvector that describes transformation of light intensity between the fibers as follows.

\[
\begin{bmatrix}
A_1(z) \\
A_2(z) \\
A_3(z) \\
\vdots \\
A_N(z)
\end{bmatrix}
= -j
\begin{bmatrix}
\kappa_{12} & 0 & \cdots & 0 \\
0 & \beta_3 & \kappa_{34} & \cdots & \kappa_{3N} \\
0 & 0 & \kappa_{45} & \cdots & \kappa_{4N} \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & 0 & \kappa_{N(N-1)}
\end{bmatrix}
\begin{bmatrix}
A_1(0) \\
A_2(0) \\
A_3(0) \\
\vdots \\
A_N(0)
\end{bmatrix}
\]
where \( M_{pq} = \sum_{j=1}^{n} \sin\left(\frac{pm\pi}{n+1}\right) \sin\left(\frac{qn\pi}{n+1}\right) e^{j\lambda z} \) is the transfer matrix. In this case, the power transfers among the fibers consider that the coupling coefficient is much smaller than the propagation constant. The Propagation constant is given by the following equation.

\[
\beta = \left( \frac{2\pi n_m}{\lambda} \right)^2 \left( \frac{U^2}{a^2} \right)^{1/2}
\]

(23)

Where \( \lambda_m = -2jK \cos\left(\frac{m\pi}{n+1}\right) \), \( m,q,s,t = 1,2,3,\ldots,n \) is the modes of light which represents the eigenvalue of coupled-mode differential equation, and \( K \) is the modified of Bessel function which called as Hankel function. The value of \( \delta = 1 - \left(\frac{n_e}{n_{eo}}\right)^2 \), \( \nu = \frac{2\pi n_m \sqrt{\delta}}{\lambda} = U^2 + W^2 \) are defined as normalized frequency, where as \( U \approx 2.405 e^{-\nu/2} \) is the progression of phase, and \( W \) is the first kind of Bessel function that represents the transverse decay of amplitude.

The power exchanges among fibers during fabrication process are experimentally exhibited as a function of pulling time. It can be seen in Figure 5.1a that the fiber coupler with coupling ratio 50:50 is fabricated by heating two SMFs. As the heating process is started, while the coupling region is also pulled, the initial power shown by red line is detected in fiber 1. When the fusion and elongation are increased, the fibers become melting. The cladding of fiber 1 and fiber 2 are joined, and the distance between the cores becomes closer. At this condition, the coupling coefficient is exponentially increased.

When the coupling region is more pulled, power in fiber 1 is exponentially decreased, and transferred to fiber 2. This power exchange was automatically stopped when the preset coupling ratio is attained. Since the coupling region is carefully twisted, pulled and heated with the suitable parameters, the pre-set coupling ratio will be obtained with high accuracy. Otherwise, inappropriate parameters cause the fabrication process stopped when the preset coupling ratio is slightly changed. The modeling has been purposed by constructing a matrix transfer of 2X2 which it can be written as follows.

\[
A(z) = \begin{bmatrix}
\cos \sigma z - j \frac{\delta}{\sigma} \sin \sigma z & -j \frac{K_\nu}{\sigma} \sin \sigma z \\
-j \frac{K_\nu}{\sigma} \sin \sigma z & \cos \sigma z - j \frac{\delta}{\sigma} \sin \sigma z
\end{bmatrix} A(0)
\]

(24)

By inserting the initial parameter of our coupling coefficient, and input power launched to the fiber 1, the transfer matrix component of Equation (24) is yielded as follows.

\[
M_{2,1} = \begin{bmatrix}
1.0000 - 0.0000i & 0 - 0.0112i \\
0 - 0.0112i & 1.0000 + 0.0000i
\end{bmatrix}
\]

(25)

This matrix exhibits as transfer power between fiber 1 and fiber 2. Rows and columns of this matrix describe the condition of power at the coupling and the number of \( n \)-th fibers. If the first column is revealed as the fiber 1, and the second column is revealed as the fiber 2, it is clear that the first row indicates power at initial condition when the input power is launched into fiber 1, while power at fiber 2 is a minimum. The power exchange is being preceded by calculating the iteration of the inner matrix until the final condition where the maximum power at fiber 2 is denoted by the second row of that matrix. As the results, good agreement between modeling and experimental results of power exchanges between fibers in 1X2 fiber coupler is depicted in Figure 7 (a and b).
It can be evidently seen in Figure 7(a) that the input power is gradually transferred from fiber 1 to fiber 2 as a function of the pulling time which is also visualized during fusion. Since pulling time is defined as the time which is recorded during heating and pulling processes and by knowing the elongation speed, the pulling length can be determined. Modeling of power exchanges between fiber 1 and fiber 2 is then carried out as function of pulling length as depicted by Figure 7(b).

A good approximation of power exchanges between modeling and experimental results shows that power in fiber 1 is periodically transferred to fiber 2. Here, the fiber coupler of 1X2 is considered having the coupling ratio 50:50. During experiment, the coupling ratio is often difficult to be attained. This is due to the time delay of fusion temperature goes down to the thermal equilibrium from working temperature, when the process is suddenly stopped, causes the coupling length is still elongated in micrometer.

Multi directional fiber couplers of 1X3 and 1X4 for various coupling parameters have been also fabricated. Nevertheless, the fabrication process is more difficult than basic of 1X2 fiber coupler. The pulling stages must hold all fibers with uniform fiber strain by vacuum suction so that it can pull all fibers at the vacuum holder. The fibers may be broken if this is not carefully done. In addition, the twisting method also gives a significant effect to the performance of fiber coupler. The both three and four fibers must be correctly twisted with identical bending of all fibers to reduce the power loss at coupling length.

Various coupling ratios of 1X3 directional fiber coupler have been obtained during fabrication. The configuration of coupling ratio is set up before fusion and elongation process. This shows that the power is gradually transferred from fiber 1 to fiber 2. The power at fiber 2 is then transferred symmetrically to the fiber 1 and fiber 3. The transfer powers among the three fibers are obtained during experiment as visualized in Figure 8.

It can be seen that when the pulling and heating process started, the power is distributed and it remains constants until the desired coupling ratio is reached. Since the interaction of electric field occurs among the nearest fibers only, first power at fiber 1 has to be transferred to fiber 2, and then this power is transferred to fiber 3 due to coupling coefficient between fiber 2 and fiber 3, $\kappa_{23}$.

As a comparison, the power exchanges in 1X3 directional fiber coupler are modeled by constructing a set of transfer matrix 3X3 as follows.

$$
\begin{pmatrix}
A_1(z) \\
A_2(z) \\
A_3(z)
\end{pmatrix} = e^{-j\beta l} \\
\begin{pmatrix}
\frac{1}{2} (\cos \sqrt{2\kappa_2 z} + 1) - j \frac{1}{\sqrt{2}} \sin \sqrt{2\kappa_2 z} & \frac{1}{2} (\cos \sqrt{2\kappa_2 z} - 1) \\
- j \frac{1}{\sqrt{2}} \sin \sqrt{2\kappa_2 z} & \cos \sqrt{2\kappa_2 z} - j \frac{1}{\sqrt{2}} \sin \sqrt{2\kappa_2 z} \\
\frac{1}{2} (\cos \sqrt{2\kappa_2 z} - 1) - j \frac{1}{\sqrt{2}} \sin \sqrt{2\kappa_2 z} & \frac{1}{2} (\cos \sqrt{2\kappa_2 z} + 1)
\end{pmatrix}
$$

(26)

There are three possible input power can be launched for 1X3 fiber coupler, it is dependent on the desired coupling ratio. The identical output powers can be obtained by launching optical input power into fiber 2. In this case, power is launched into fiber 1, $A_1(0) = 1$ mW due to get coupling ratio 25:25:50. After run the MATLAB code, the transfer component of the Equation (26) becomes a transfer power operator at coupling length of 1X3 fiber coupler.

$$
M_{1X3} = 
\begin{pmatrix}
0.9998 - 0.0165i & -0.0002 - 0.0112i & -0.0001 + 0.0000i \\
-0.0002 - 0.0112i & 0.9997 - 0.0165i & -0.0002 - 0.0112i \\
-0.0001 + 0.0000i & -0.0002 - 0.0112i & 0.9998 - 0.0165i
\end{pmatrix}
$$

(27)

Since the input power is launched to the fiber 1, the matrix transfer given by Equation (27) represents power distribution at the coupling region. It shows that the first row denotes the condition where the maximum power is at fiber 1. At point $M_{11}$, power is mainly distributed inside the fiber 1. But, the coupling coefficient between fiber 1 and fiber 2, $\kappa_{12}$ as represents by $M_{12}$, causes the power is transferred into the fiber 2. Concurrently the value of $\kappa_{13}$ is very small and then it is neglected.
When the optical power completely transferred to the fiber 2, the power distribution is exhibited by the second row. This power is denoted by $M_{22}$. Both $M_{21}$ and $M_{23}$ have the same values of -0.0002-0.0112i. The negative sign indicates that the power at fiber 2 will propagate symmetrically to the fiber 1 and fiber 3.

The third rows of transfer matrix represented in Equation (26) depicts that the power is completely transferred into the fiber 3. In this condition, the maximum power is inside of the fiber 3 which is denoted by $M_{33} = 0.9998 - 0.0165i$. However the powers at fiber 2 and fiber 1 are $M_{32} = -0.0002 - 0.0112i$ and $M_{31} = -0.0001 - 0.0000i$ respectively which are much smaller than $M_{33}$.

Fig. 9. Theoretical power exchanges of 1X3 fiber coupler with input power at fiber 1, $A_1(0) = 1$ mW

Figure 9 shows a good approximation of theoretical result compared to the experimental result shown in Figure 5.2, where the initial power at fiber 1 is gradually transferred into the fiber 2, and then fiber 3. Figure 8 shows that fabrication process stopped when the coupling ratio is 25:25:50 which is denoted by the power distribution remains constant at all fibers.

Based on the modeling results shown in Figure 9, it explains that the easiest configuration of coupling ratio of 1X3 fiber coupler is to obtain 25:25:50, and 45:45:10 since the input power is launched into fiber 1 or fiber 3. But, the identical output powers can be set by launching the input power into the fiber 2. Power at fiber 2 will be symmetrically transferred to the fiber 1 and fiber 3 as a function of the coupling length.

For directional fiber coupler consisting of four fibers joint, the coupling region might be more complicated. The main problem coming during experiment is a mechanical process such as twisting technique involving four fibers at coupling region. However, the sample of 1X4 fiber coupler for various coupling ratios have been fabricated with low exertion loss, EL1 = 0.341 dB as shown in Appendix Q.

The power exchanges among four fibers of 1X4 fiber coupler were recorded while the coupling region is pulled and heated as visualized in Figure 5.4. In this case the input power was launched into the fiber 1 ($A_1(0) = 1$). Before fusion process, the initial input power at fiber 1 detected by the photo detector is depicted by the red line. This power then propagates into its nearest fiber which is fiber 2 as depicted by the blue line due to the coupling coefficient between them $\kappa_{12}$. While the power at fiber 2 is increased, the coupling coefficient between fiber 2 and fiber 3 $\kappa_{23}$ causes this power propagates into fiber 3 as depicted by the yellow line. It is then transferred into the fiber 4 as given by the green line due to the coupling coefficient between fiber 3 and fiber 4 $\kappa_{34}$.

Fig. 10. Experimental results of power exchanges in 1X4 fiber coupler, with input power at fiber 1 and pulling speed = 100 µm/s, EL1 = 0.341 dB

Figure 10 shows that the fusion process stopped when the preset coupling ratio of 1X4 fiber coupler 50:30:10:10 was obtained. The power exchanges among the fibers in 1X4 directional fiber coupler were also modeled based on the transfer matrix method. The transfer matrix equation for 4X4 was constructed by determining the matrix MXN given by Equation (22). It yields as follows,
\[
\begin{bmatrix}
A_1(z) \\
A_2(z) \\
A_3(z) \\
A_4(z)
\end{bmatrix} =
\begin{bmatrix}
M_{11} & M_{12} & M_{13} & M_{14} \\
M_{21} & M_{22} & M_{23} & M_{24} \\
M_{31} & M_{32} & M_{33} & M_{34} \\
M_{41} & M_{42} & M_{43} & M_{44}
\end{bmatrix}
\begin{bmatrix}
A_1(0) \\
A_2(0) \\
A_3(0) \\
A_4(0)
\end{bmatrix}
\] (28)

where

\[
M_{11} = M_{44} = \frac{\gamma_1^2 (\gamma_1^2 - 1)}{\gamma} \cos(\gamma_1 \kappa z) + \frac{\gamma_2^2 (\gamma_2^2 - 1)}{\gamma} \cos(\gamma_2 \kappa z)
\]

\[
M_{12} = M_{43} = -\frac{\gamma_1 (\gamma_1^2 - 1)}{\gamma} \sin(\gamma_1 \kappa z) - \frac{\gamma_2 (\gamma_2^2 - 1)}{\gamma} \sin(\gamma_2 \kappa z)
\]

\[
M_{13} = M_{42} = -\frac{1}{\gamma} \cos(\gamma_1 \kappa z) - \frac{1}{\gamma} \cos(\gamma_2 \kappa z)
\]

\[
M_{14} = M_{41} = \frac{\gamma_1 \gamma_2^2}{\gamma} \sin(\gamma_1 \kappa z) - \frac{\gamma_2 \gamma_1^2}{\gamma} \sin(\gamma_2 \kappa z)
\]

\[
M_{21} = M_{34} = \frac{\gamma_1 (\gamma_1^2 - 2)}{\gamma} \sin(\gamma_1 \kappa z) - \frac{\gamma_2 (\gamma_2^2 - 2)}{\gamma} \sin(\gamma_2 \kappa z)
\]

\[
M_{22} = M_{33} = \frac{\gamma_1^2 (\gamma_1^2 - 1)}{\gamma} \cos(\gamma_1 \kappa z) - \frac{\gamma_2^2 (\gamma_2^2 - 1)}{\gamma} \cos(\gamma_2 \kappa z)
\]

\[
M_{23} = M_{32} = \frac{\gamma_2}{\gamma} \sin(\gamma_1 \kappa z) - \frac{\gamma_1}{\gamma} \sin(\gamma_2 \kappa z)
\]

\[
M_{24} = M_{31} = -\frac{1}{\gamma} \cos(\gamma_1 \kappa z) + \frac{1}{\gamma} \cos(\gamma_2 \kappa z)
\]

and \( \gamma = \sqrt{5} \), \( \gamma_1 = \left[ \frac{\sqrt{5} + 3}{2} \right]^{1/2} \), and \( \gamma_2 = \left[ \frac{-\sqrt{5} + 3}{2} \right]^{1/2} \). Since the input power is launched to the fiber 1, \( A_1(0) = 1, A_2(0) = 0, A_3(0) = 0 \).

Similarly, this matrix transfer of 4X4 also has the same principle of power distribution with the transfer matrix of 3X3. The maximum values of power distribution are given by the diagonal matrix. The first row describes power at fiber 1 where \( M_{11} \) is maximum. It is then transferred to the nearest fiber which is fiber 2 as given by the second row, where \( M_{22} \) contains the maximum power. The third row describes the condition where a maximum power is at \( M_{33} \) or fiber 3. Almost of all powers is then transferred into fiber 4 as given by the fourth row of Equation (28) which \( M_{44} \) has the maximum value.

The modeling results of power exchange among four fiber of 1X4 directional fiber coupler using above transfer matrix is shown in Figure 11.

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Fig. 11. Theoretical result of power exchange in 1X4 fiber coupler

Good agreement between the modeling results compared to the experimental result is obtained by assuming all fibers are in linear arrangement with identical fiber cross section, refractive index and the propagation constant. Figure 11 shows the propagation of the power transfer in fiber 1, fiber 2, fiber 3, and fiber 4. The dash-black straight line which is vertically taken at coupling length 2.5 µm shows that the pre-set coupling ratio 50:30:10:10.

From the analysis of experimental and theoretical works of multi directional fiber couplers shows that the power at initial fiber cannot be completely returned to itself after distributing its power periodically to the other fibers, or it cannot be also transferred completely to other fibers due to power loss. Otherwise the fiber coupler consisting of two or three coupled fibers, optical power is able to be transferred periodically from fiber 1 to fiber 2, and to fiber 3. But this cannot be reached in the fiber coupler consisting of more than three fibers, such as 1X4. It is also due to periodic function of the coupling coefficients.

Power at fiber 3 is not only transferred to the fiber 4 due to the coupling coefficient between fiber 3 and fiber 4 \( \kappa_{34} \), but it is also transferred into the fiber 2 and returned into the initial fiber 1 due to the coupling coefficient between them which are \( \kappa_{32} \) and \( \kappa_{21} \). It can be seen clearly in Figure 5.5 that the maximum power transfer in fiber 4 is only about 0.8 mW, while power in fiber 1 remains increased. By increasing the coupling length subject to time of pulling length, more power is transferred to other fibers.
6. Conclusion

It can be summarized that coupling coefficient is an important parameter to control the coupling ratio at the output ports. The coupling coefficient not only determined by the geometrical coupling region, but significantly affected by the separation distance between fiber axes. Our theoretical purposed shows a good agreement with the empirical calculation from the experimental data. The coupling coefficient is exponentially increased when the degree of fusion is increased which can be adjusted by controlling heating temperature and pulling speed during experiment. However, the previous coupling coefficient from some references cannot reach the maximum value when the distance between fibers are reduced. By using the proposed coupling coefficient, we obtained good agreement between experiment and theoretical consideration of power exchange.

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References