Modelling and resolution of a linear problem in integer numbers for an optimal air hub location in the WAEMU zone

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Abstract: In this paper, we propose two linear models in integer numbers for the optimal air traffic hub location in the WAEMU(West African Economic and Monetory Union) zone. The first one minimizes the distances and takes into account the flow of passengers recorded in the different airports. The second one, in addition of the transportation costs, minimizes the extension costs for the fact that the airports do not have the same scale. We elaborate two algorithms of resolution and use Python for the simulations.

Key–Words: optimal location, hub, PLNE, NAS algorithm, WAEMU.

1 Introduction

WAEMU West African Economic and Monetary Union (also known by its French acronym, UEMOA) was created by January 10, 1994. WAEMU members (Benin, Burkina Faso, Cote d'Ivoire, Guinea-Bissau, Mali, Niger, Senegal, and Togo) are working toward greater regional integration.
In the air traffic, WAEMU has several objectives including :

- Create between WAEMU states, common market based on the free movements of goods and people
- So it wants to create an air company. But an air company requires, among other, a hub airport.

A hub is an airport correspondence platform. The principle of the hub is to permit the connection of a big number of small flows of traffic to the more important flows. This organization by beaches multiplies the opportunities of correspondence with minimum waiting time between flights, makes profitable travels and the aircraft using. It also avoid too important turning to passengers. In hub airports, maintenance problems are also resolved. The hub must be located in a country generating itself an important demand in air transport, this in order to contribute to the replenishment of aircrafts and the geographical diversity of the links proposed. Of this fact, most hubs are located close to important urban agglomerations, especially if these concentrate some international economic functions. The airport hub must have a sufficient capacity, so much to develop the offer as to organize the timetables as freely as possible and so to coordinate the flights efficiently.
The problem consists in determining the best location for the hub, optimizing an economic function This one can depend on distances between the airports, costs of airport extension, costs of transportation, it integrates the cost of exploitation, of kerosene, the number of passengers recorded in the airport . . . Several researchers worked on the hubs location. A state of the art on hubs location problem have been studied by A. SIBEL and Y. KARA[1] and by CAMPELL J.F [2]. A linearization of this formulation is given by CAMPELL in [3]. IVAN Contreras and al. used the branch and price [4] and the lagrangean relaxation[5] for the capacitated hub location problem with single assignment. C Diallo and al. [6] worked on the scheduling aircraft landing at LSS (Leopold Sedar SENGHOR) airport. Tanguy and al. [7] elaborated a mathematical model for scheduling staff with assignment job and in [8] use the location to present an industrial problem. O'Kelly[9] gives a quadratic integer program for the location of interacting hub facilities. Rodíguez and al. [10] solved the hub location and assignment problem under capacity constraints by the branch and price. All these authors whose list is not exhaustive brought a contribution in air traffic system. But until now we have no specific contribution for the WAEMU zone. In this paper, we bring our contribution in the management of air traffic system for the WAEMU zone by proposing and solving models of air hub location.

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2 Problem position

Here, we consider the eight countries of the WAEMU zone: BENIN, BURKINA, COTE D'IVOIRE, GUINEA BISSAU, MALI, NIGER, SENEGAL and TOGO, represented by the international airport of their Capital. Our objective is to select one airport from the eight to make it as a hub. First we assume that the hub is located taking into account the flow traffic and the distances traveled . Then we consider the cost of airports extension because the airports in WAEMU have not the same dimensions.

3 Mathematical formulation

For the mathematical formulation, we propose two models. The first model minimizes the distance between the different airports. The second model minimizes the sum of the extension cost and the total transportation cost. The physical network is considered as a graph $G = (N, A)$, where N is the set of nodes that correspond to origin/destinations and representing the capitals airports of WAEMU countries. A is the set of edges linking the different airports.

3.1 Mathematical model minimizing distances

We shall use the following Parameters:

- N: the number of WAEMU countries
- d_{ij}: the distance between nodes i and j
- δ_{ikj}: the distance between nodes i and j via node k
- P_i: the average number of passengers recorded in one year at the node(airport) i (the average is on three years)
- $\overline{X}$: the arithmetic mean of the passengers number recorded in the WAEMU zone $\overline{X} = \frac{1}{N}\sum_{i=1}^{N} P_i$

The decision variable is

$$x_k = \begin{cases} 1 & \text{if node k is the hub} \\ 0 & \text{otherwise} \end{cases}$$

The function to minimize is:

$$\begin{aligned} f(x) &= \sum_{k=1}^{N} x_k\left(\sum_{i=1}^{N} d_{ik} + \sum_{j=1}^{N} d_{kj}\right) \\ &= \sum_{k=1}^{N}\sum_{i=1}^{N}\sum_{j=1}^{N} x_k \delta_{ikj} \end{aligned}$$

where $\delta_{ikk} = d_{ik}$; $\delta_{kkj} = d_{kj}$ et $\delta_{kkk} = 0$

Our mathematical model is :

$$min \sum_{k=1}^{N}\sum_{i=1}^{N}\sum_{j=1}^{N} \delta_{ikj} x_k \quad (1)$$

$$\sum_{k=1}^{N} x_k = 1 \quad (2)$$

$$\sum_{k=1}^{N} P_k x_k \geq \overline{X} \quad (3)$$

$$x_k \in \{0,1\} \forall k \in \{1, ..., N\} \quad (4)$$

The objective (1) minimizes the sum of distances between different airports and the hub airport.

Constraint (2) means that one hub exactly is to locate.

Constraint (3) shows that if the hub is located in node k then the number of passengers recorded in one year in this airport, must be upper or equal to the average of passengers recorded in WAEMU zone.

Constraints (4) mean that x_k are binary variables.

3.2 Mathematical model integrating the extension cost

In this model, in addition to minimize the total cost of distances covered, we minimize the airports extension costs.

In addition to the above parameters we have:

- c_{ikj}: transportation cost of airport i to airport j via airport k
- $c_{ikj} = \alpha\delta_{ikj}$ where α is the transportation cost on distance unit.
- $c_{ikk} = \alpha d_{ik}$ is the transportation cost between airports i and k
- $c_{kkj} = \alpha d_{kj}$ is the transportation cost between airports k and j
- $c_{kkk} = 0 f_{ikj}$: passengers flow coming from all airport i via the airport k to the destination $j \neq i$.
- f_{ikj} : passengers flow coming from all airport i via the airport k to the destination $j \neq i$.

- f_k : the extension cost of airport k
- m_i: the number of trade movements recovered in one year at the node (airport).
- $\overline{M}$:the arithmetic mean of the movements recorded in the WAEMU zone

The decision variable is always

$$x_k = \begin{cases} 1 & \text{if node k is the hub} \\ 0 & \text{otherwise} \end{cases}$$

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Definition 1 *We call weight of the transportation cost the real* λ_{ikj} *defined by* $\lambda_{ikj} = \frac{1}{f_{ikj}}$

The objective function is the following:

$$f(x) = \sum_{k=1}^{N} f_k x_k + \sum_{k=1}^{N} x_k \sum_{i=1}^{N} \sum_{j=1}^{N} \lambda_{ikj} c_{ikj}$$

Substituting c_{ikj} by its expression, we obtain :

$$f(x) = \sum_{k=1}^{N} f_k x_k + \sum_{k=1}^{N} \sum_{i=1}^{N} \sum_{j=1}^{N} \lambda_{ikj} \alpha \delta_{ikj} x_k$$

we can also write :

$$f(x) = \sum_{k=1}^{N} \sum_{i=1}^{N} \sum_{j=1}^{N} (f_k + \lambda_{ikj} \alpha \delta_{ikj}) x_k$$

Our mathematical programming model is :

$$\begin{aligned} (PLNE) z &= \min \sum_{k=1}^{N} \sum_{i=1}^{N} \sum_{j=1}^{N} (f_k + \lambda_{ikj} \alpha \delta_{ikj}) x_k && (5) \\ \sum_{k=1}^{N} x_k &= 1 && (6) \\ \sum_{k=1}^{N} P_k x_k &\geq \overline{X} && (7) \\ \sum_{k=1}^{N} m_k x_k &\geq \overline{M} && (8) \\ x_k &\in \{0,1\} \forall k \in \{1, ..., N\} && (9) \end{aligned}$$

The first terms of the objective function represent the extension costs of the airports and the last terms, the total cost of transportation weighted by a coefficient. (5) minimizes the sum of the two sets of costs. Constraint (6) ensures that one hub exactly is to locate. Constraint (7) shows that if the hub is located in node k then the number of passengers recorded in one year in this airport, must be upper or equal to the average of passengers recorded in WAEMU zone. Constraint (8) shows that if the hub is located in node k then the number of movements recorded in one year in this airport, must be upper or equal to the average of movements recorded in WAEMU zone. Constraint (9) are the integrality constraints.

4 Resolution

For the resolution, we elaborated an algorithm for each model. Algorithm NAS for the first one and algorithm SYNA for the second.

4.1 NAS algorithm

Stage 1

- Enter the value of N
- Enter the $P_i \forall i \in \{1, ..., N\}$
- Calculate $\overline{X}$

Stage 2

- Enter the values d_{ij}
- Give matrix of distances
- Calculate the sum $SL_i (i \in \{1, ..., N\})$ of every line of the matrix

Stage 3

- Initialize a sum (to take a value upper $\max SL_i$)
- For $i \in \{1, ..., N\}$,
  if $P_i \geq \overline{X}$, to give line i and SL_i
  $SL_i < somme \forall i \in \{1, ..., N\}$ then $solution = SL_i$
- Give the minimal value SL_i and index i
- Optimal location hub is i.

4.2 SYNA algorithm

Stage 1

- Enter the value of N
- Enter the $P_i \forall i \in \{1, 2, \cdots, N\}$
- Enter the m_i
- Calculate $\overline{X}$
- Calculate $\overline{M}$

Stage 2

- Enter the value d_{ij}
- Enter the matrix of distances
- Calculate $\delta_{ikj} = d_{ik} + d_{kj}\ \forall i, k, j, \in 1, 2, \cdots, N$.

Stage 3

- Enter flows between different airports origin/destination f_{ikj}
- Calculate $\lambda_{ikj} = 1/f_{ikj}, f_{ikj} \neq 0$.

Stage 4

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- Enter the airports extension cost f_k.
- Enter the transportation cost on one distance unit α Calculate $S_k = f_k + \lambda_{ikj}\alpha\delta_{ikj}$.

Stage 5

- For $i \in \{1, 2, \cdots, N\}$ If $P_i \geq \overline{X}$ then to give the values of S_i following the ascending order The optimal hub location is the smallest value index.

4.3 Simulations and results of the first model

For the simulation, we have used the software Python version 2.7, on a computer HP PROBOOK which characteristics are the following:
Processor: Intel(R), CPU CORE(TM)i5-421OU@170 GHz 240GHz
Memory installed(RAM): 6.00Go (5.88Go usable)
Type of system: operating system 64 bits, processor 64
The data concerning the distances between different airports are calculated by a software of distances determination to bird flight between the airports that is on the site: www.ephemeride.com.
For the number of passengers recorded in the airports during one year, we use the data of the world bank that are on the site: www.banquemondiale.org consulted February 04, 015. The data are those of 2010 to 2013. We have made the average.

Capitals	COO	OUA	ABJ	OXB
passengers	76820	131352	269662	0
Capitals	BKO	NIM	DKR	LFW
passengers	220686	103792	476869	770524

Table 1: number of the air passengers transported

The air passengers transported include passengers of interior and international flights of air company authorized in the country.
In these data of the world bank as those of the ICAO (International Civil Aviation Organization), the number of passengers of the airport of Bissau which code IATA is (International Air Transport Association) OXB is not represented.
After simulation by the software Python, the average of the number of the passengers recorded in the WAEMU zone is $\overline{X} = 256213.125$ either 256214 passengers and the sum of the distances of every airport to all others airports is in the table 2

In this table:

- From the first column to the eight column, we have the matrix of distances.

	COO	OUA	ABJ	OXB	BKO	NIM	DKR	LFW	Sum	V.V.C
COO	0	789,734	710,061	2073,561	1324,38	789,308	2363,256	127,041	8117.341	
OUA	789,734	0	828,943	1539,151	699,712	418,901	1749,6	748,308	6774.349	
ABJ	710,061	828,943	0	1483,128	918,532	1130,158	1818,472	583,025	7472.319	7472.319
OXB	2073,561	1539,151	1483,128	0	840,934	1942,843	373,591	1961,805	10215.013	
BKO	1324,38	699,712	918,532	840,934	0	1102,383	1061,459	1231,227	7178.627	
NIM	789,308	418,901	1130,158	1942,843	1102,383	0	2127,439	815,437	8326.469	
DKR	2363,256	1749,6	1818,472	373,591	1061,459	2127,439	0	2258,757	11752.574	11752.574
LFW	127,041	748,308	583,025	1961,805	1231,227	815,437	2258,757	0	7725.6	7725.6

Table 2: Table of simulation results.

- The ninth column gives the sum of the distances of every airport to all others.
- The last column gives the V.V.C (Values verifying the constraints). They are three.

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Among the three values, the minimal value is 7472.319 Km and corresponds in the ABJ airport. The Abidjan airport is therefore the optimal location.

Remark 2 *The airport of Ouagadougou doesn't respect the constraint (3). Without this constraint, instead of Abidjan, it is the airport of Ouagadougou that would have been chosen as hub.*

Remark 3 *For the second model, the expansion costs are not yet known, we made simulations with estimated values compared to the classification of airports. According to the values, the hub is located either Abidjan or Dakar.*

5 Conclusion

This paper is a first contribution to the establishment of an airline within the WAEMU. This work was requested by ANACIM (National Agency of Civil Aviation and Meteorology). We have proposed two integer linear models for the hub location problem, and have elaborated an algorithm for each. The simulations have been performed with Python. After computational experiments, three airports out of eight, namely Abidjan, Dakar and Lome met constraints imposed for the first model. And the optimal hub location for a company of the WAEMU is the airport of Abidjan. As hub can be displaced for one reason or another, as perspectives we intend to make a model of hub location that will take into account:

- the stability of the zone.
- The capacity problems and the fleetsize.
- A model for two hubs..

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