# Research of Influence of Vibration Impact of the Basis in the Micromechanical Gyroscope 

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#### Abstract

Analytical and experimental studies of the azimuthal module of two-component vibrational micromechanical gyroscope were conducted. The algorithms of operation were proposed and the comparative analysis was carried out. The influence of mechanical disturbances on the movement of azimuthal module in the form of translational and angular oscillations is shown; the errors of determining of the azimuth are defined.


Key-Words: - Micromechanical gyroscope, Vibration, Amplitude, Frequency, Errors, Sensing element

## 1 Introduction

For developers there is a problem of decrease of dimensions of the modern systems of orientation (SO) and movement control of objects for various purposes, and, thereby, of reduce of expenses at operation and prime cost. One of the promising directions in the development of SO is application of micromechanical gyroscopes (MMG); they are made from the modern engineering materials, according to the new technologies in the micromechanics [1-2]. These devices fully conform to the required criteria, although the MMG are inferior to other types of gyroscopes for precision [3-4].

With due regard for the high rate of improvement of accuracy parameters of MMG the authors consider the dynamic characteristics of twocomponent micromechanical gyroscope and automatic setting of system parameters of MMG, which provide the best characteristics of SO, based on these gyroscopes [5].

SO are used for determining of position of moving objects and devices relatively of given basic directions (the directions of the midday line and local vertical at orientation in near-earth space).

## 2 Experimental

Researched system has two measurement channels: the azimuth channel (for measuring of azimuth of the object $\alpha$ ) and the horizontal channel (for measuring of angles of deviation of the object from the plane of the horizon $\gamma, \delta$ ).

The sensing element (SE) of the azimuth channel is a two-component oscillatory vibratory micromechanical gyroscope [6]; its scheme is shown in Fig.1.


Fig.1: Scheme of two-component micromechanical gyroscope

The micromechanical gyroscope of the azimuthal module contains an inertial body 1 in the corps 7 ; it is fixed by means of two-axis elastic subweight 2 and 3 . Electrostatic vibrational drive 6 excites the harmonic oscillations of the gyroscope $z(t)=z_{0} \sin \omega t \quad$ along the $z$-axis, which is perpendicular to the plane of its sensing element.

Coriolis forces arise as a result of Earth's rotation and translational oscillations of the gyroscope at
speed $\dot{z}(t)$; these forces are changed according to harmonic law with frequency of vibration exciter $\omega$. It leads to the appearance of translational oscillations $x(t), y(t)$ of inertial mass along the axes OX and OY. The amplitudes of these oscillations $x_{a}$, $y_{a}$ are measured by displacement sensors 4,5 and are fed to the computing device for calculating the azimuth $\alpha$, latitude $\varphi$ and signal conditioning of resonant setting of system $U_{R S}$.

The amplitude of the output signal decreases as a result of last increase of stiffness. The system of locked loop of frequency decides to return on a step backwards on basis of this information and to stop work. The system of locked loop of frequency is implemented by a microcontroller.

## 3 Results and Discussion

Mechanical disturbances of the form of translational and angular oscillations are on the object at the place of installation of SO; these disturbances will distort the information movement of the azimuthal module and cause to errors of definition of azimuth.

The equations for calculation of the movement SE on the basis at the forward vibration have an appearance:

$$
\begin{gather*}
m_{1} \ddot{x}+\mu \dot{x}+k_{2} x-H_{1} \dot{y}= \\
=F_{1} \sin \alpha \cos \omega t+f_{1} \sin \alpha \cos q t-f_{2} \sin q t \\
m \ddot{y}+\mu \dot{y}+k_{1} y+H_{1} \dot{x}=  \tag{1}\\
=F_{1}^{\prime} \cos \alpha \cos \omega t+f_{1}^{\prime} \cos \alpha \cos q t-f_{2}^{\prime} \sin q t ;
\end{gather*}
$$

where $m, m_{1}$ - inertial mass of the body and internal frame; $x, y$ - moving of the inertial mass along the axes $\mathrm{X}, \mathrm{Y} ; \mu_{1}, \mu_{2}$ - coefficients of forces of viscous friction; $\Omega_{x}, \Omega_{y}{ }^{-}$ projections of the speed of Earth's rotation on the axis which are associated with the gyroscope; $k_{1}=c_{1}+m g(\ell+\mathrm{z}) ; \quad f_{1}^{\prime}=2 m \Omega_{H} c q ;$ $k_{2}=c_{2}+m g(\ell+\mathrm{z}) ; f_{1}=2 m_{1} \Omega_{H} c q ; \quad f_{2}^{\prime}=m b q^{2} ;$ $f_{2}=m_{1} a q^{2} ; ; \quad H_{1}=2 m \Omega_{V} ; \quad F_{1}=2 m_{1} z_{0} \omega_{H} ;$ $\Omega_{H}=\Omega_{E} \cos \varphi ; \Omega_{V}=\Omega_{E} \sin \varphi ; \Omega_{E}$ - speed of daily rotation of the Earth; $c_{1}, c_{2}$ - angular stiffness of elastic connections; $a, b, q-$ amplitude and frequency of translational vibration of the basis.

At the condition of resonant setting and lack of vibration the algorithm of determining of the azimuth in the semi-analytical SO has form [1]

$$
\alpha=k \pi \pm \operatorname{arctg} \frac{\left(x_{a} \mu-y_{a} H_{1}\right)}{\left(x_{a} H_{1}+y_{a} \mu\right)} \cdot d
$$

where $d=\frac{m}{m_{p}+m}, H_{1}=2 m \Omega_{E} \sin \varphi$.
Micromechanical gyroscope is a vibrating system with forces of viscous friction and two partial frequencies:

$$
\begin{gathered}
\omega_{1,2}^{2}=\frac{-\left(m+m_{p}\right) K_{2}-m K_{1}}{2\left(m+m_{p}\right) m} \pm \\
\pm \frac{\sqrt{\left.\left(m+m_{p}\right) K_{2}+m K_{1}\right]^{2}-4\left(m+m_{p}\right) m\left(K_{1} K_{2}-F_{2}^{2}\right)}}{2\left(m+m_{p}\right) m}
\end{gathered}
$$

where $K_{1}=2 k_{2}-\left(m+m_{p}\right) \Omega_{y}^{2} ; K_{2}=2 k_{1}-m \Omega_{x}^{2}$; $F_{2}=m \Omega_{x} \Omega_{y}$.

These frequencies depend on the speed $\Omega_{x}, \Omega_{y}$. However, this effect can be neglected at the excitation of oscillations of inertial mass at high frequencies; the natural frequencies are determined by the expression

$$
\begin{gather*}
\omega_{1,2}^{2}=\frac{-\left(m+m_{p}\right) 2 \kappa_{1}-m 2 \kappa_{2}}{2\left(m+m_{p}\right) m} \pm \\
\pm \frac{\sqrt{\left[\left(m+m_{p}\right) 2 \kappa_{1}+m 2 \kappa_{2}\right]^{2}-16\left(m+m_{p}\right) m \kappa_{1} \kappa_{2}}}{2\left(m+m_{p}\right) m} \tag{2}
\end{gather*}
$$

At excitation of oscillations of the inertial body on one of the partial frequencies of the system the amplitude of one coordinate reaches the maximum values and the amplitude of the other coordinate is by two times less (Fig.2).

Given the acute nature of the resonance due to the high quality factor of silicon MMG, it is necessary a precise resonant setting. The system of auto-adjust of frequency is entered for this purpose; this system can be realized with regulating of "stiffness" of subweight by means of the vibrodrive. "Stiffness" of the system changes at the voltage on the electrodes of the drive; and, therefore, the natural frequency of the gyroscope changes too.

The algorithm of the system of locked loop of frequency is designed in such a way that the initial value of the stiffness $k_{0}$ is set on the first step. The value of this stiffness should be such that the system worked obviously not in resonance [7].

After then the current value of the stiffness $k$ begins to strive for resonant value $k_{r}$, and it increases on size $\Delta k$ at each step. The change of amplitude of the output signal of the gyroscope is also controlled in each step.

If the amplitude did not increase at some step as at resonance, but it decreased, it means that the resonance is passed and it is necessary take a step back.


Fig.2: Amplitude-frequency characteristics of MMG

The work of system of locked loop of frequency is simulated using software Simulink MATLAB; the results are presented in Fig. 3.

The first two graphs show the amplitudes of output signals of the gyroscope along the axes $O X$ and $O Y$. The third graph represents the dependence of the output signal of Microcontroller block, which implements the algorithm of work of system of the auto setting, from time. On these graphs show, system of the auto setting of the frequency gradually increases stiffness, bringing the system to resonance.


Fig.3. The output signals of MMG at auto-adjust of its own frequencies

The solution of the system of equations (1) we defined by the form of the right parts (in the case of the translational vibration of the basis):

$$
\begin{gather*}
x(t)=A \cos \omega t+B \sin \omega t+A_{1} \cos q t+B_{1} \sin q t \\
y(t)=M \cos \omega t+N \sin \omega t+  \tag{2}\\
+M_{1} \cos q t+N_{1} \sin q t
\end{gather*}
$$

If the frequency of vibration of the basis $q$ does not coincide with working frequency of $\operatorname{SE} \omega$, the amplitudes of the useful informative oscillations of SE $A, B, M, N$ at the resonant setting are determined by the expressions

$$
\begin{aligned}
& x_{a}=\frac{f\left(m_{1} \mu \sin \alpha+m H_{1} \cos \alpha\right)}{\omega\left(H_{1}^{2}+\mu^{2}\right)}, \\
& y_{a}=\frac{f\left(m \mu \cos \alpha-m_{1} H_{1} \sin \alpha\right)}{\omega\left(H_{1}^{2}+\mu^{2}\right)},
\end{aligned}
$$

where $f=2 z_{0} \omega \Omega_{E} \cos \varphi$.
Then the amplitudes of vibrations of inertial mass (taking into account the translational vibration of the basis) are of the form

$$
\left.\begin{array}{c}
B_{1}=\frac{f_{2}\left[\left(m_{1} q^{2}-k_{2}\right)\left(m q^{2}-k_{1}\right)^{2}+H_{1}^{2} q^{2}\left(m q^{2}-k_{1}\right)\right]}{\Delta}- \\
-\frac{f_{2}\left[\mu^{2} q^{2}\left(m_{1} q^{2}-k_{2}\right)\right]}{\Delta}+ \\
+\frac{f_{1} \mu q \sin \alpha\left[\left(m q^{2}-k_{1}\right)^{2}+\left(H_{1}^{2}+\mu^{2}\right) q^{2}\right]}{\Delta}- \\
+\frac{f_{1}^{\prime} H q \cos \alpha\left[-\left(m_{1}^{\prime} q^{2}-k_{2}\right)\left(m q^{2}-k_{1}\right)+q^{2}\left(m_{1}^{2} q^{2}-k_{2}\right)+\left(m q^{2}-k_{1}\right)\right]}{\Delta}+ \\
+\frac{f_{1}^{2} \cos \alpha\left[H_{1}^{2} q^{2}\left(m_{1} q^{2}-k_{2}\right)-\mu^{2} q^{2}\left(m q^{2}-k_{1}\right)\right]}{\Delta}+ \\
M_{1}=\frac{f_{1}^{\prime} \cos \alpha\left[-\left(m_{1} q^{2}-k_{2}\right)^{2}\left(m q^{2}-k_{1}\right)\right]}{\Delta}+ \\
\\
\quad+\frac{f_{1} H_{1} \mu q^{2}\left[\left(m q^{2}-k_{1}\right)^{2}+\left(m_{1} q-k_{2}\right)\right]}{\Delta}- \\
\\
\quad-\frac{f_{2}^{\prime} \mu q\left[\left(m_{1} q^{2}-k_{2}\right)^{2}+q^{2}\left(H_{1}^{2}+\mu^{2}\right)\right]}{\Delta}+ \\
+\frac{f_{2} H q\left[\left(m_{1} q^{2}-k_{2}\right)\left(m q^{2}-k_{1}\right)-q^{2}\left(H_{1}^{2}+\mu^{2}\right)\right]}{\Delta} \\
+\frac{-f}{\Delta}+\frac{\sin _{2} \alpha\left[H_{1}^{2} q^{2}\left(m q^{2}-k_{1}\right)-\mu^{2} q^{2}\left(m_{1} q^{2}-k_{2}\right)\right]}{\Delta}+ \\
+\frac{f_{2} \mu q\left[\left(m q^{2}-k_{1}\right)^{2}+q^{2}\left(H_{1}^{2}+\mu^{2}\right)\right] H_{1} \mu q^{2}}{\Delta}- \\
+\left(m_{1} q^{2}-k_{2}\right)\left(m q^{2}-k_{1}^{2}\right] \\
\Delta
\end{array}\right]
$$

$$
\begin{gathered}
-\frac{f_{1}^{\prime} \cos \alpha\left[\left(m q^{2}-k_{1}\right)+\left(m_{1} q^{2}-k_{2}\right)\right] H_{1} \mu q^{2}}{\Delta}+ \\
+\frac{f_{2}^{\prime} H_{1} q\left[-\left(m_{1} q^{2}-k_{2}\right)\left(m q^{2}-k_{1}\right)+q^{2}\left(H^{2}+\mu^{2}\right)\right]}{\Delta}, \\
+\frac{\left.N_{1}=\frac{f_{2}^{\prime}\left[\left(m_{1} q^{2}-k_{2}\right)^{2}\left(m q^{2}-k_{1}\right)\right]}{\Delta}+H_{1}^{2} q^{2}\left(m_{1} q^{2}-k_{2}\right)+\mu^{2} q^{2}\left(m q^{2}-k_{1}\right)\right]}{\Delta}+ \\
+\frac{f_{1} H_{1} q \sin \alpha\left[\left(m_{1} q^{2}-k_{2}\right)^{2}+\left(m q^{2}-k_{1}\right)\right]}{\Delta}+ \\
+\frac{f_{1} H_{1} q \sin \alpha\left[-\left(H_{1}^{2}+\mu^{2}\right) q^{2}\right]}{\Delta}+ \\
+\frac{-f_{2} H_{1} \mu q^{2}\left[\left(m_{1} q^{2}-k_{2}\right)+\left(m q^{2}-k_{1}\right)\right]}{\Delta}+ \\
+\frac{f_{1}^{\prime} \mu q \cos \alpha\left[\left(m_{1} q^{2}-k_{2}\right)^{2}+q^{2}\left(H^{2}+\mu^{2}\right)\right]}{\Delta} \\
+\frac{\Delta=\left(m_{1} q^{2}-k_{2}\right)^{2}\left(m q^{2}-k_{1}\right)^{2}-}{}+ \\
+\mu^{2} q^{2}\left[\left(m_{1}^{2} q^{2}-k_{2}^{2}\left(m_{1} q^{2}-k_{2}\right)\left(m q^{2}-k_{1}\right)+\right.\right. \\
\left.+\left(m q^{2}-k_{1}\right)^{2}\right]+ \\
+q^{4}\left(H^{2}+\mu^{2}\right)^{2}
\end{gathered}
$$

From these expressions and results of computer simulation (Fig. 4) it turns out that the oscillations with frequency of translational vibration $q$ are superimposed on the useful informative oscillations $x(t), y(t)$ with frequency at the presence of translational vibration of the basis. These oscillations distort the informative movement of SE and cause the calculation errors of the azimuth $\alpha$.

Oscillations of the SE occur at the frequency of basis vibration $q$; these oscillations are due to the translational vibration. Informative oscillations occur at the operating frequency of vibrational excitation of SE $\omega$.

If frequency of vibration of basis does not coincide with the operating frequency of the SE $\omega$, the band filters are used for eduction of the useful signals. Then all signals caused by vibration of the basis will be filtered out.

Therefore the azimuth of object is calculated by useful information signals. These signals $\mathrm{x}^{*}(\mathrm{t})$ and $\mathrm{y}^{*}(\mathrm{t})$ remain at the exit of band filters (Fig.4).

It should be noted that the errors of calculation of the azimuth will remain the same as on fixed basis.

The presence of angular vibration of basis, the movement of SE of azimuthal module is characterized by the equations

$$
\begin{aligned}
& m_{1} \ddot{x}+\mu \dot{x}+k_{2} x-\left(H_{1}+H_{2} \sin \chi t\right) \dot{y}= \\
& =F_{1} \sin \alpha \cos \omega t+f_{4} \sin \chi t- \\
& -\frac{F_{2}}{2} \sin (\chi+\omega) t-\frac{F_{2}}{2} \sin (\chi-\omega) t, \\
& m \ddot{y}+\mu \dot{y}+k_{1} y+\left(H_{1}+H_{2} \sin \chi t\right) \dot{x}= \\
& \quad=F_{1}^{\prime} \cos \alpha \cos \omega t-f_{4}^{\prime} \sin \chi t- \\
& -\frac{F_{2}^{\prime}}{2} \sin (\chi+\omega) t-\frac{F_{2}^{\prime}}{2} \sin (\chi-\omega) t .
\end{aligned}
$$

where $H_{2}=2 m \Omega_{H}\left(\sin \alpha \cdot \delta_{0}+\cos \alpha \cdot \gamma_{0}\right)$,
$F_{2}=2 m_{1} \Omega_{V} z_{0} \omega \delta_{0}, \quad F_{2}^{\prime}=2 m_{1} \Omega_{V} z_{0} \omega \gamma_{0}$,
$f_{4}=m_{1} g \gamma_{0}, \quad f_{4}^{\prime}=m g \delta_{0}, \quad \gamma_{0}, \delta_{0}, \chi \quad$ are the amplitudes and frequency of angular oscillations of object.

From the equations it is seen that at rotational vibration of basis on SE are useful forces $F_{1} \sin \alpha \cos \omega t, F_{1}{ }^{\prime} \cos \alpha \cos \omega t$ with frequency $\omega$; the other forces are obstructions and distort the useful information of SE.


Fig. 4: Characteristics of the SE at the translational vibration of the basis

In addition, SE is a system with periodically varying coefficients $H_{2} \sin \chi t \cdot \dot{y}, \quad H_{2} \sin \chi t \cdot \dot{x}$. Parametric oscillations can occur in such systems at certain relations between natural frequency $\omega$ and the vibration frequency $\chi$. It can be avoided by appropriate choice of the coefficient of viscous
friction forces $\mu$; it is carried out in the considered SE. Therefore, periodic members $H_{2} \sin \chi t \cdot \dot{y}$, $H_{2} \sin \chi t \cdot \dot{x}$ aren't considered by consideration of influence of angular vibration of the basis on SE.
"Moments-disturbances" will cause oscillations of SE along the axes X and Y with vibration frequency $\chi$ and frequencies $(\chi+\omega),(\chi-\omega)$. These oscillations distort the useful informational oscillations of SE occurring at the operating frequency $\omega$ of SE. The presence of band filters in the channels of conversion of the output signals of SE also eliminates the influence of the angular vibration of the basis on the accuracy of the calculation of azimuth.

## 4 Conclusion

The information movement of SE is carried out with frequency of vibrational excitation of inertial weight. All interferences at the mechanical indignations are filtered out at placing of band filters at the exit of angular sensors for measuring of amplitudes of oscillations of SE. Thus the accuracy of calculation of an azimuth remains same as well as at the absence of vibration.

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