SYNTHESIS AND ANALYSIS OF A MEMRISTOR-BASED RECTIFYING CIRCUITS IN MATLAB AND SIMSCAPE

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Abstract: *The main purpose of this paper is to propose a new memristor-based rectifying circuits for hard-switching mode. It is specified that when the memristor operates in hard-switching mode for high-level signals with low frequency then the current-voltage characteristics are non-symmetrical and the element can rectify sine-wave and other bipolar signals. For hard-switching mode the state variable of the memristor element reaches the boundaries of the memristor and the state-flux characteristic is multi-valued. The rectifying properties of the memristor element depends primarily on the ratio between the resistances of the memristor in fully-open and fully–closed states. For the analysis of several rectifying memristor-based circuits a modified MATLAB and SIMSCAPE memristor model with nonlinear dopant drift is proposed. It is based on the Joglekar Model and on Boundary-Condition Memristor Model (BCM) respectively.*

Keywords: *memristor, window function, rectifying circuit, hard-switching mode*

1. INTRODUCTION

The memristor element was predicted in 1971 by Prof. Leon Chua as the fourth nonlinear circuit element [1]. Its prototype based on titanium-dioxide was invented in 2008 by Stanley Williams in the HP research labs [2]. Several prototypes based on different technologies were also invented – like polymeric, spintronic and other realizations [2, 3, 4, 5]. The asymmetric current-voltage relation for hard-switching mode is a precondition for investigation of the memristor-based rectifiers [6, 7, 8, 9].

In Section 2 a modified memristor model with nonlinear dopant drift appropriate for simulating hard-switching mode is proposed and the pseudo-code based algorithm for its description is presented. In Section 3 two rectifying circuits based on the classical diode realizations are analyzed using the new SIMSCAPE memristor model, based on the previous pseudo-code, and the new rectifying circuits are compared with their diode analogs. In Section 4 the concluding remarks are presented.

2. A NEW MEMRISTOR MODEL WITH NONLINEAR DOPANT DRIFT

The structure scheme of the Williams' memristor is presented in Fig. 1. The length of the doped region is denoted with *w* and the length of the memristor is *D* [2].

The state variable *x* is equal to the ratio between the length of the doped region *w* and the length of the whole memristor *D* [2]:

$$x = \frac{w}{D} \tag{1}$$

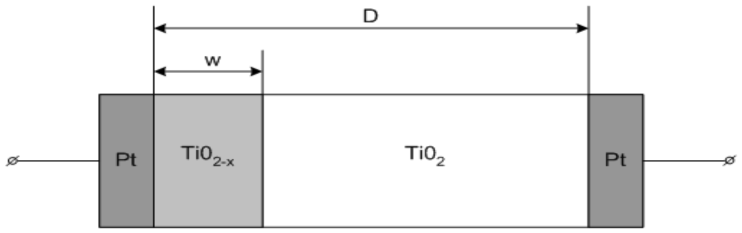

Fig. 1. Structure scheme of the Williams' memristor element

The minimal value of the state variable x according to [2] and [3] should have been x_{min} = 0. This announcement will be mathematically true if the minimal length of the doped region is w = 0. One of the new ideas in the present paper is based on physical and chemical considerations. Actually when we apply a voltage in reverse direction for a long time then the doped region will shrink but the oxygen vacancies will remain in the same section. The oxygen vacancies are generated in the initial process of electroforming the memristor and they cannot vanish. Due to this the length of the doped region could have a very small minimal length but this length cannot be zero. So the minimal value of the state variable x will have, for example, a value of x_{min} = *0.05*. The maximal value of the state variable x_{max} could be equal to unity. The last statement is physically possible because the oxygen vacancies could reach the right boundary of the memristor. According to this reason we have: $x \in [x_{\min}, 1]$. The derivative of the state variable x with respect to time t is [2, 3]:

$$\frac{dx}{dt} = kif\left(x\right) \tag{2}$$

The coefficient k depends on memristor parameters and it is [2]:

$$k = \frac{\mu R_{ON}}{D^2} \tag{3}$$

The quantity R_{ON} = 100 **Ω** is the resistance of the memristor in a fully closed state. The quantity μ is the average value of the oxygen vacancies mobility. It could be different for the special realizations and materials of the memristor element. The window function used in (2) is proposed by Joglekar [3]:

$$f\left(x\right) = 1 - \left(2x - 1\right)^{2p} \tag{4}$$

The Joglekar window function in the nonlinear ionic drift memristor model is used for presenting the nonlinear relationship between the velocity of the oxygen vacancies v and the memristor current i for high-intensity electric field. The exponent in this case is p = 1. If we choose p > 1 the problem is analytically unsolvable but only in Gaussian hyper-geometric forms. The basic current-voltage relation is [2]:

$$v = Mi = \left[R_{ON} x + R_{OFF}\left(1 - x\right)\right] i \tag{5}$$

The quantity M is called "memristance" and it presents the resistance of the memristor. The quantity $R_{OFF} = 16\ k\Omega$ is the resistance of the element in a fully open state. Actually $R_{OFF} >> R_{ON}$. Then the following approximation is used for (5) [2]:

$$R_{ON} - R_{OFF} \approx -R_{OFF} \tag{6}$$

After expressing the current i from (2) and substituting in (5) we obtain the basic differential equation of the memristor:

$$vdt = \frac{R_{OFF}}{4k} \frac{1}{x} dx \tag{7}$$

The solution of (7) with respect to state variable x is:

$$x = x_0 \exp\left[\frac{4k}{R_{OFF}} \int_{t_0}^{t} v(t')\, dt' \right] \tag{8}$$

It is important to denote that formula (8) is valid for the soft-switching operating mode. Then the state variable x does not reach its limit values: $x \in (x_{\min}, 1)$. The memristor element operates in soft-switching mode when the memristor voltage has a low-level magnitude and comparatively high frequency. The state-flux characteristic of the memristor element for this mode is a single-valued function and the operating point is moving on the curve described by (8). The current-voltage relation is symmetrical. Let's the initial value of the state variable is x_0. When we have a signal with high magnitude and low frequency the flux linkage has high level. Then the state variable x could reach its limiting positions [5, 6, 7]. The memristor element operates in the so called hard-switching mode. For this mode the current-voltage characteristic is asymmetrical. Follows the pseudo code algorithm for simulating the memristor model with nonlinear dopant drift, based on Eq. (1) – (8). In the code presented below the basic memristor and voltage source parameters are given. The basic formula used for the code generation is the differential equation (7). For the pseudo-code the differential of time dt is substituted with the limited step value Δt.

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Procedure: A modified memristor model with nonlinear dopant drift
 Given: Voltage source with instantaneous value v(t)
 Given: The duration of the simulation T = tmax - tmin
 Given: The number of samples N
 Given: The sample step time: deltat = T / N
 Given: Memristor parameters: Ron = 100; Roff = 16e3;
        mu = 1e-13; D = xmax = 10e-9; xmin = 0.05;
        k = mu*Ron/(D^2); const = (4*k) / Roff;
 Given: The initial value of the state variable x0
      set x = x0  for t = 0
       for t = tmin:deltat:tmax
          psi = int(v(t)dt,-∞,t);
       for n=1,  x1=x0;
       for n=2:1:N+1, x1(n)=x1(n-1)+const*u(n-1)*x1(n-1)*deltat;
         if x1(n)<=xmin  AND  u(n)<=0,  then  x1(n)=xmin;
         if x1(n)>=xmax  AND  u(n)>=0,  then  x1(n)=xmax;
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            else  x1(n)=x1(n-1)+const*u(n-1)*x1(n-1)*deltat;
          end cycle
         return u,i,psi, x, M;
end procedure
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Verifying the modified memristor model in MATLAB

The state-flux characteristic for hard-switching mode in given in Fig. 2. The current-voltage relation of the memristor is presented in Fig. 3.

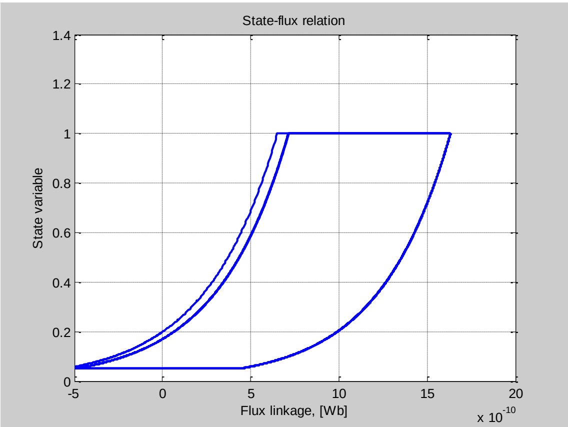

Fig. 2. State-flux relation for hard-switching mode

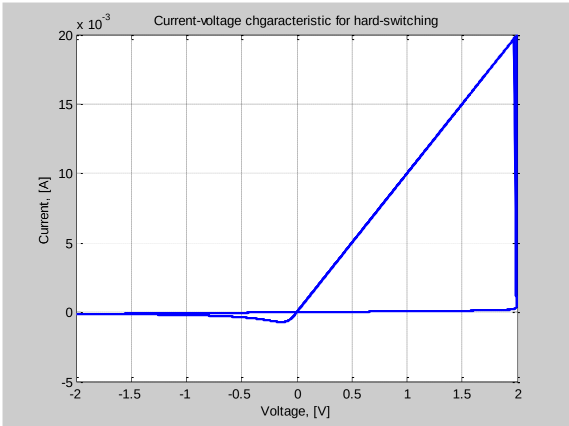

Fig. 3. Current-voltage relationship for hard-switching mode

The state-flux and current-voltage characteristics are compared with previous-published results in [3, 4, 6] and the comparison verifies the abilities of the modified memristor model for using both in soft-switching and hard-switching simulation.

3. ANALYSIS OF MEMRISTOR-BASED RECTIFYING CIRCUITS IN MATLAB

The classical half-wave diode rectifier with a capacitive filter is given in Fig. 4. The new memristor-based half-wave rectifier is presented in Fig. 5. The SIMULINK model of the circuit presented in Fig. 5 is given in Fig. 6. The SIMSCAPE modified memristor library model is based on the pseudo-code presented above. The SIMULINK model of the diode rectifier is similar to the model of the memristor rectifier [6, 7, 8, 9]. The capacitors have a capacitance of 100 μF and the resistors have a resistance of 2,2 $k\Omega$.

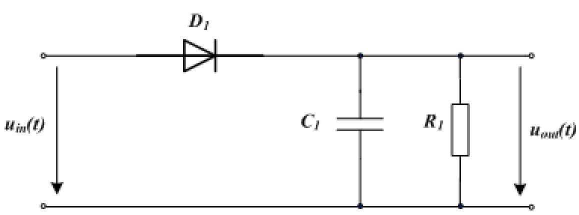

Fig. 4. A half-wave diode rectifier

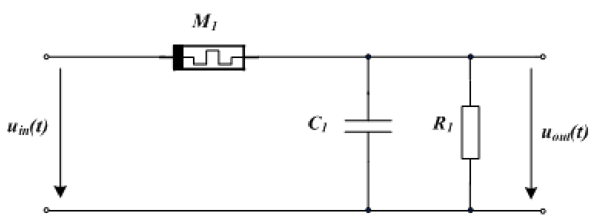

Fig. 5. A half-wave memristor rectifier

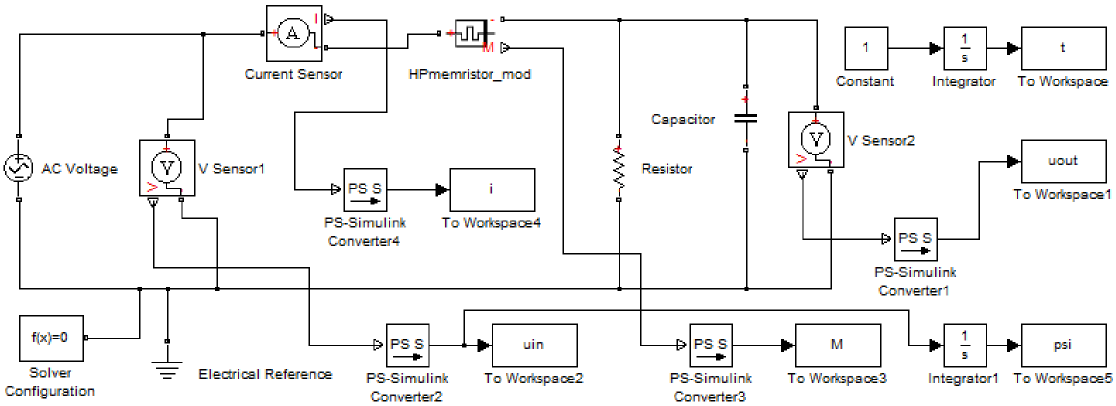

Fig. 6. SIMSCAPE and SIMULINK schematic of the circuit given in Fig. 5

The time diagrams of the output voltage for the diode and memristor rectifying circuits are given in Fig. 7. It is clear that the transient in the memristor circuit has a long duration and the output voltage is less than the output voltage of the diode rectifier. The memductance-flux relation of the memristor element is given in Fig. 8.

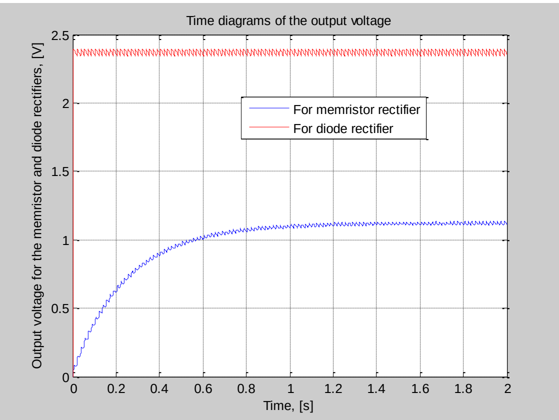

Fig. 7. Time diagram of the output voltage of the half-wave rectifying circuits

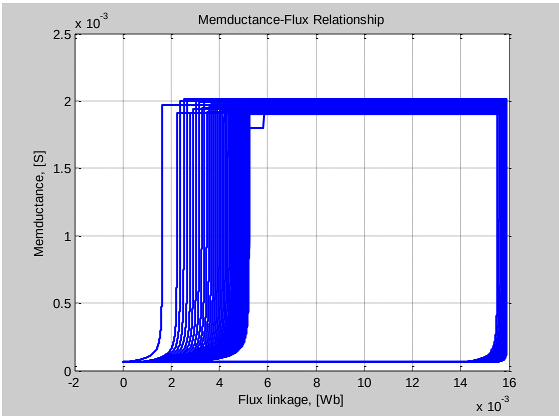

Fig. 8. Memductance-flux relationship for hard-switching mode

Follows the Graetz bridge rectifying circuits with diodes and memristors, which circuits are given in Fig. 9 and Fig. 10, respectively [8, 9]. The capacitors have a capacitance of 100 μF and the resistors have a resistance of 2,2 $k\Omega$. The output voltages for sine wave input signal with amplitude 10 V are given in Fig. 11.

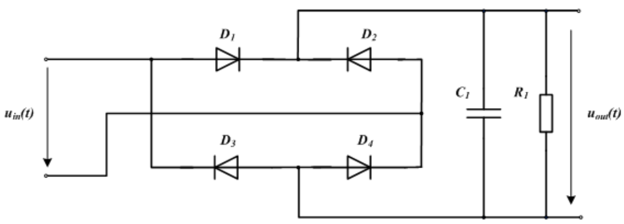

Fig. 9. A Graetz rectifying circuit with diodes

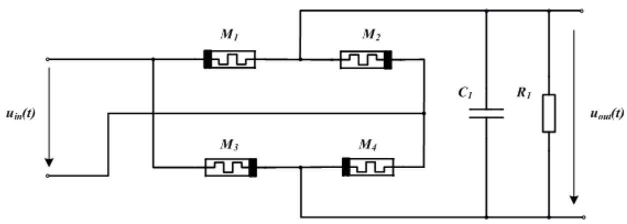

Fig. 10. A Graetz rectifying circuit with memristors

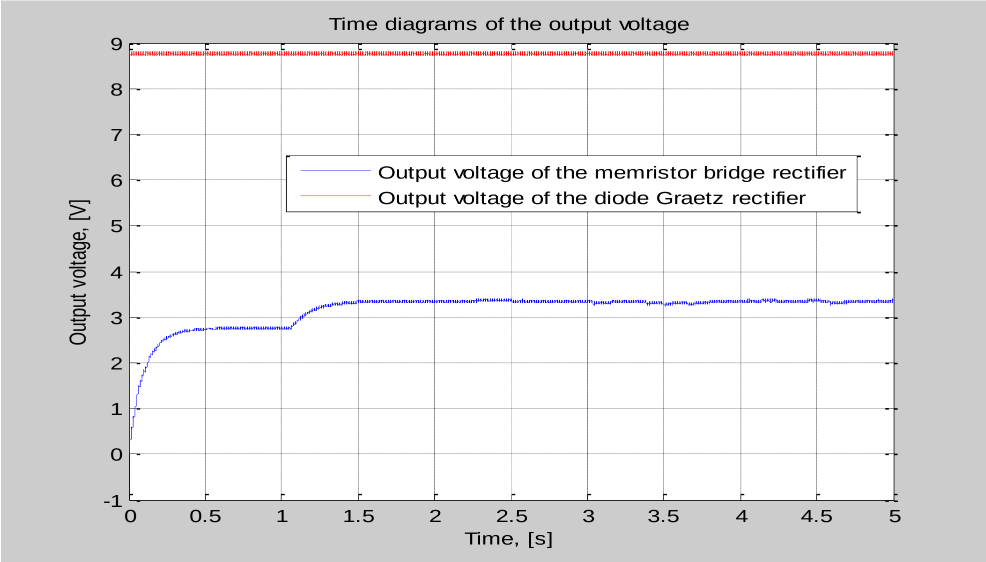

Fig. 11. Time diagrams of the output voltages of the Graetz rectifying circuits

4. CONCLUSIONS

The results from the simulations of the memristor-based circuits presented above confirm that the memristor elements could be used in rectifying circuits if they are operating in hard-switching mode. Really the ratio between the high-resistance and low-resistance states for the memristor is less than the same ratio for the classical semiconductor diode. The output voltage of the memristor-based rectifiers has about two times less value than the value of the output voltage of the diode rectifiers. But the memristor element has several different advantages – good scalability and possibility for integrating with the CMOS technology.

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References

[1] Chua, L. O. *Memristor – The Missing Circuit Element*. IEEE Trans. on Circuit Theory, Vol. CT-18, pp. 507-519, September (1971), DOI 10.1109/TCT.1971.1083337.

[2] Strukov, D. B., G. S. Snider, D. R. Stewart, R. S. Williams. *The missing memristor found*. Nature, 06932, Vol. 453, pp. 80 – 83, (2008), DOI:10.1038/nature06932.

[3] Joglekar, Y., S. Wolf. *The elusive memristor – properties of basic electrical circuits*. European Journal of Physics, 30, pp. 661 – 675, (2009), DOI:10.1088/0143-0807/30/4/001.

[4] Corinto, F., A. Ascoli. *A boundary condition-based approach to the modeling of memristor nanostructures*. IEEE Transactions on circuits and systems – I, Regular papers, Vol. 59, Issue 11, ISSN 1549 – 8328, pp. 2713 – 2726, November (2012).

[5] Ascoli, A., R. Tetzlaff, F. Corinto, M. Gilli. *PSpice switch-based versatile memristor model*. IEEE, 2013, pp. 205 – 208.

[6] "Simscape." [Online]. Available: http://www.mathworks.co.uk/ products/simscape/.

[7] Walsh, A., Carley, R., Feely, O., Ascoli, A. *Memristor circuit investigation through a new tutorial toolbox*. 2013 European Conference on Circuit Theory and Design (ECCTD), DOI: 10.1109/ECCTD.2013.6662261, pp. 1 – 4.

[8] Stefanov, N. *Power-supply devices*. S., Technica, 2005, ISBN 954 – 03 – 0564 – 0.

[9] Pabst, O., T. Schmidt. *Sinusoidal analysis of memristor bridge circuit: rectifier for low frequencies*. Arxiv, February 2013, pp. 1 – 6.

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