

# MODELLING AND INVERSE PROBLEMS IN ELECTROMAGNETIC SYSTEMS

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**Abstract:** *The development of high technology, energy-efficient and reliable electromagnetic devices, requires high level of knowledge not only for the electromagnetic processes in the devices, but also for interrelated with electromagnetic field thermal, mechanical physicochemical conditions. Such kind of profound studies gives possibility for determination and control of the parameters and characteristics of the systems even in inaccessible for experimental investigation areas.*

*In spite of the differences in the type and the purpose of the devices, modeling of the processes is based on the precise numerical analysis of the electromagnetic and coupled with them fields and subsequent optimization of the constructions and working parameters of the studied systems. This determines the necessity for correct formulation and solving two basic tasks: forward problem – problem for numerical analysis of the field and inverse problem – optimization, identification and synthesis of field structures. Several examples of modeling and optimization of processes in electromagnetic system have been presented in the paper, using 2D and 3D FEM analysis.*

**Keywords:** *electromagnetic processes, coupled field problems, finite element method, modeling and inverse problems*

## 1. INTRODUCTION

Development of electronic and electromagnetic systems is crucial to the operation of modern society. Nowadays the most important task of the design engineers is to produce a high-quality, market competitive products, which ensure effective work with respect to given requirements and limitations. Obtaining of the final technical product is complicated synthesis process of detailed investigations, optimisations, expert consultations and ranking [1]. The bibliographic survey shows that in over the last two decades, the focus in research related to electromagnetic devices has moving from analysis of the device performance towards the automatic optimisation of a particular device to meet the needs and requirements specified by an user [2].

The creation of high technology, energy-efficient and reliable electromagnetic devices that do not harm the environment is of significant importance. The development and improvement of such systems requires high level of knowledge not only for the electromagnetic processes taking place in the devices, but also of complex interrelated with electromagnetic field thermal and mechanical processes and physicochemical conditions [4, 5, 6, 7 and 8]. The profound study and numerical modelling and optimisation gives possibility for determination and control of the parameters and characteristics of the system even in inaccessible for experimental investigation areas.

## **2. MAIN STEPS AND SPECIFIC FEATURES IN MODELING AND OPTIMISATION OF ELECTROMAGNETIC SYSTEMS**

### **2.1. Main steps in modeling and optimization of the processes**

In spite of the differences in the type and the purpose of the studied objects, achieving the aims of modeling and optimization is obtained on the basis of precise numerical analysis of the electromagnetic and coupled with them thermal, mechanical and physicochemical processes and subsequent optimization of the constructions and working parameters of the studied systems. This determines the necessity for correct formulation and solving two basic tasks:

- *Forward problem* – problem for numerical analysis of the field (electromagnetic, thermal and mechanical) of the studied device. This problem is connected with precise diagnostics of the state of existing devices and obtaining significant for the studied process characteristics and parameters. Different numerical methods are used for the determination of the field quantities and characteristics –finite difference method, finite element method, boundary element method, some hybrid methods and so on. The most often used for the field analysis is finite element method (FEM) It has been successfully applied for solving the analysis in 2D and 3D linear and non-linear coupled problem.

- *Inverse problem* – optimization, identification and synthesis of field structures. The problem is connected with the necessity of control of processes in existing devices or with the design of new ones with preliminary set desired parameters and characteristics. The solving of the problem is based on optimization procedure. The choice of optimization method to a great extent depends on the specifics and complexity of the forward problem to be solved at each step of the optimization procedure. requirements: The analysis and comparison of different approaches and techniques for solving inverse problems in electrical engineering, concerning their robustness, speed and convergence shows that there is no universal optimisation strategy which can guarantee "the best of the best solutions" and the solution of the optimisation problem usually depends on the researcher's experience and available programme tools. But developments in numerical field analysis methods, as well as powerful computers offer the opportunity to attack realistic problems of technical importance.

### **2.2. Efforts and investigations for improving the models**

The main goal of computer modeling is to help the design engineers to develop high-quality, market competitive products, which ensure effective work with respect to given requirements and limitations. Therefore, the efforts and investigations are focused on the following main directions:

1. Précising the solution of the forward problem. It automatically improves formulation and solution of the inverse problem. It can be realized by:

- Applying for the analysis of the processes in the investigated systems precise numerical methods, taking into account real geometry (2D or 3D) and material properties (linear, nonlinear or presence of hysteresis);
- Solving the problem as a coupled electromagnetic-thermal-mechanical, taking into account mutual dependencies taking place in coupled field modeling.

2. Precising the solution of inverse problem. It can be achieved by:

- Proper formulation of the inverse problem as problem of:
  - optimisation
  - parameter extraction
  - identification
  - synthesis
- Using (according to the specific futures of the investigated device) modern, powerful optimisation methods such as:
  - Simulated annealing
  - Evolution strategies
  - Genetic algorithms
  - Neural networks
  - Response surface supply methodology

3. Experimental investigations and adjustments of the numerical models, obtained during the solution of forward problem. The analysis of the processes and experimental verifications of the mathematical model are carried out for different combinations of the designed parameters, according the Design of experiment theory (DOE).

### **2.3. Specific features of modeling and inverse problem solutions**

The analysis and comparison of different approaches and techniques for solving inverse problems in electrical engineering, concerning their robustness, speed and convergence shows that there is no universal optimisation strategy which can guarantee "the best of the best solutions" and the solution of the optimisation problem usually depends on the researcher's experience and available programme tools. But developments in numerical field analysis methods, as well as powerful computers offer the opportunity to attack realistic problems of technical importance.

A numerical optimisation combined with modern field computation methods numerical methods is found to be an important and powerful engineering tool for the design of electronic and electromagnetic systems. During the past decade important researcher's efforts have been devoted to the development of efficient optimisation techniques, which could amplify the power, brought by modern numerical analysis tools such FEM. Many publications have been reported about successful solutions applying new modern stochastic optimisation technique, combined with FEM and with circuit simulation programs.

In order to select and develop effective approach for solving inverse problems in the region of electrical engineering it is very important to point out the properties of typical optimisation problem and then to estimate the application possibilities.

The main common features of the inverse problems in the region of electrical engineering are:

- Complexity, which means a high number of design parameters.
- Constrained – there are constraints concerning device behaviour, as well as constraints concerning device geometry. Any of them can be determined as equality or inequality constraints.
- The objective function is non-linear.
- Design variables are real.
- There is no derivative information available (interdependencies between design variables and quality function are unknown).
- Type of the optimisation problem is parameter or static optimisation. Nowadays optimisations are performed mainly as static problems. Optimisation of dynamic system behaviour, combined with detailed numerical field and processes analysis will be too time consuming.
- The quality function is disturbed by stochastic errors caused by the truncation errors of numerical field computation.

Generally, thousands of different optimisation methods exist, but unfortunately many of them can be successfully applied only for certain types of problems. Despite their huge number, optimisation methods have many common features and even they can be considered as improved versions of each other. Using these common features many different classifications are possible. The most popular classifications of the optimisation algorithms are:

- Deterministic or Stochastic methods
- Direct or Indirect methods

The main features of the methods and approaches for solving inverse problems have to be analysed regarding the mentioned specific problems, taking into account their:

- Reliability
- Robustness
- Insensibility to stochastic disturbances
- Application range
- Stable solutions
- Performance

The literature observation on this topic shows the following situation: The deterministic methods such as Conjugate Gradient (CG), Newton, Quasi Newton and etc. are classical optimisation methods.

- They are basically local optimisation methods and often are converging to a local minimum

- They are often based on the construction of the derivatives or approximations of the derivative of the objective function. But when using FEM for electromagnetic field analysis it is often difficult to obtain derivatives. It requires human interaction in the optimisation procedure.
- They are very sensitive to stochastic disturbances, especially contained in the derivative information they are based on.
- These gradient based methods are very popular and very effective and converge to the local minimum in a small number of steps when analytical objective function exists.

Stochastic optimisation methods, such as Simulated annealing (SA), evolution strategy (ES) and genetic algorithms (GA) are modern methods, based on statistical analysis.

- These methods utilise random processes to scan the whole parameter space. That is way they can reach the global optima with high probability.
- They accept deterioration in the objective function during the solving process and this enables them to escape local minimum and find the region of the global optimum no matter where the strategy is started from.
- They do not require derivative information
- They are rather simple to implement.
- They are stable in convergence.

The main drawback of these methods is the fact that they need a very high number of function evaluations during the solving process. It means that when applying FEM for electromagnetic field analysis the process becomes too time consuming for the complex realistic problems. There are also reports for combined optimisation strategy. It begins with stochastic method and after localisation of the global optimum region searching process continues with deterministic method. The problem is how to choose the switching point between the two methods.

### **3. EXAMPLES**

#### **3.1. 2D modeling of coupled electromagnetic-thermal field in induction heating device**

The motivation for carrying out the presented work is necessity of development, analyzing and estimation of the efficiency of real induction heating system used for heating of flat details, usually of regular shape – discs of ferromagnetic and nonferromagnetic materials [9]. After heating and plastic deformation, hardened discs are used for producing instruments. For this reason it is necessary to develop system, creating such electromagnetic field, which ensures thermal field with specified distribution. To design such a system it is necessary to analyse electromagnetic and thermal field for number of different possible variants and then choose the proper system parameters. Thus the main goal of the work is precise investigation of the coupled electromagnetic and temperature field distributions for a number of parame-

ters defining different design of the system. Investigations are based on the finite element method (FEM).

The principal geometry of the system is shown in Fig. 1. It consists of disc-type inductor and heated detail. The inductor is multi-sectioned and consists of four sections – first three with three and the last one with two turns. Each turn consists of two square cross section (15mm x 15mm) hollow conductors and 1mm insulation between turns. The inner inductor radius is 35mm. The distance between the inductor and the detail is 7÷15 mm. The frequency is 8 kHz and applied voltage varies from 100 to 700V. The heated detail is ferromagnetic disc with diameter 550÷700mm and height 4÷8mm. The final heating temperature must be 950<sup>0</sup>C.

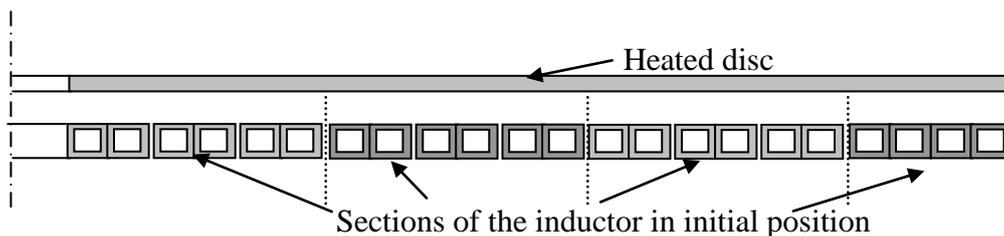


Fig. 1. Principal geometry of the induction system

There is a possibility to change the position of the different sections in z- direction. These changes depend on distances  $h_1$ ,  $h_2$ ,  $h_3$  and  $h_4$ . This design variant is shown in Fig. 2. Thus changing the section positions we can change the field distribution.

Another possibility for obtaining different field distribution is that we can supply different section combinations - for example first, third and fourth sections, or any other possible combination. In this way it is again possible to obtain rather different distribution from the initial temperature field.

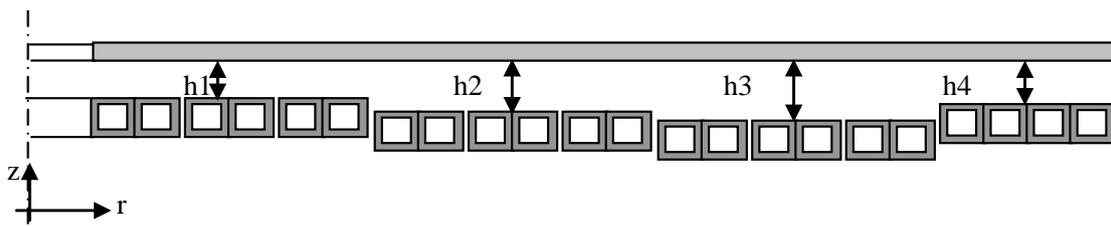


Fig. 2. Variant of possible geometry of the induction system

### ***Coupled Field Model – Theoretical Basis***

The temperature field determination in the considered induction heating systems can be obtained by solving the coupled – quasistatic electromagnetic and transient thermal field problem. Due to the geometry the problem is considered as axisymmetrical one. Numerical simulation of the heating process consists of analysis of time harmonic electromagnetic problem coupled with transient thermal problem, taking into consideration change of physical properties during the heating.

The investigated region is shown in Fig.2. Both the electromagnetic and transient thermal problems are solved in a domain consisting of the whole system and a wide buffer zone around it. It includes domains:  $\Omega_1$ - heated disc,  $\Omega_2$ - inductor,  $\Omega_3$ - buffer zone with air. The time harmonic electromagnetic field is modelled by equation:

$$\nabla_{\mathbf{x}}(\mu^{-1}\nabla\vec{A}) = \vec{J} - \sigma \frac{\partial\vec{A}}{\partial t} \quad (1)$$

$\vec{A}$  is magnetic vector potential,  $\sigma$  is electric conductivity,  $\mu$  is magnetic permeability,  $\vec{J}_e$  is current density. Taking into account that  $\vec{A} = A_{\varphi}(r, z)\vec{1}_{\varphi}$  and  $\vec{J}_e = J_{\varphi}(r, z)\vec{1}_{\varphi}$  the Eq. 1 can be written as:

$$\nabla_{\mathbf{x}}(\mu^{-1}\nabla A_{\varphi}) = J_{\varphi} - j\omega\sigma A_{\varphi}, \quad (2)$$

where  $\omega$  is angular current frequency. The boundary conditions are  $A_{\varphi} = 0$  for the axis of symmetry b1, as well for the buffer zone boundaries b2, b3 and b4.

The time varying electromagnetic field produces eddy currents  $\vec{J} = j\omega\sigma\vec{A}$  and corresponding Joule losses – source of the heating in the region:

$$Q = \frac{[\sigma]^{-1}\vec{J}\vec{J}^*}{2} \quad (3)$$

The transient thermal field is modeled by equation:

$$\rho C \frac{\partial T}{\partial t} + \nabla(-k\nabla T) = Q \quad (4)$$

where  $k$  is thermal conductivity,  $\rho$  is density, and  $C$  is heat capacity.

The boundary conditions are imposed:  $\frac{\partial T}{\partial n} = 0$  along the symmetry line b1; Dirichlet boundary conditions  $T = 20^{\circ}\text{C}$  along the buffer zone boundary b2, b3 and b4. The water cooling of the inductor along boundaries b5 was taking into account using convection boundary condition:

$$-k \frac{\partial T}{\partial n} = h(T_{\text{inf}} - T), \quad (5)$$

–  $h$  is heat transfer coefficient,  $T_{\text{inf}}$  is the external temperature. The initial temperature is  $T_0 = 20^{\circ}\text{C}$ .

### ***FEM Analysis***

Numerical investigations of coupled field distribution have been made using FEM and COMSOL 3.3 software package [11]. The change of physical properties during the heating process is possible by using Materials/Coefficients library in the software.

Study was carried out when varying different design parameters, like distances between inductor sections and heated detail, supply voltage and supply inductor sections. Analysis of the results gives possibilities to make reasonable choice of design parameters. Some results for the field distribution during the heating process for the system with initial design ( $h_1=h_2=h_3=h_4$ ), are given in Fig. 3, Fig. 4 and Fig. 5. Temperature distribution after 200 seconds heating is shown in Fig. 3. In Fig. 4 temperature increase during the heating process is observed in case of supply of all four sections. The temperature in radial direction is nonuniform, has one maximum and varies from 600 to 1080°C.

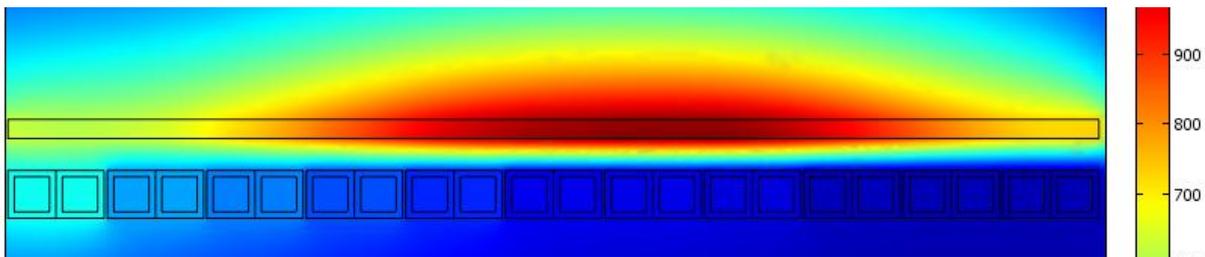


Fig. 3. Temperature distribution for the initial ( $h_1=h_2=h_3=h_4$ ) geometry of the system,  $t = 200$  sec.

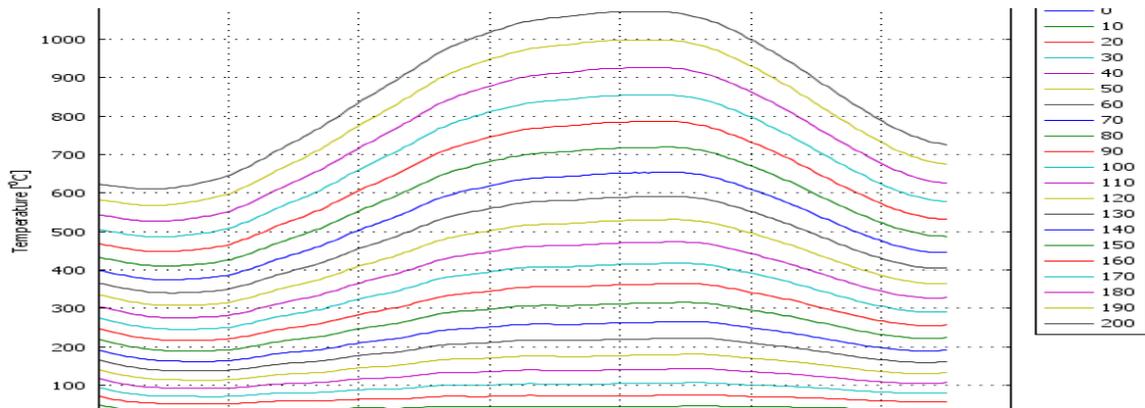


Fig. 4. Temperature increase during 200 seconds heating process ( $t=10,20,30,\dots,200s$ )

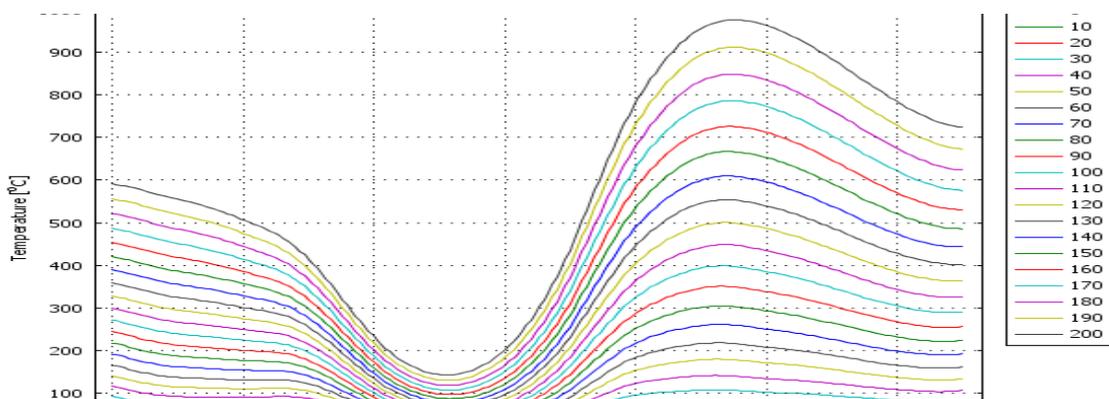


Fig. 5. Temperature increase during 200 seconds heating process ( $t=10,20,30,\dots,200s$ ).  
Second section of the inductor is not supplied.

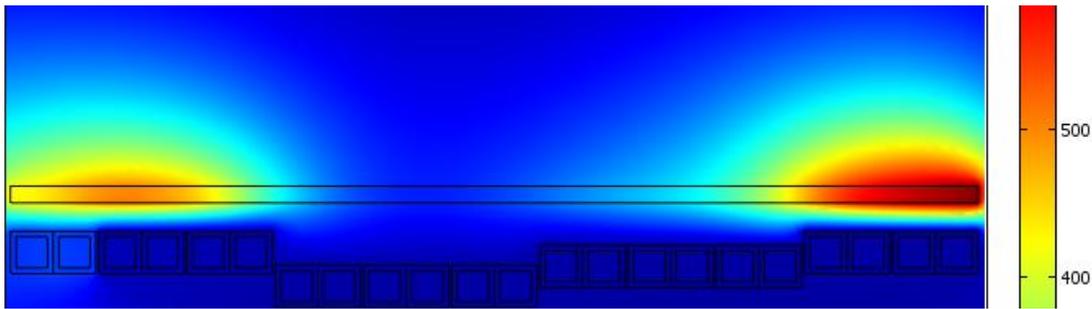


Fig. 6. Temperature distribution at moment  $t=200$ s. The inductor sections are in different positions ( $h_1 \neq h_2 \neq h_3 \neq h_4$ )

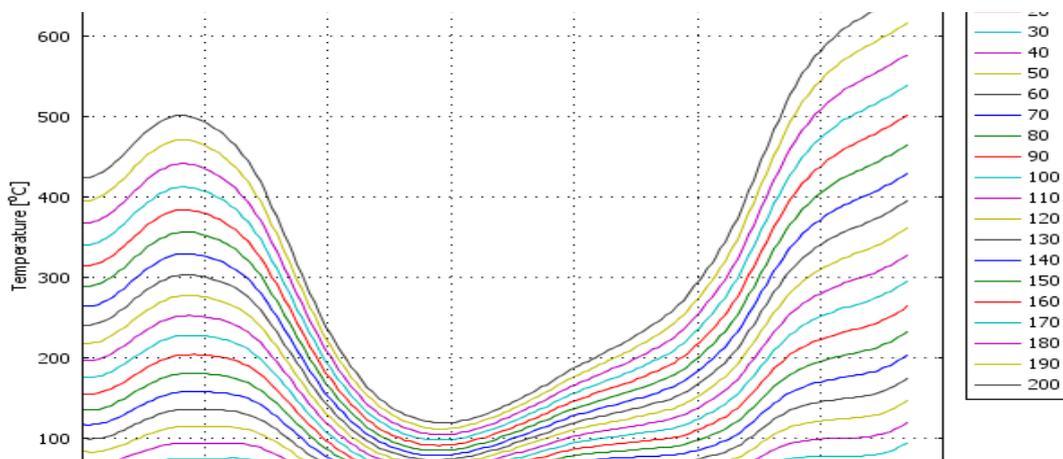


Fig. 7. Temperature increase at different moments during 200 seconds heating process when inductor sections are in different positions ( $h_1 \neq h_2 \neq h_3 \neq h_4$ )

In Fig. 6 the process of heating is shown, when inductor sections are again in the initial position, but second section of the inductor is not supplied. Temperature distribution for moment  $t=200$  second in case of different distances between inductor sections and heated detail ( $h_1 \neq h_2 \neq h_3 \neq h_4$ ) is shown in Fig.6. In Fig.7 process of disc heating for different moments  $t=0 \div 200$  seconds can be observed. It is evidently that heating process and its result at moment  $t=200$  seconds is quite different for this different design.

### 3.2. 3D modeling of coupled electromagnetic-thermal field in induction heating system for high frequency welding

The aim of the research is investigation of induction heating system used for high frequency longitudinal pipe welding [10]. The main task is to determine optimal factors and parameters influencing on quality of the welding process and required energy: frequency, welding speed, 'vee' angle, ferrite impeder presence (inner and outer).

The principal geometry of the investigated system is shown in Fig. 8. It consists of high frequency spiral inductor, which induced a voltage across the edges of the

open steel pipe. The induced voltage causes high frequency currents, concentrated on the surface layer due to the skin and proximity effects. The currents flow along the edges to the point where they meet, causing rapid heating of the metal. The weld squeeze rolls are used to apply pressure, which forces the heated metal into contact and forms welding bond. Inner ferrite impeder is also used, which concentrates magnetic flux and improves the welding efficiency.

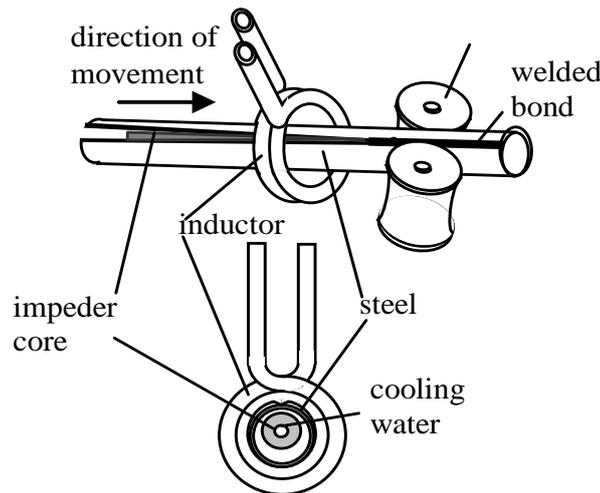


Fig. 8. Geometry of the investigated induction system

The parameters of investigation are: applied current -1000A; voltage - 500V;  $\cos\varphi = 0,1$  and  $f = 200 \text{ kHz} \div 500 \text{ kHz}$ . The end heating temperature must be about  $1300 \div 1450^\circ\text{C}$ . The cooling water flowing inside the conductors is with temperature  $40^\circ\text{C}$ .

The field problem is considered as coupled - electromagnetic and thermal. The electromagnetic field is analysed as time harmonic and eddy current losses in conductive parts of the system are considered as heat sources in analysis of transient thermal field problem. Both the electromagnetic and transient thermal problems are solved in a domain consisting of the whole system and a wide buffer zone around it. The investigated region is shown in Fig.9. It includes domains:  $\Omega_1$ - inductor;  $\Omega_2$ - impeder;  $\Omega_3$ - welded pipe;  $\Omega_4$ - cooling water;  $\Omega_5$ - buffer zone with air.

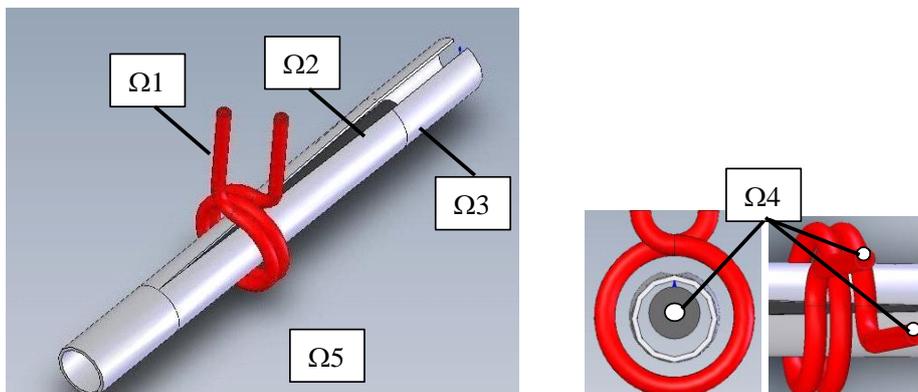


Fig. 9. Investigated region

### FEM Analysis

3D-coupled electromagnetic and temperature field problem and has been solved using finite element method and COMSOL 4.2 software package [11].

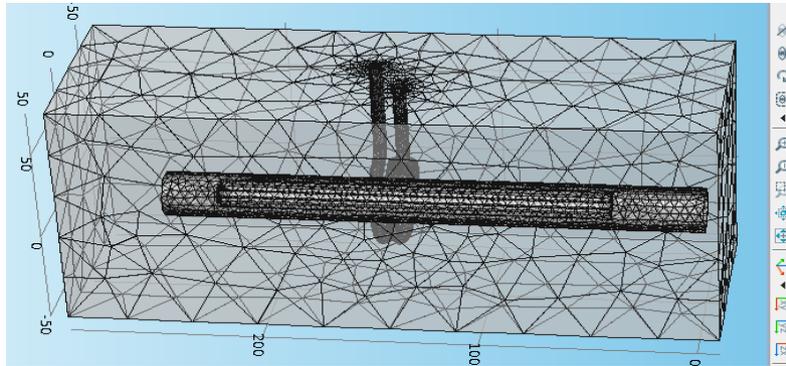


Fig. 10. 3D FE mesh in the investigated region

The finite element mesh used in investigation is shown in Fig. 10 and current density distribution in two specific for the problem cross sections in Fig. 11. The obtained temperature value around the “point of closure” (Fig. 12) is about 1400 0C.

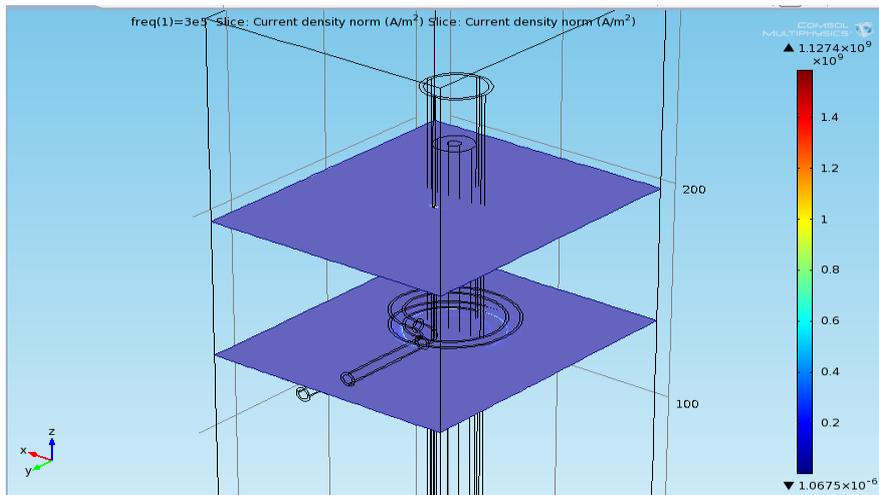


Fig. 11. Current density in two cross sections: around “point of closure” and spiral inductor.

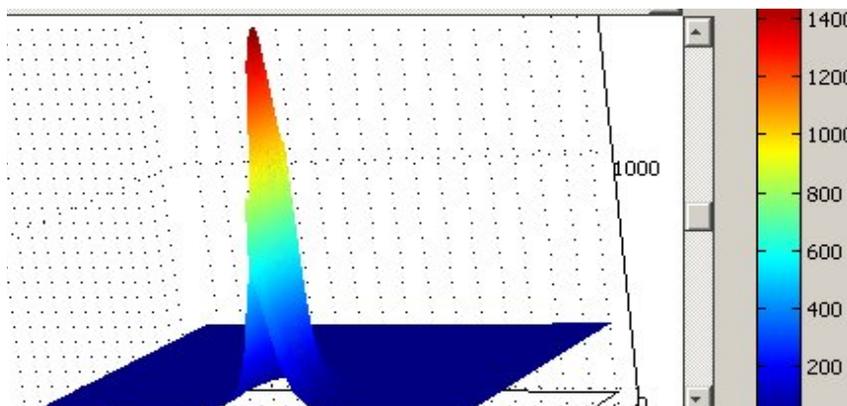


Fig. 12. Temperature increase around the “point of closure”

#### 4. CONCLUSION

The paper presents the main steps, problems and possibilities in numerical modelling and optimisation of the high technology, energy-efficient and reliable electromagnetic devices that do not harm the environment. The development and improvement of such systems is based on profound, complex field study and subsequent optimisation. Such modelling gives possibility for determination and control of the parameters and characteristics of the systems even in inaccessible for experimental investigation areas.

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