Statistics of η-μ Random Variable

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Abstract: - The η-μ distribution, which can be used to describe signal envelope variation in channels where in-phase and quadrature components have different powers, is considered in this paper. The closed form expressions for probability density function (PDF), cumulative distribution function (CDF) and moments are calculated. Also, statistics of product, ratio and maximum of two η-μ random variables is studied. The obtained statistical functions are shown graphically by using simulations in MATLAB. The influence of parameters of η-μ distribution on statistics of the product, ratio and maximum of two η-μ random variables is analyzed.

Key-Words: - Cumulative distribution function, η-μ distribution, Probability density function, random variable

1 Introduction

The η-μ distribution can be used to describe small scale signal envelope variation in multipath fading channels where in-phase component power and quadrature component power are different. This distribution has two parameters η and μ. The parameter η is equal to ratio of in-phase component power and quadrature component power and parameter μ is in relation to the number of clusters in propagation environment [1] [2]. The η-μ distribution is general distribution and from this distribution can be derived Nakagami-*m*, Rayleigh and Nakagami-*q* distribution as special cases. Nakagami-*m* distribution can be derived from η-μ distribution by setting η=1, the η-μ distribution reduces to Nakagami-*q* from μ=1 and Rayleigh distribution approximation η-μ distribution by setting η=1 and μ=1 [3].

There are several works in open technical literature considering statistical characteristics of η-μ random process and performance of wireless communication system operating over η-μ multipath fading channels [4] [5]. In paper [4], the η-μ random variable is considered and probability density function is determined. PDF of η-μ distribution is determined by using can convolution integral for two Nakagami-*m* density function. The joint probability density function of in-phase component and quadrature component of η-μ random process is evaluated in work [5]. In paper [6], wireless communication system with equal gain combining (EGC) receiver operating over Nakagami-*q* short term fading channel is analysed. The sum of N Nakagami-*q* random variables is approximated by using η-μ distribution. In [7], the co-channel interference effect on average error rates in Nakagami-q (Hoyt) fading channels is observed.

The second order statistical measures of wireless system in the presence of η-μ multipath fading are studied in [8]. The level crossing rate of proposed system is calculated and by using this expression average fade duration is evaluated. In this paper, joint probability density function of η-μ random variable and its the first derivative is calculated. Sum of non-identical squared η-μ variates and applications in the performance analysis of DS-CDMA systems is determined in [9].

The influence of Maximal Ratio Combining (MRC) receiver to mitigation of η-μ fading is processed in [10] [11]. The performance analysis of Switch and Stay Combining (SSC) diversity reception over η–μ fading channel in the presence of Co-channel interference (CCI) is given in [12].

In this paper η-μ random variable is considered. Probability density function, cumulative distribution function and moments of η-μ random variable are calculated and derived expressions are closed form. The obtained formulas can be used for performance analysis of wireless communication systems subjected to η-μ small scale fading. Further, in this paper product of two η-μ random variables is analysed and probability density function of proposed product is calculated. In this paper also the ratio of two η-μ random variables is analysed. Probability density function and cumulative

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distribution function of considered ratio are calculated. Those formulas can be used in performance analysis of wireless communication system operating over *η-μ* multipath fading channel in the presence of co-channel interference subjected to *η-μ* multipath fading. Further, probability density function of maximum of two *η-μ* random variables is calculated as closed form expression.

2 The *η*-μ Random Variable

Probability density function of *η-μ* random variable is:

$$p_x(x)=\frac{4\sqrt{\pi}\,\mu^{\mu+1/2}h^{\mu}x^{2\mu}}{\Gamma(\mu)H^{\mu-1/2}\Omega^{\mu+1/2}}e^{-\frac{2\mu h}{\Omega}x^2}\cdot$$

$$\cdot I_{\mu-1/2}\left(\frac{2\mu H}{\Omega}x^2\right),\quad x\geq 0. \tag{1}$$

where:

$$H=\frac{\eta^{-1}-\eta}{4},\quad h=\frac{2+\eta^{-1}-\eta}{4},\quad \eta\geq 0, \tag{2}$$

and the variances of independent Gaussian in-phase and quadrature processes are arbitrary with their ratio defined as *η*.

$$p_x(x)=\frac{4\sqrt{\pi}\,\mu^{\mu+1/2}h^{\mu}}{\Gamma(\mu)H^{\mu-1/2}\Omega^{\mu+1/2}}$$

$$\cdot\sum_{i_1=0}^{\infty}\left(\frac{\mu h}{\Omega}\right)^{2i_1+\mu-1/2}\frac{1}{i_1!\Gamma(i_1+\mu+1/2)}x^{4i_1+4\mu-1}e^{-\frac{2\mu h}{\Omega}x^2},$$

$$x\geq 0 \tag{3}$$

The cumulative distribution function of *η-μ* random variable is :

$$F_x(x)=\int_0^x dt\,p_t(t)\,dt=$$

$$=\frac{4\sqrt{\pi}\,\mu^{\mu+1/2}h^{\mu}}{\Gamma(\mu)H^{\mu-1/2}\Omega^{\mu+1/2}}\frac{1}{2}\left(\frac{\mu h}{\Omega}\right)^{i_1+\mu-1/2}\cdot$$

$$\cdot\frac{1}{i_1!\Gamma(i_1+\mu+1/2)}\gamma\left(2i_1+2\mu,\frac{2\mu h}{\Omega}x^2\right). \tag{4}$$

The moment n-th order of *η-μ* random variable is [13]:

$$m_n=\overline{x^n}=\int_0^{\infty}dx\,x^n p_x(x)=\frac{4\sqrt{\pi}\,\mu^{\mu+1/2}h^{\mu}}{\Gamma(\mu)H^{\mu-1/2}\Omega^{\mu+1/2}}\cdot$$

$$\cdot\sum_{i_1=0}^{\infty}\left(\frac{\mu h}{\Omega}\right)^{2i_1+\mu-1/2}\frac{1}{i_1!\Gamma(i_1+\mu+1/2)}\frac{1}{2}\left(\frac{\mu h}{\Omega}\right)^{i_1+\mu-1/2}\Gamma(2i_1+2\mu) \tag{5}$$

3 Product, Ratio and Maximum of Two *η*-μ Random Variables

The *η-μ* random variables x_1 and x_2 follow distribution:

$$p_{x_j}(x_j)=\frac{4\sqrt{\pi}\,\mu_j^{\mu_j+1/2}h_j^{\mu_j}}{\Gamma(\mu_j)H_j^{\mu_j-1/2}\Omega_j^{\mu_j+1/2}}\cdot$$

$$\cdot\sum_{i_1=0}^{\infty}\left(\frac{\mu_j h_j}{\Omega_j}\right)^{2i_1+\mu_j-1/2}x_j^{4i_1+4\mu_j-1}\frac{1}{\Gamma(i_1+\mu_j+1/2)}e^{-\frac{2\mu_j h_j}{\Omega_j}x_j^2}$$

$$x_j\geq 0,\quad j=1,2 \tag{6}$$

The product of two *η-μ* random variable is:

$$x=x_1\cdot x_2,\quad x_1=\frac{x}{x_2}. \tag{7}$$

The probability density function of *x* is:

$$p_x(x)=\int_0^{\infty}dx_2\frac{1}{x_2}p_{x_1}\left(\frac{x}{x_2}\right)p_{x_2}(x_2)=$$

$$=\frac{4\sqrt{\pi}\,\mu_1^{\mu_1+1/2}h_1^{\mu_1}}{\Gamma(\mu_1)H_1^{\mu_1-1/2}\Omega_1^{\mu_1+1/2}}\cdot$$

$$\cdot\sum_{i_1=0}^{\infty}\left(\frac{\mu_1 h_1}{\Omega_1}\right)^{2i_1+\mu_1-1/2}\frac{1}{i_1!\Gamma(i_1+\mu_1+1/2)}\cdot$$

$$\cdot\frac{4\sqrt{\pi}\,\mu_2^{\mu_2+1/2}h_2^{\mu_2}}{\Gamma(\mu_2)H_2^{\mu_2-1/2}\Omega_2^{\mu_2+1/2}}$$

$$\cdot\sum_{i_2=0}^{\infty}\left(\frac{\mu_2 h_2}{\Omega_2}\right)^{2i_2+\mu_2-1/2}\left(\frac{\mu_1 h_1 x^2}{\Omega_1}\frac{\Omega_2}{\mu_2 h_2}\right)^{2i_2+2\mu_2-2i_1+2\mu_1-1}\cdot$$

$$\cdot\frac{1}{i_2!\Gamma(i_2+\mu_2+1/2)}K_{4i_2+4\mu_2-4i_1+4\mu_1-2}\left(2\sqrt{\frac{4\mu_1 h_1 x^2}{\Omega_1}\frac{\mu_2 h_2}{\Omega_2}}\right) \tag{8}$$

Previous expression can be used for evaluation of the outage probability of wireless relay communication system with two sections operating over *η-μ* multipath fading environment. By using the expression for PDF, the cumulative distribution function of product of two *η-μ* random variables can be calculated. Random variable denoted with (*η-μ*)*(*η-μ*) can be studied as a product of two *η-μ* random variables. By setting *η*=1, the (*η-μ*)*(*η-μ*) random variable reduces to Nakagami-*m**Nakagami-*m* random variable. Nakagami-*q**Nakagami-*q* random variable can be derived from

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(*η*-*μ*)*(*η*-*μ*) random variable for $\mu=1$. For $\eta=1$ and $\mu=1$ Rayleigh*Rayleigh random variable approximates (*η*-*μ*)*(*η*-*μ*) random variable.

The ratio of two *η*-*μ* random variable is:

$$x=\frac{x_1}{x_2},\quad x_1=x\cdot x_2. \tag{9}$$

The probability density function of the ratio of two *η*-*μ* random variable is:

$$p_x(x)=\int_0^\infty dx_2 x_2 p_{x_1}(xx_2)p_{x_2}(x_2)=$$
$$=\frac{4\sqrt{\pi}\,\mu_1^{\mu_1+1/2}h_1^{\mu_1}}{\Gamma(\mu_1)H_1^{\mu_1-1/2}\Omega_1^{\mu_1+1/2}}\cdot$$
$$\cdot\sum_{i_1=0}^{\infty}\left(\frac{\mu_1h_1}{\Omega_1}\right)^{2i_1+\mu_1-1/2}\frac{1}{i_1!\,\Gamma(i_1+\mu_1+1/2)}\cdot$$
$$\cdot\frac{4\sqrt{\pi}\,\mu_2^{\mu_2+1/2}h_2^{\mu_2}}{\Gamma(\mu_2)H_2^{\mu_2-1/2}\Omega_2^{\mu_2+1/2}}\cdot$$
$$\cdot\sum_{i_2=0}^{\infty}\left(\frac{\mu_2h_2}{\Omega_2}\right)^{2i_2+\mu_2-1/2}\left(\frac{\Omega_1\Omega_2}{2\mu_1h_1x^2\Omega_2+2\mu_1h_1\Omega_1}\right)^{2i_1+2\mu_1+2i_2+2\mu_2-2}\cdot$$
$$\cdot\frac{1}{i_2!\,\Gamma(i_2+\mu_2+1/2)}x^{4i_1+4\mu_1-1}\Gamma(2i_1+2\mu_1+2i_2+2\mu_2-2). \tag{10}$$

The cumulative distribution function of the ratio of two *η*-*μ* random variable is:

$$F_x(x)=\int_0^x dt\,p_x(t)=\frac{2\sqrt{\pi}\,\mu_1^{\mu_1+1/2}h_1^{\mu_1}}{\Gamma(\mu_1)H_1^{\mu_1-1/2}\Omega_1^{\mu_1+1/2}}\cdot$$
$$\cdot\sum_{i_1=0}^{\infty}\left(\frac{\mu_1h_1}{\Omega_1}\right)^{2i_1+\mu_1-1/2}\frac{1}{i_1!\,\Gamma(i_1+\mu_1+1/2)}\cdot$$
$$\cdot\frac{4\sqrt{\pi}\,\mu_2^{\mu_2+1/2}h_2^{\mu_2}}{\Gamma(\mu_2)H_2^{\mu_2-1/2}\Omega_2^{\mu_2+1/2}}\cdot$$
$$\cdot\sum_{i_2=0}^{\infty}\left(\frac{\mu_2h_2}{\Omega_2}\right)^{2i_2+\mu_2-1/2}\frac{1}{i_2!\,\Gamma(i_2+\mu_2+1/2)}\cdot$$
$$\cdot(\Omega_1\Omega_2)^{2i_1+2\mu_1+2i_2+2\mu_2-2}\cdot$$
$$\cdot\int_0^x dt\frac{t^{4i_1+4\mu_1-1}}{\left(2\mu_1h_1\Omega_2t^2+2\mu_2h_2\Omega_1\right)^{2i_1+2\mu_1+2i_2+2\mu_2-2}}=$$
$$=\frac{2\sqrt{\pi}\,\mu_1^{\mu_1+1/2}h_1^{\mu_1}}{\Gamma(\mu_1)H_1^{\mu_1-1/2}\Omega_1^{\mu_1+1/2}}\cdot$$
$$\cdot\sum_{i_1=0}^{\infty}\left(\frac{\mu_1h_1}{\Omega_1}\right)^{2i_1+\mu_1-1/2}\frac{1}{i_1!\,\Gamma(i_1+\mu_1+1/2)}\frac{4\sqrt{\pi}\,\mu_2^{\mu_2+1/2}h_2^{\mu_2}}{\Gamma(\mu_2)H_2^{\mu_2-1/2}\Omega_2^{\mu_2+1/2}}\cdot$$
$$\cdot\sum_{i_2=0}^{\infty}\left(\frac{\mu_2h_2}{\Omega_2}\right)^{2i_2+\mu_2-1/2}\frac{1}{i_2!\,\Gamma(i_2+\mu_2+1/2)}\cdot$$
$$\cdot(\Omega_1\Omega_2)^{2i_1+2\mu_1+2i_2+2\mu_2-2}\cdot$$
$$\cdot\frac{(2\mu_2h_2\Omega_1)^{-(2i_1+2\mu_1+2i_2+2\mu_2-2)}}{2}\left(\frac{\mu_2h_2\Omega_1}{\mu_1h_1\Omega_2}\right)^{2i_1+2\mu_1}\cdot$$
$$\cdot B_z(2i_1+2\mu_1,2i_2+2\mu_1-2)$$
$$z=\frac{\mu_1h_1\Omega_2x^2}{\mu_2h_2\Omega_1+\mu_1h_1\Omega_2}. \tag{11}$$

The ratio of two *η*-*μ* random variables is important performance measure wireless communication system operating over *η*-*μ* multipath fading channel in the presence of co-channel interference subjected to *η*-*μ* multipath fading. Probability density function, cumulative distribution function and moment nth order are calculated. The ratio of two *η*-*μ* random variables is random variable denoted with (*η*-*μ*)/(*η*-*μ*). For $\eta=1$, the (*η*-*μ*)/(*η*-*μ*) variable reduces to Nakagami-*m*/Nakagami-*m* random variable and for $\mu=1$ (Nakagami-*q**Nakagami-*q*) random variable is derived from (*η*-*μ*)/(*η*-*μ*) random variable. Rayleigh/Rayleigh approximates (*η*-*μ*)/(*η*-*μ*) random variable for $\eta=1$ and $\mu=1$. By derived formulaes for PDF and CDF of ratio of two *η*-*μ* random variables can be calculated outage probability and bit error probability wireless communication system operating over *η*-*μ* multipath fading channel in the presence of co-channel interference affected to *η*-*μ* multipath fading.

Moment of n-th order of the ratio of two *η*-*μ* random variables is [13]:

$$m_n=\overline{x^n}=\int_0^\infty dx\,x^n p_x(x)=\frac{2\sqrt{\pi}\,\mu_1^{\mu_1+1/2}h_1^{\mu_1}}{\Gamma(\mu_1)H_1^{\mu_1-1/2}\Omega_1^{\mu_1+1/2}}\cdot$$
$$\cdot\sum_{i_1=0}^{\infty}\left(\frac{\mu_1h_1}{\Omega_1}\right)^{2i_1+\mu_1-1/2}\frac{1}{i_1!\,\Gamma(i_1+\mu_1+1/2)}\cdot$$

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$$\cdot\frac{4\sqrt{\pi}\,\mu_2^{\mu_2+1/2}h_2^{\mu_2}}{\Gamma(\mu_2)H_2^{\mu_2-1/2}\Omega_2^{\mu_2+1/2}}\cdot\sum_{i_2=0}^{\infty}\left(\frac{\mu_2 h_2}{\Omega_2}\right)^{2i_2+\mu_2-1/2}\cdot$$

$$\cdot\frac{1}{i_2!\Gamma(i_2+\mu_2+1/2)}\Gamma(2i_1+2\mu_1+2i_2+2\mu_2-2)\cdot$$

$$\cdot(\Omega_1\Omega_2)^{2i_1+2\mu_1+2i_2+2\mu_2-2}\cdot$$

$$\cdot\int_0^{\infty}dx\frac{x^{4i_1+4\mu_1-1+\eta}}{\left(2\mu_1h_1\Omega_2x^2+2\mu_2h_2\Omega_1\right)^{2i_1+2\mu_1+2i_2+2\mu_2-2}} \tag{12}$$

$$\int_0^{\lambda}\frac{x^m}{\left(a+bx^n\right)^{\rho}}=\frac{a^{-\rho}}{n}\left(\frac{a}{b}\right)^{\frac{m+1}{n}}B_z\left(\frac{m+1}{n},\rho-\frac{m+1}{n}\right),$$

$$z=\frac{b\lambda^n}{a+b\lambda^n} \tag{13}$$

$$B_z(a,b)=\frac{z^a}{a}2F_1(a,1-b,1+a,z),$$

$$a=2\mu_2h_2\Omega_1,\quad b=2\mu_1h_1\Omega_2 \tag{14}$$

$$n=2,\quad \rho=2i_1+2\mu_1+2i_2+2\mu_2-2,$$

$$m=4i_1+4\mu_1-1,\quad \lambda=x \tag{15}$$

Probability density function of maximum of two η-μ random variable is:

$$p_x(x)=p_{x_1}(x)F_{x_2}(x)p_{x_2}(x)F_{x_2}(x)=$$

$$=\frac{4\sqrt{\pi}\,\mu_1^{\mu_1+1/2}h_1^{\mu_1}}{\Gamma(\mu_1)H_1^{\mu_1-1/2}\Omega_1^{\mu_1+1/2}}\cdot$$

$$\cdot\sum_{i_1=0}^{\infty}\left(\frac{\mu_1h_1}{\Omega_1}\right)^{2i_1+\mu_1-1/2}\frac{1}{i_1!\Gamma(i_1+\mu_1+1/2)}\cdot$$

$$\cdot x^{4i_1+4\mu_1-1}e^{-\frac{2\mu_1h_1}{\Omega_1}x^2}\frac{4\sqrt{\pi}\,\mu_2^{\mu_2+1/2}h_2^{\mu_2}}{\Gamma(\mu_2)H_2^{\mu_2-1/2}\Omega_2^{\mu_2+1/2}}\cdot$$

$$\cdot\sum_{i_2=0}^{\infty}\left(\frac{\mu_2h_2}{\Omega_2}\right)^{2i_2+\mu_2-1/2}\frac{1}{2}\left(\frac{\Omega_2}{\mu_2h_2}\right)^{2i_2+2\mu_2}\cdot$$

$$\cdot\frac{1}{i_2!\Gamma(i_2+\mu_2+1/2)}\gamma\left(2i_2+2\mu_2,\frac{2\mu_2h_2}{\Omega_2}x^2\right)+$$

$$+\frac{4\sqrt{\pi}\,\mu_2^{\mu_2+1/2}h_2^{\mu_2}}{\Gamma(\mu_2)H_2^{\mu_2-1/2}\Omega_2^{\mu_2+1/2}}\sum_{i_2=0}^{\infty}\left(\frac{\mu_2h_2}{\Omega_2}\right)^{2i_2+\mu_2-1/2}\cdot$$

$$\cdot\frac{1}{i_2!\Gamma(i_2+\mu_2+1/2)}x^{4i_2+4\mu_2-1}e^{-\frac{2\mu_2h_2}{\Omega_2}x^2}\cdot$$

$$\cdot\frac{4\sqrt{\pi}\,\mu_1^{\mu_1+1/2}h_1^{\mu_1}}{\Gamma(\mu_1)H_1^{\mu_1-1/2}\Omega_1^{\mu_1+1/2}}\sum_{i_1=0}^{\infty}\left(\frac{\mu_1h_1}{\Omega_1}\right)^{2i_1+\mu_1-1/2}\cdot$$

$$\cdot\frac{1}{i_1!\Gamma(i_1+\mu_1+1/2)}\frac{1}{2}\left(\frac{\Omega_1}{2\mu_1h_1}\right)^{2i_1+2\mu_1}\gamma\left(2i_1+2\mu_1,\frac{2\mu_1h_1}{\Omega_1}x^2\right). \tag{16}$$

The previous expression is probability density function of the maximum of two η-μ random variables. This formula can be used in performance analysis of wireless communication system with selection combining diversity receiver with two branches, operating over η-μ multipath fading channels.

Selection Combining (SC) diversity receiver reduces η-μ short term fading effects on system performance [15]. SC receiver output signal envelope can be calculated as the maximum of the signal envelopes of its inputs. SC receiver selects the branch with the highest signal envelope. The performance measures as the outage probability, the bit error probability and the system capacity of proposed communication system can be calculated by using the obtained expression. The random variable which is equal to the maximum of two η-μ random variables can be denoted as max(η-μ, η-μ) and is general random variable. By setting η=1, max(η-μ,η-μ) reduces to max(Nakagami-*q*, Nakagami-*q*); max(Nakagami-*m*, Nakagami-*m*) random variable can be derived from max(η-μ, η-μ) random variable by setting μ=1; the random variable max(Rayleigh, Rayleigh) approximates max(η-μ, η-μ) random variable for η=1 and μ=1.

4 Numerical Results

In Fig. 1, the histogram of η-μ random process is shown. The abscissa of the histogram is the amplitude values of η-μ random process; the ordinate is the number of samples in the interval of abscissa.

In Fig. 2, the cumulative distribution function of product of two η-μ random variables for several values of parameters η and μ=2 is presented.

The cumulative distribution function of product of two η-μ random variables is outage probability of wireless relay communication system with two sections operating over η-μ multipath fading. The outage probability increases as output signal envelope increases. The output signal envelope has

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higher influence on the outage probability for lower values of the output signal envelope. This figure shows that the outage probability increases when parameter η decreases for lower values of parameter η.

The histogram of product of two η-μ distribution is shown in Fig. 3.

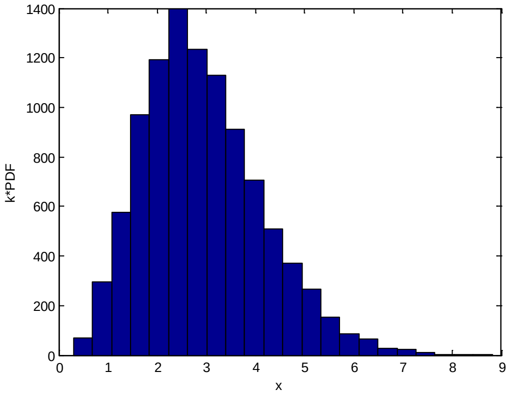

Fig.1. Histogram of η-μ random process

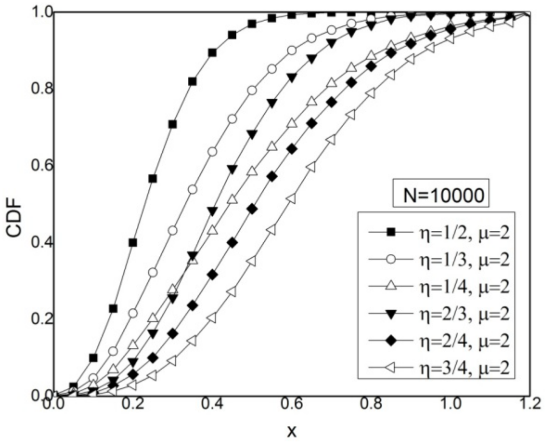

Fig.2. Cumulative distribution function of product of two η-μ random variables

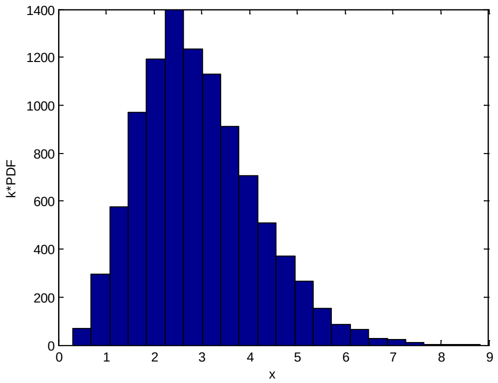

Fig.3. Histogram of product of two η-μ random variables

The cumulative distribution function of the ratio of two η-μ random variables is shown in Fig. 4. The cumulative distribution function of the ratio of two η-μ random variables is the outage probability of wireless communication system operating over η-μ short term fading in the presence of co-channel interference subjected to η-μ multipath fading. The influence of the output signal envelope is higher for lower values of the output signal envelope x.

The histogram of the ratio of two η-μ random variables is shown in Fig. 5. The cumulative distribution function of the maximum of two η-μ random variables is shown in Fig. 6.

The cumulative distribution function of the maximum of two η-μ random variables is actually the outage probability of wireless communication system with selection combining diversity receiver with two inputs in the presence of η-μ multipath fading.

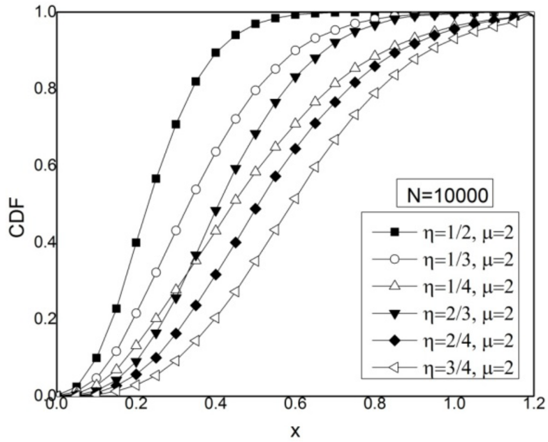

Fig.4. Cumulative distribution function of the ratio of two η-μ random variables

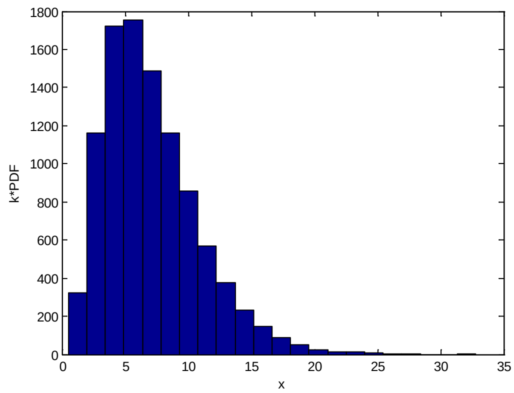

Fig.5. Histogram of the ratio of two η-μ random variables

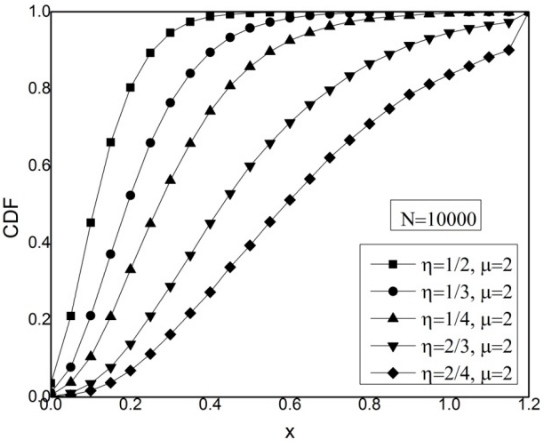

Fig.6. Cumulative distribution function of maximum of two η-μ random variables

The influence of the output signal envelope is higher for lower values of the output signal envelope. For lower values of the output signal envelope x, the outage probability increases as parameter η increases, and the outage probability stagnates for higher values of the output signal envelopes.

4 Conclusion

In this paper, η-μ random variable is considered. The η-μ distribution can be used to describe small scale signal envelope variation in fading channels where powers of in-phase and quadrature components are different. The parameter η can be calculated as the ratio of in-phase and quadrature component powers and parameter μ is in relation with the number of clusters in propagation environment. The closed from expressions for probability density function, cumulative distribution function and moments of η-μ random variable are evaluated. These expressions can be used for calculation the outage probability, the bit error probability and the channel capacity of wireless communication system operating over η-μ multipath fading channel. Farther, in this paper, probability density function of the product of two η-μ random variables, the ratio of two η-μ random variables and the maximum of two η-μ random variables are calculated as the expressions in the closed form. PDF of product of two η-μ random variables can be applied in performance analysis of wireless relay communication system with two sections. Ratio of two η-μ random variables can be used in performance analysis of wireless communication system operating over η-μ small scale fading channel in the presence of η-μ co-channel interference. Maximum of two η-μ random variables can be used in the performance analysis of wireless communication system which use SC receiver to reduce η-μ fading effects on the system performance. In this paper, $(\eta$-$\mu)$*$(\eta$-$\mu)$, $(\eta$-$\mu)/(\eta$-$\mu)$ and max$(\eta$-μ, η-$\mu)$ random variables are formed. The $(\eta$-$\mu)$*$(\eta$-$\mu)$ random variable can be calculated as the product of two η-μ random variables, the $(\eta$-$\mu)/(\eta$-$\mu)$ random variable can be calculated as the ratio of two η-μ random variables and max$(\eta$-μ, η-$\mu)$ random variable can be calculated as a maximum of two η-μ random variables.

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