DIAGNOSIS OF CHAOTIC PROCESSES IN NONLINEAR DYNAMICAL SYSTEMS

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Abstract. *The paper presents an algorithm proposed for diagnosis of chaotic process in a nonlinear dynamic system. The algorithm is illustrated by examination of a classical Lorenz system.*

Keywords: *nonlinear dynamical systems, chaotic processes, phase portrait, chaotic attractor*

1. INTRODUCTION

To study the irregular processes in nonlinear dynamical systems (NDS) different methods are used: physical experiments, numerical and analytical methods, computer modeling and simulation. Some of them are related to visual diagnosis of the process [1,2] and are applicable only at initial diagnosis and prediction of chaotic behavior.

More accurate evaluation of the process is given by analytical and numerical methods [3], the result is objective and convenient also for creating algorithms for synthesis of chaotic systems.

Undoubtedly, the diagnosis methods and the choice of diagnostic criteria depend on the particular system and tasks assigned for solving. It is typical for all of them that they are built based on the main properties of chaotic fluctuations [5,6].

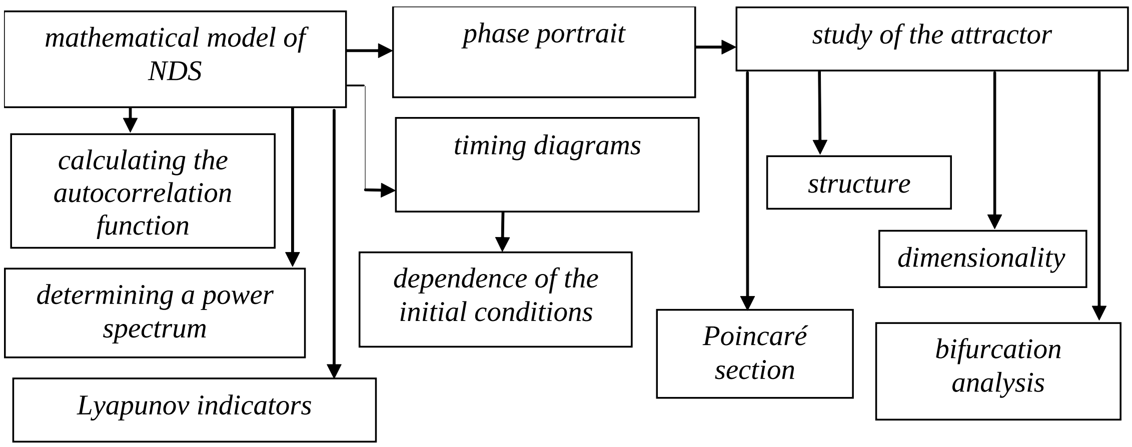

Fig. 1

Considering these properties, the research methods of the NDS, given in Fig. 1, are systemized and the paper presents an algorithm proposed for the diagnosis of chaotic processes in NDS. The algorithm is illustrated on the basis of the Lorenz system [4], which is a classical example of the existence of chaotic fluctuations.

2. ALGORITHM FOR DIAGNOSIS OF CHAOTIC PROCESSES IN NDS

The examination on the nature of the processes in NDS starts with construction of a phase portrait as shown in Fig. 2.

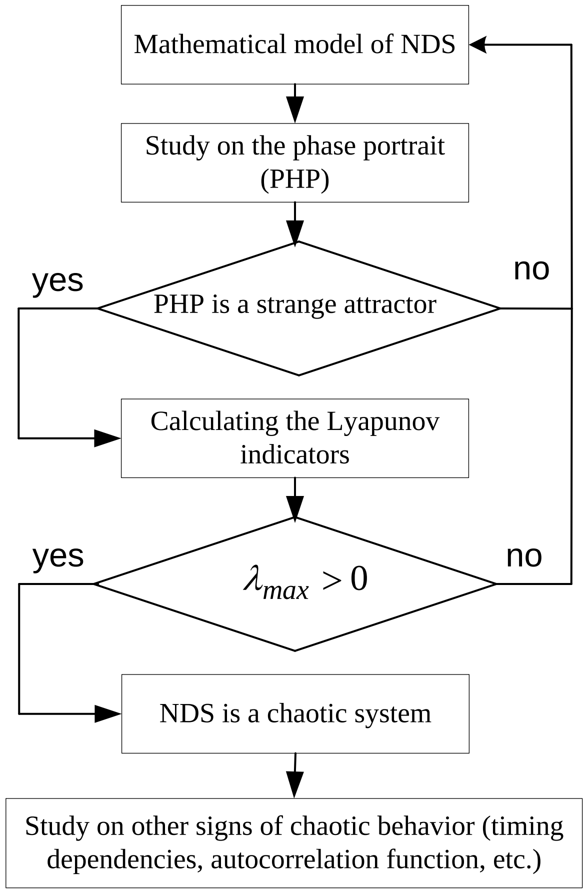

Fig. 2

The analysis of the phase portrait allows to judge about the topological structure of the chaotic process. Here it is useful to examine also the qualitative transformations of the phase portrait in altering parameters of the examined system, i.e. the bifurcation diagram. In regard to the attractor characteristics, it is appropriate to determine also its dimensionality, as the fractal dimensionality [6] indicates for existence of a fractal structure, i.e. "Strange" attractor.

Undoubtedly, however, the most important feature of the attractor is the presence of Lyapunov indicators and their spectrum [5]. They determine the stability of the attractor orbit. The presence in the spectrum of minimum one positive Lyapunov indicator means instability of the examined phase trajectory. The largest Lyapunov indicator being positive is a necessary and sufficient condition for chaotic of the system [5].

After examining NDS by these basic criteria and detecting the signs of chaotic behavior, the other properties of irregular processes are tracked as well: the nature of time dependencies, sensitivity to initial conditions and changes of the autocorrelation function.

3. ILLUSTRATIVE EXAMPLE

The proposed algorithm for diagnosis of chaotic regime is discussed on the basis of the Lorenz system with classical parameters $\sigma = 10$, $\rho = 28$ и β =8/3 [4]:

$$\begin{aligned} \dot{x} &= \sigma(y - x) \\ \dot{y} &= \rho x - y - xz \\ \dot{z} &= xy - \beta z \end{aligned} \tag{1}$$

For the purpose of research, a simulation model is developed in Matlab/Simulink environment (Fig. 3) to obtain the phase portrait (Fig. 4) and timing diagrams (Fig. 6).

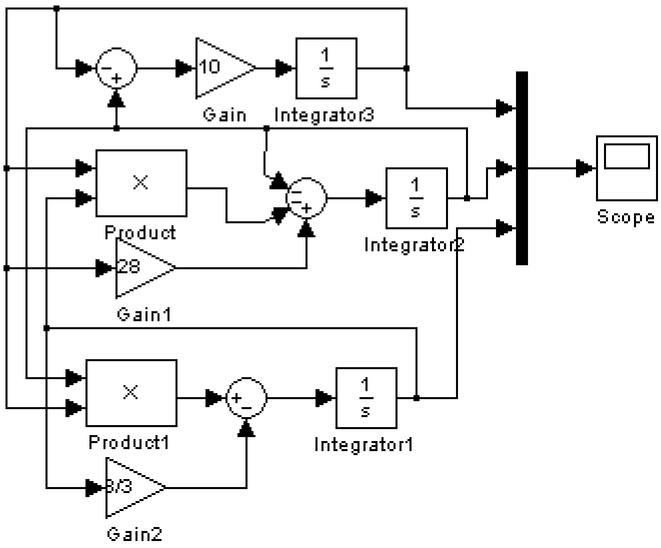

Fig. 3

The spectrum of Lyapunov indicators is calculated using Matlab and shown in Fig. 5. The chaotic of the system is confirmed with the existence in the spectrum of maximum positive indicator $\lambda_{max} > 0$.

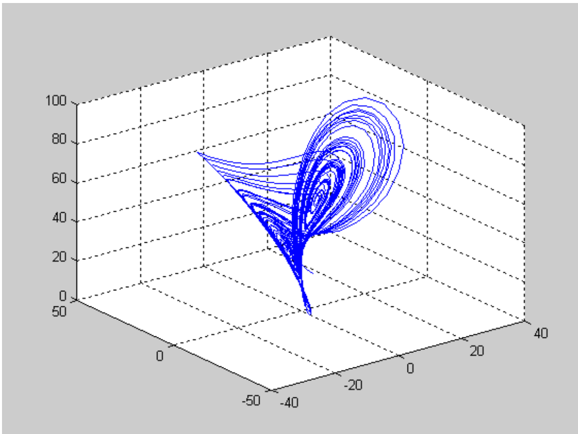

Fig. 4

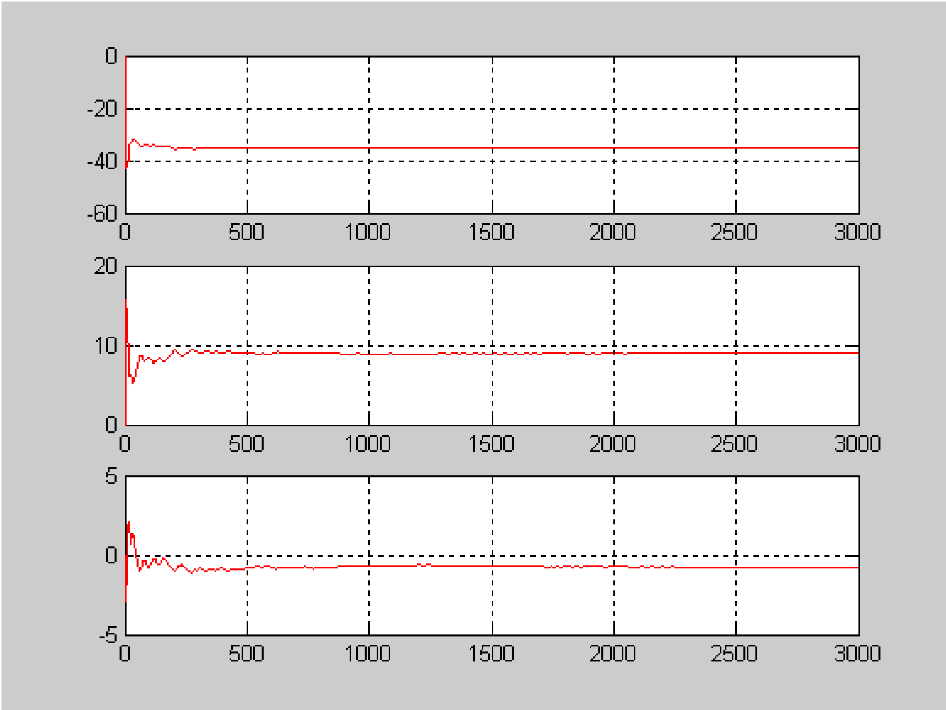

Fig. 5

Fig. 6 shows the timing diagrams illustrating the complete lack of periodicity in the amplitude of phase variables *x, y, z*.

One of the most significant features of the chaotic process, sensitivity to initial conditions, is evident by the dependencies shown in Fig. 7 where the solution with two different initial values is given: $x_1(0)=10$, $x'_1(0)=9$.

Through numerical study in Matlab, the amendment of the autocorrelation function (Fig. 8) for the variable *x* of system is built (1). It can be seen that the autocorrelation function strongly decreases, which is another evidence for chaotic of the process.

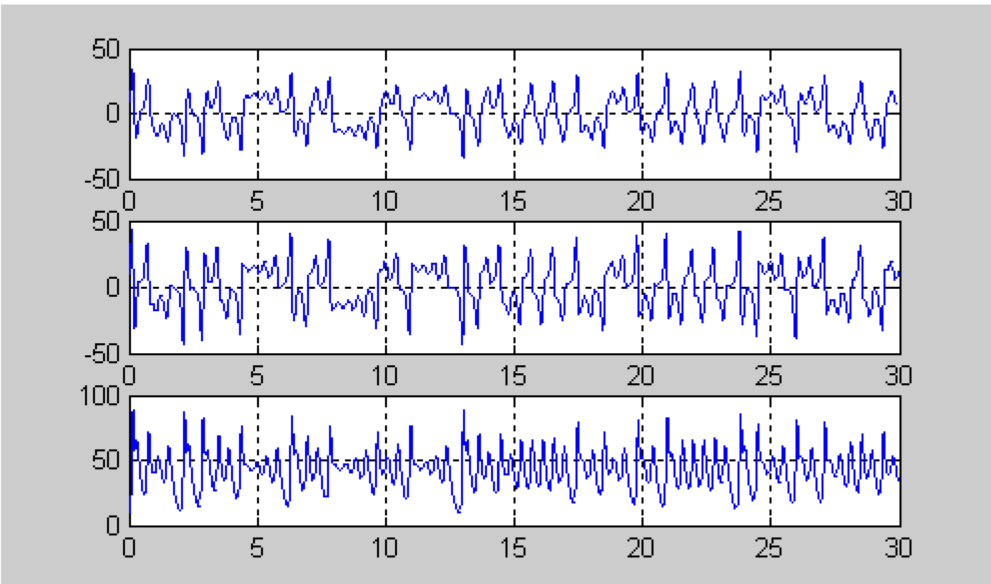

Fig. 6

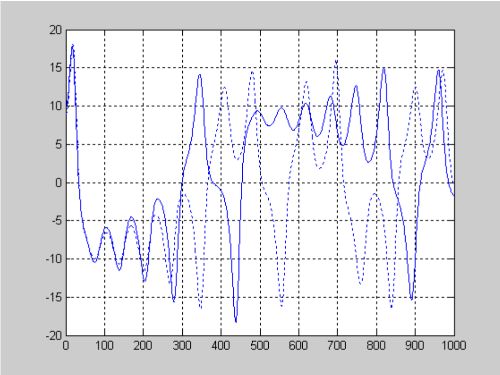

Fig. 7

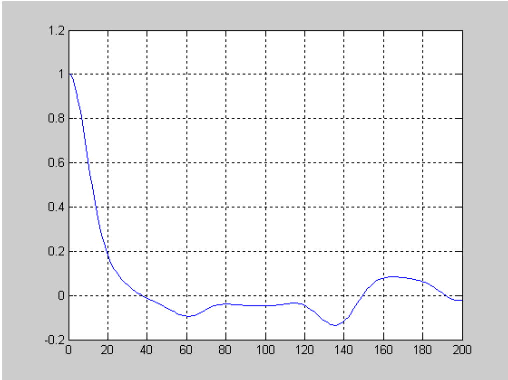

Fig. 8

4. CONCLUSION

While studying the chaotic in NDS, it is not necessary to check all criteria and features of the irregular processes, since many of them are interchangeable. The complex of diagnostic methods for a chaotic process is sufficient to include examination on the phase space and calculation of the spectrum of Lyapunov indicators as illustrated by the algorithm proposed in this paper.

References

[1] Cherneva G., E. Dimkina, A Pspice-aided Simulation and Examination of Lorenz-Based Chaotic Circuit, XXXIV International Conference on Fundamentals of Electrotechnics and Circuit Theory IC-SPETO-2011, 18-21.05.2011, Gliwice, Poland, p.73-74.

[2] Cherneva G., E. Dimkina. Simulation and Examination of a Signal Masking Chaotic Communication System, Based on the Duffing Oscillator. Journal „Communications”. Zilina, Slovakia. ISSN 1335-4205. № 2A/2013. Vol.15, pp. 6-10.

[3] D.W. Jordon and P. Smith. Nonlinear Ordinary Differential Equation. N.Y. Oxford University Press, 1987.

[4] Baghious E. H., P. Jarry, Lorenz attractor from differential equations with piecewise-linear terms, International Journal of Bifurcation and Chaos, Vol. 3, No. 1,1993

[5] Baker G. L. and J. G. Gollub: Chaotic dynamics – an introduction, Cambridge University Press, 1996.

[6] Бутенин Н. В., Неймарк Ю. И., Фуфаев Н. А. Введение в теорию нелинейных колебаний. М. Наука, 1976.

[7] Морозов А. Д., Драгунов Т. Н. Визуализация и анализ инвариантных множеств динамических систем. Москва, Ижевск: Институт компьютерных исследований, 2003.

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