

# MODELING OF SENSORS OF ELECTRICALLY CONDUCTIVE ELASTOMERS

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**Abstract.** *This article clarifies the key issues in the development of models of sensors of electrically conductive elastomers (SECE). Mathematical models of SECE are composed operating at direct and alternating current mode. Based on them two simulation models are realized in Matlab environment. These models allow study of the properties of the sensors and the impact of input influences prior to their practical realization. This article offers an analysis of the results obtained in the simulations and experimentally. The article aims to support the construction works of sensors to which are applied ever – increasing requirements for improvement of their metrological characteristics.*

**Keywords:** *elastomers, strength measuring sensors, mathematical models, simulations*

## 1. INTRODUCTION

With the development of electronic technology and the increasing desire to replace manual labor with automated production activities, the design and implementation of specialized industrial robots is required. To properly perform their tasks the robots need a large amount of different types of sensors.

A number of requirements are placed on the sensors: small size and weight, high reliability, sensitivity, repeatability of statements, minimum equipment costs, immunity from interference and aggressive and moist environments, thermal stability, etc [5].

The tactile sensors are a special class of sensors detecting force and pressure that are characterized by their compactness and low cost.

The most commonly used construction of tactile sensors with electrically conductive rubbers (elastomers) is the "sandwich" type (fig. 1). In it the elastomeric material is placed between two metal electrodes. This structure is embedded as the base used in test samples and their models. The pressure is applied statically, perpendicular to the sensor, by means of a suitable mechanical system and weights, and the volume resistance of the material changes in result. We are rarely interested in surface resistance. It is measured and constitutes one of the criteria for quality in production of elastomeric blends [1].

The purpose of this work is the mathematical modeling of sensors of electrically conductive elastomers (SECE) and the creation of simulation models.

Matlab Simulink programming environment allows the description and study of the selected object. The model, thus constructed, allows investigation of the sensor prior to its practical implementation, which shortens the design time [3].

## 2. THEORETICAL JUSTIFICATION

The used electrically conductive elastomers are polymers, for which glass transition temperature is below the ambient (room) temperature, their melting point is above the ambient temperature and they are in a highly elastic condition. They have properties similar to those of natural rubber and are known as "rubbers".

All real objects under the influence of external forces change their shape and volume. The aspiration of the objects to recover their volume and shape is called elasticity [2].

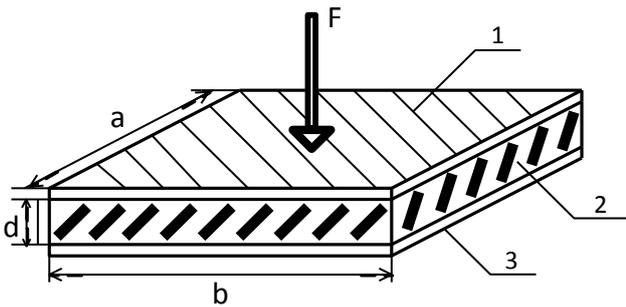


Fig. 1. Structural Scheme of SECE

Fig. 1 shows the structure of the SECE. The external impact -the slowly changing static force  $F$ - is applied perpendicular to the metal plates. With increasing the force  $F$  the active internal resistance of the sensor  $R_r$  reduces, which is due to the reduction in the volume resistance and to the increase of the contact area between the electrodes 1 and 3, and the elastomer 2. The dependence  $R_r = f(F)$

is nonlinear. For its analytical display the theory of the elastic properties of bodies is used [6].

For an evenly deformed object from perpendicular directed force  $F$ , the elastic forces are evenly distributed and directed perpendicular to the entire cross-section  $S$ .

Then  $\sigma = \frac{F}{S}$  is called normal elastic stress.

The dimensionless parameter  $\varepsilon = \frac{\Delta d}{d}$  is called relative deformation. When the external forces shrink the object, the relative deformation is negative ( $\Delta L < 0$ ) and the elastic stresses are called pressure stresses.

For relatively small deformations the elastic pressure stresses are directly proportional to the relative deformation  $\varepsilon$  - Hooke's law

$$\sigma = k\varepsilon. \tag{1}$$

The proportionality factor  $k$  is a constant which does not depend on the dimensions of the object, it is characterized only by the elastic properties of its building material. It is called modulus of elasticity. During stretching or shrinking the elastic modulus is called Young's modulus and is marked with  $E$ . For this type of deformation the Hooke's law has the form

$$\sigma = E\varepsilon. \tag{2}$$

Then, the equation for the operating force is:

$$F = \sigma S = E_r \varepsilon S = E_r S \frac{\Delta d}{d}. \tag{3}$$

The following expression is derived for the relative deformation:

$$\varepsilon = \frac{\Delta d}{d} = \frac{1}{E_r S} F. \quad (4)$$

### 2.1. SECE working at direct current mode

For the purposes of modeling it is assumed that all the elements that make up the scheme of the sensor are ideal.

The electrical circuit for studying the SECE is shown in fig. 2.

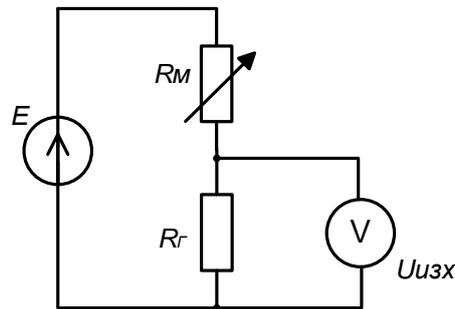


Fig. 2. Scheme for connecting SECE with a direct-current source

With the resistors  $R_r$  the volume and contact resistance of the sensor are defined. The load resistance is denoted by  $R_M$ , which allows for realization of different conditions for sensor operation. The output value from the work of the sensor is the voltage on  $R_r$ .

The equation for the output voltage is

$$U_V = \frac{R_r}{R_r + R_M} E = \frac{E}{1 + \frac{R_M}{R_r}}. \quad (5)$$

Considering the relative deformation  $\varepsilon$  (2) and a correction factor  $\beta$  for the function of voltage variation, the following expression is obtained:

$$U_V = \frac{E}{1 + \frac{R_M}{R_{r_0} e^{-\frac{\beta}{E_r S} F}}}. \quad (6)$$

The expression (6) is used to create a simulation model in Matlab environment, as shown in fig. 3.

As a controlling example the experimental results (fig. 1.b) are compared graphically with those from the simulation in Matlab (fig. 1.a). A very good match is observed. This is confirmed by the comparison of the baseline values determined in both ways. The relative error is within the boundaries of  $\delta_U = \pm 5\%$ .

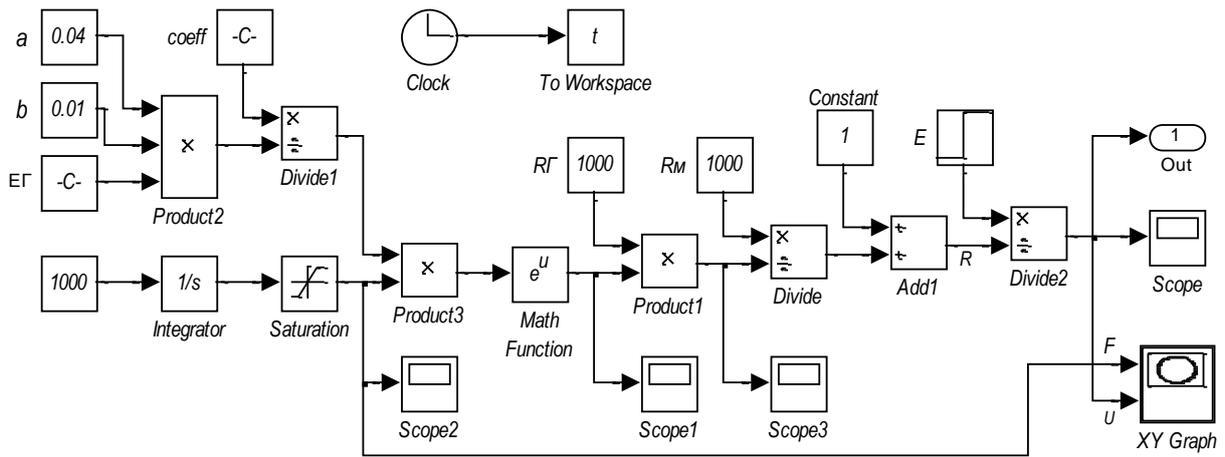


Fig. 3. Simulation scheme in Matlab environment

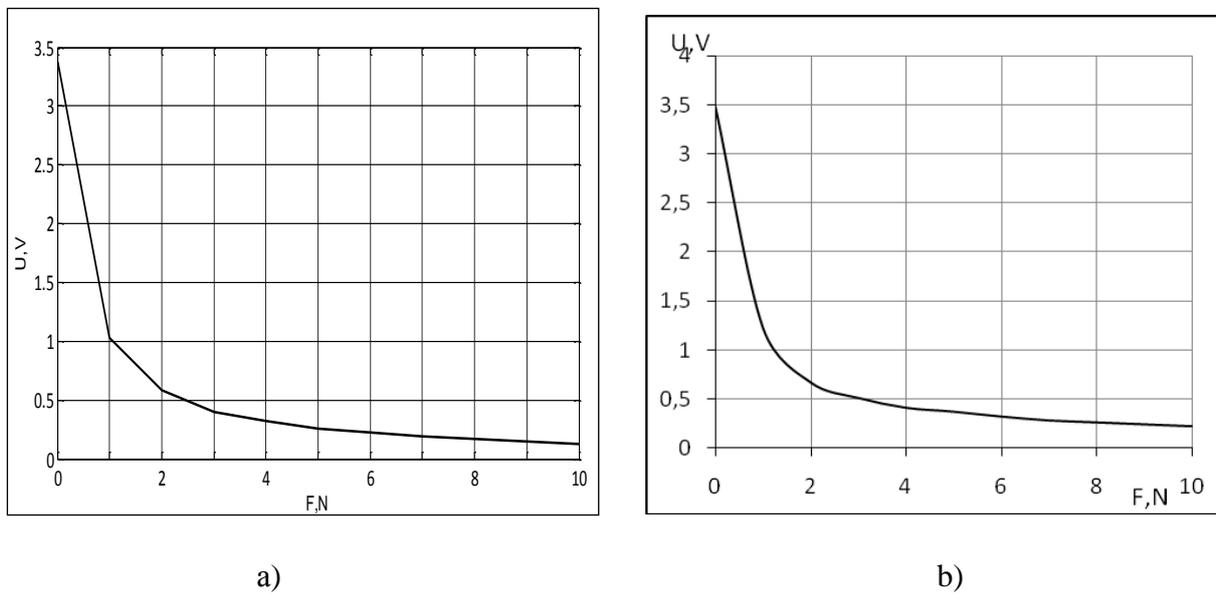


Fig. 4. Change in the output voltage as a function of the force measured

## 2.2. SECE operating at an alternating current mode

Fig. 5 shows the electrical scheme for examination of SECE with and AC-current source.

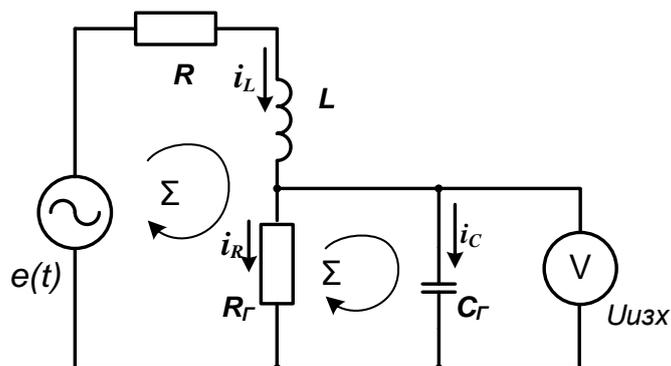


Fig. 5. Scheme for connecting SECE with an AC-current source

The electrical parameters of SECE are presented with the help of a resistor  $R_\Gamma$  and capacitor  $C_\Gamma$ , connected in parallel. With the resistor again the volume and contact resistance of the sensor is expressed, and by  $C_\Gamma$  the capacitor, which is obtained from the presented constituent structure of the sensor (metal - electrically conductive elastomer – metal) is denoted. SECE is connected in series with the coil with inductance  $L$ .

For the electric circuit (fig. 5) the following system of equations under the laws of Kirchhoff in a snapshot is composed:

$$\begin{cases} i_L(t) = i_R(t) + i_C(t) \\ i_R(t) = \frac{1}{R_\Gamma} \left[ e(t) - Ri_L(t) - L \frac{di_L(t)}{dt} \right] \\ i_C = C_\Gamma R_\Gamma \frac{di_R(t)}{dt} \\ u_C(t) = \frac{1}{C_\Gamma} \int i_C(t) dt \end{cases} \quad (7)$$

The variation of the resistance and the sensor capacity as a result of external, deforming, force impact is determined by the following expressions:

$$R_\Gamma = R_{\Gamma_0} e^{-\frac{\beta}{E_r S} F} \quad \text{and} \quad C_\Gamma = \epsilon_0 \epsilon_\Gamma \frac{S}{de^{-\frac{\beta}{E_r S} F}} \quad (8)$$

The system (7) and the equations (8) are used for the creation of a simulation model in Matlab environment, shown in fig.6.

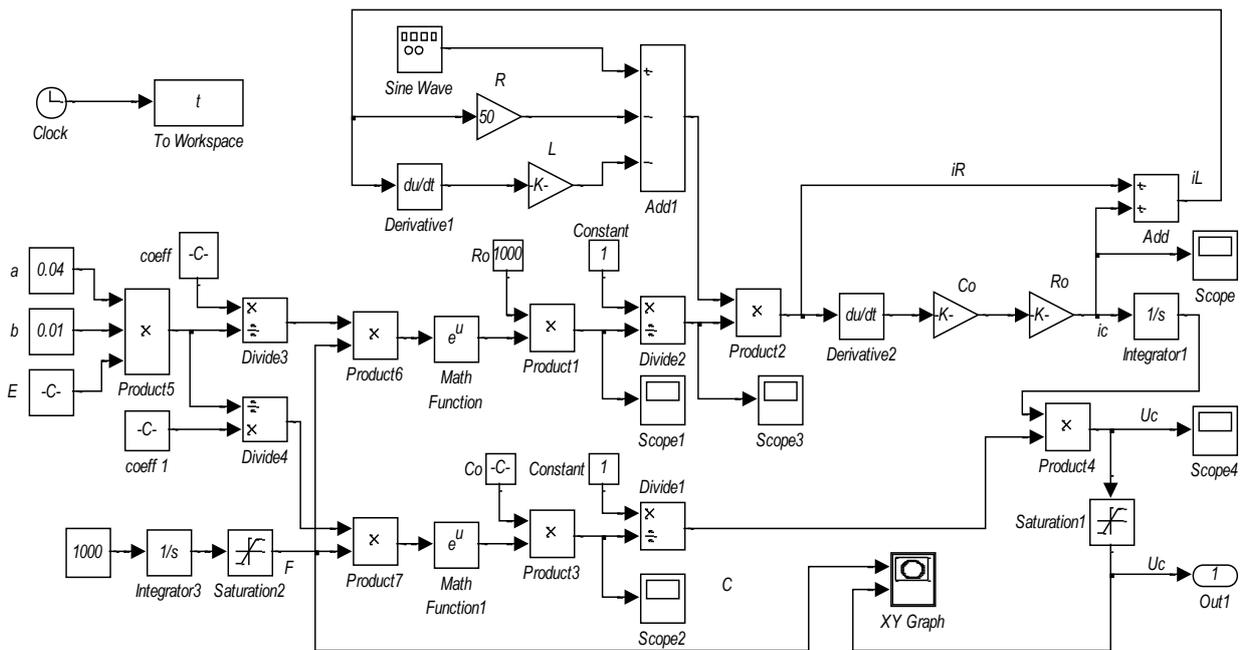


Fig. 6. Simulation scheme in Matlab environment

The graphs with variations in the output voltage as a function of the force  $F$  are given in fig. 7 (a - simulation, b - experimental data).

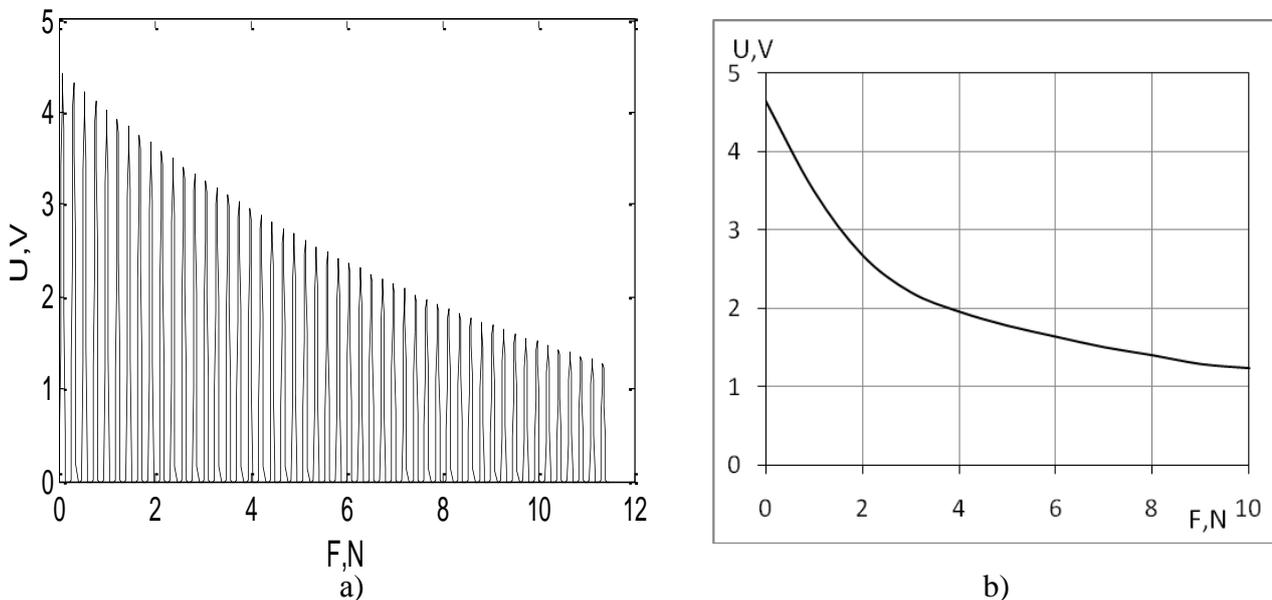


Fig. 7. Change in the output voltage as a function of the force measured

## 5. CONCLUSIONS

Analytical expressions are obtained for the effect of SECE in two modes of operation: direct and alternating current. The parameters of the models are defined. Simulations in Matlab environment are created. The simulation results overlap with the experimental ones with a maximum relative error  $\delta_U = \pm 5\%$ .

The results can be used to derive dependencies related to the other parameters of the sensor by means of which the overall design could be afterwards executed.

The presented modeling approach can be applied to other types of sensors with different size, different elastomers with different resistivity, for determining the type, possessing the best metrological characteristics according to the specific pre-set requirements.

## References

- [1] P. Georgiev, "Sensory Engineering", University Press, Technical University of Gabrovo, 1999.
- [2] Harper, Charles A., Edward M. Petrie. "Plastics Materials and Processes", John Wiley & Sons, 2003.
- [3] Y. Yordanov, "Application of Matlab in engineering studies", Part I and II, Ruse 2004.
- [4] M. Kostova, "Electrically Conductive Elastomeric Composition for Rubber Electrodes, Copyright Certificate, No 2603, RB, 1993.
- [5] Y. Marinov and V. Dimitrov, "Sensor devices for robots" Technique, Sofia, 1982, pp.9-15.
- [6] [http://web.uni-plovdiv.bg/marudova/Polymers/Polimeri\\_7.pdf](http://web.uni-plovdiv.bg/marudova/Polymers/Polimeri_7.pdf).

Reviewer: Assoc. Prof. PhD A. Chervenkov