Assessment of Bhaleth Bridge using Mechanism Method

PARDEEP KUMAR
Department of Civil Engineering
National Institute of Technology Hamirpur (Himachal Pradesh) India 177 005
Pardeepkumar.nit@gmail.com, www.nith.ac.in/~pk/

ABSTRACT: The arch bridges built in ancient times are an integral part of infrastructure. For many reasons including changing trend in the use of materials, design philosophy, relative costs and maintenance, the masonry arches are almost out of use. However there is a renewed interest in performance of existing masonry arch bridges. The paper presents a brief review of the existing methods of assessment and analysis including the analysis of two masonry arch bridges using these methods. The bridges have been analysed using the proposed mechanism procedure by the author. The results obtained are in agreement with other methods of analysis.

Key-Words: - Masonry, Arch, Bridges, FEM, Mechanism,

1 Introduction
The bridges are the pillars of the railway and the roadway network. In India, there are about 3,60,000 major and minor bridges and aqueducts catering to the transportation facilities. Out of these approximately 1,30,000 are railway bridges with masonry arch bridges numbering to approximately 19,600[1,2]. The remaining 2,30,000 are the road bridges, out of which a substantial number comprise of masonry arch bridges.

The assessment of existing arch bridges is imperative because of the need to:
- Consider all parameters effecting load carrying capacity.
- Allow modeling of all types of the service loads.
- Allow the modeling of the strengthening and rehabilitation works.
- Be improvable with the future developments in the field.

The aim of the paper is to present the developments in the methods of assessment and analysis, newly developed as well as the refinement of the older ones [3,4,5,6].

2 Arch Behaviour
Every masonry arch bridge is a unique three-dimensional (3D) structure. These may be frequently skewed and are composed of composite materials surrounded by the fill. The arch ring is the main load-bearing element of the masonry arch bridges. The spandrel walls, wing walls, haunch fillings and fill contribute to the overall strength of the bridge. The extent of contribution depends upon their condition, which makes every masonry arch bridge a unique structure. Further, the term masonry applied to both brick as well as stone constructions complicates the overall behaviour of the masonry arch bridges.

2.1 Failure Modes
2.1.1 Compression
The axial thrust and moment resulting from eccentricity of thrust may cause excessive
2.1.2 Shear

The normal thrust in the arch ring can be resolved into radial shear and axial thrust. When the radial shear in the arch exceeds the shear bond strength of the masonry, it is said to fail in shear. With increase in load the ratio of radial shear to axial force increase but as the failure occurs the ratio is still quite low.

2.1.3 Mechanism

This type of failure results from formation and rotation at hinges forming a mechanism. The line of thrust in a well-proportioned arch would be near central, but due to increase in loads the moments in sections increase which is balanced by the increase in eccentricity of line of thrust. Wherever the eccentricity of line of thrust increases the maximum value corresponding to safe tensile strength, the progressive hinge formation takes place. In fixed arches the formation of the fourth hinge invokes a three bar mechanism and hinges rotate to failure.

2.1.4 Snap Through

In shallow thin arches the successive formation and rotation of hinges takes place rapidly. This type of failure is snap through. Full-scale experimental studies have identified snap through as a failure mode of the arches.

3 Assessment Methods

3.1 Elastic Method

Pippard’s elastic method is based on his observation that a very slight spread of the abutments of the voisoir arch tends to produce pins or hinges at abutments. Pippard ignored the possibility of the formation of the third hinge and analyzed the arch as a two-hinged arch. The arch assumed for analysis was parabolic, loaded with point load W at the crown. Using the strain energy approach the horizontal thrust at the abutments and the bending moment at the crown can be found out (fig.1). The section of arch is assumed to vary as

\[ I = I_o \frac{d_f}{d_s} \quad H_L = \frac{25WL}{128a} \quad (1) \]

where

\[ V_L = \frac{W}{2} \quad (2) \]

\[ H_D = \frac{\gamma L^2 h}{a} \left( \frac{a}{21} + \frac{h + d}{4} \right) \quad (3) \]

\[ M_D = \frac{1}{168} \mu L^2 ah \quad (4) \]

The combined effect of dead and live load gives

\[ H = \frac{L}{a} \left[ \gamma L h \left( \frac{a}{21} + \frac{h + d}{4} \right) + \frac{25}{128} W \right] \quad (5) \]

\[ M_c = \frac{1}{4} L \left( \frac{\gamma L ah}{42} - \frac{7}{32} W \right) \quad (6) \]

Pippard used value of the bending moment to derive rules of assessment. As the value of W goes on increasing 2nd term in equation (6) increases and tend to develop tensile stresses. Based upon middle third rule, which he argued is less restrictive criteria; the limiting value of W was given by rather middle half rule by the solution of

\[ \frac{M_c}{H} = \frac{1}{4} \quad (7) \]
Which leads to
\[ W_1 = \frac{32\gamma L h \left[ 2a^2 + 4ad + 2Ld(h + d) \right]}{21(28a - 25d)} \]  
(8)

Similarly based on the maximum compressive stress criteria for arch ring having depth d and effective width 2h.

\[ f = \frac{H}{2dh} - \frac{3M_h}{h^2} \]
\[ W_2 = \frac{256 fhd}{L} + 128\gamma L h \left( \frac{a}{28d} - \frac{1}{2L} - \frac{h + d}{4a} \right) \left( \frac{25}{a} + \frac{42}{d} \right) \]
(10)

Pippard compared the results with full scale test conducted by Building Research Station and discarded the use of equation no. 8 and recommended the use of equation no. 10 which even violated the middle half rule. Pippard proposed that within the barrel of actual arch two ribs could be thought of comprising the actual barrel. Hence

\[ W_a = 2W_2 \]  
(11)

The method was later updated by Department of Transport in the light of MEXE method, using modern frame analysis approach. The geometrical factor was dropped while material factor and condition factor were retained. The tensile stress, which appears in analysis, is ignored in the procedure. Separate dead and live load analysis are done and superimposed. The results when compared with other assessment methods indicate that the method significantly overestimated the strength of the arches. Also there is no allowance for the different masonry and backfill types. It is unable to distinguish the relative effects of masonry strengths.

3.2 The MEXE / MOT Method

The Military Engineering Experimental Establishment converted the Pippard’s table in to an ingenious nomograph. The idea was incorporated in Ministry of transport memorandum of 1967. The basis is described in fig. 2 wherein the entire range of values covered in Pippard’s table are supposed to be estimated sufficiently accurately by relating the permissible axle load Wa with the arch span (L) and the total crown thickness (h + d). The provisional axle load is then modified by number of modifying factors described below. The value Wa read off from nomograph is designated as provisional because of the modifying factors which are required to be applied to the value to get the Axle load.

\[ W_a(PAL) = \frac{740(d + h)^{1.3}}{L} \text{ ton} \]  
(12)

Fig. 2 Basis of MEXE nomograph

The formula is applicable for 1.5 m to 18 m span (L) and 0.25 m to 1.8 m of d+h. Following modifying factors are used:

3.2.1 Span: rise factor (Fs)

For span /rise ratio of 4 or less modifying factor is 1. For span/rise ratio greater than 4 the modifying factor is < 1.

3.2.2 Profile factor (Fp)

Parabolic profile of the arch is assumed ideal. If \( R_q \) is the rise at quarter span and \( R_c \) is the rise at crown, for

\[ \frac{R_q}{R_c} \leq 0.75 \]  
factor is 1.0 and for

\[ \frac{R_q}{R_c} \geq 0.75 \]  
factor is less than 1.

3.2.3 Material factor (Fm)

It depends upon material type and thickness of arch barrel and the fill.

3.2.4 Joint factor (Fj)

The joint factor is determined based upon width of the joint (\( F_w \)), Depth of the joint (\( F_d \)) and mortar factor (\( F_m \)).

\[ F_j = F_w * F_d * F_m \]

3.2.5 Condition factor (Fc)

The condition factor depends upon assessment of cracks and deformations that may be present in the structure. The value varies between 0 & 1.

Mod. Axle Load = Fs*Fp*Fm*Fj*Fc*WA

The method has been now adopted for civil use and is now Departmental Standard BD 21/84-
(1984) advice note BA 16/84, 1984. The approach is empirical based on elastic analysis with limits on allowable compressive (1.4 N/mm²) and tensile stress (0.7 N/mm²). The modifying factors have no interrelation and are independent of geometry, yet they must be correlated and related to geometry of arch. The method has been found to overestimate the strength.

3.3 Plastic Method

In the plastic theory postulated by Heyman [3], there is no sliding between the voussoirs. The materials do not fail by crushing i.e. has an infinite compressive strength. The materials have infinite value of modulus of elasticity and tension is not allowed to develop.

Heyman postulated that upper bound method would involve an assumed hinge configuration and lowest upper bound should give a thrust line between hinges that is fully contained within the masonry of the arch ring. In this case lower bound theorem would be satisfied if thrust line can be found for a complete arch which is equilibrium with external loading and lies anywhere within the masonry of arch ring then the arch is safe.

For the quick approximate solution, Heyman assumed that the worst loading position is the quarter point. If the position of the hinge points is known, it is possible to calculate the failure load \( P \) for an arch or alternatively the minimum thickness of the arch to sustain a value of the load and thus geometrical factor of safety can be calculated. The geometrical factor of safety for any position of the load is the ratio of thickness of the shrunk arch and that of the original arch. The load can be made to shift along the span and segmental factors of safety determined for various load positions as shown in fig. 3.

![Fig. 3 Geometrical Factor of Safety](image)

The failure load can be determined using principle of static equilibrium. Owing to the basic assumption of zero tensile strength, the mechanism methods are exclusively undertaken using computer software. For the critical position of load, estimate is made for four hinge positions. Each position is reallocated until a minimum capacity is determined. The method produces a safe prediction of strength and properly reflects the influence of both brickwork and soil types.

3.4 Proposed Mechanism Method

The moment-axial force interaction is the most important parameter to determine the load carrying capacity of the masonry arch bridges. Wherever the combination of moment and axial force developed in the section lies on this surface, a hinge shall be assumed to form at that section and the hinge will continue to rotate when further load is applied till the arch is converted to a mechanism. The proposed interaction equations by the author [7] take into account the masonry tensile strength, indicated by the presence of constant term in this equation.

\[
\frac{M}{M_p} = 0.0779 + 4.3994 \left( \frac{P}{P_p} \right) - 4.5662 \left( \frac{P}{P_p} \right)^2 \tag{14}
\]

Considering the unit width of the arch ring, it can be divided into a sufficient number of segments along the barrel centerline. Each segment can be assumed to be a straight line joining the two nodes. These segments can be represented by a beam element having appropriate material and sectional properties. The end nodes are fixed at the springing line to provide restraint against any horizontal, vertical, or rotational movement. The arch is analysed first under the dead loads imposed due to self-weight of the arch ring and the load of the overlaying fill. The weight of the fill is calculated over each segment and is applied as equivalent nodal loads at its two nodes. The arch is then analysed under a unit live load applied at quarter point. The obtained values of bending moment and axial force due to dead and live load so obtained are modified to satisfy the limit state envelope at every node. A step-by-step linear analysis is performed to locate the four hinge locations and the corresponding total load on the bridge is the failure load.

For any section, under the action of the dead loads and live loads, moments and axial forces are determined. Assuming for a particular section, \( M_{DL} \) is the moment produced due to the dead loads;
\(M^I_{LL}\) is the moment produced due to the unit live loads;
\(P^I_{DL}\) is the axial force developed due to dead loads;
\(P^I_{LL}\) is the axial force developed due to unit live loads
\(P_o\) is the maximum concentric axial force,
\(\sigma = \frac{P_o}{b d}\)
\(M_o\) is the maximum moment assuming maximum eccentricity of \(d/4\),
\(M_o = 0.125 \sigma \cdot b d^2\) and
\(\alpha\) is the factor at which interaction equation is satisfied

Then for a hinge to form at this section the above given interaction equation should be satisfied. The combinations of the moments and axial forces due to dead and live loads at all the nodes are checked to satisfy the following equation.

\[
\left(\frac{M_{DL} + \alpha M_{DL}}{M_o}\right) - A_1 \left(\frac{P_{DL} + \alpha P_{DL}}{P_o}\right) + A_2 \left(\frac{P_{LL} + \alpha P_{LL}}{P_o}\right) = 0
\]

(15)

where \(P_o\) and \(M_o\) are ultimate concentric axial force and ultimate moment. Constants \(A_0\), \(A_1\) and \(A_2\) are used as per the equations 2, 3, or 4. The iterations are made until the interaction at each end of the member is satisfied and the value of \(\alpha\) is determined at all ends of the members. Consequently, the lowest value of \(\alpha\) amongst all is the first load factor at which the first hinge is formed.

\[\alpha_{\text{first}} = \alpha_1\]

(16)

All the values of the moments, axial loads and the displacements at different sections are updated corresponding to this load factor. The location of this section within the arch is identified and the rotational restraint at this section or node is removed by insertion of a hinge. This is achieved by modifying the stiffness of the member in which the hinge is formed. The final moments and axial forces after formation of the first hinge becomes:

\[M_1 = M_{DL} + \alpha_1 M_{LL}\]

(17)
\[P_1 = P_{DL} + \alpha_1 P_{LL}\]

(18)

The modified arch, with one inserted hinge is reanalyzed under unit live load acting at the earlier position. This gives rise to additional moments \(M^2_{LL}\) and axial forces \(P^2_{LL}\) at all sections, except at the section where a hinge has already formed, where the additional moment is zero. Now, for the formation of the second hinge, \(\alpha_2\) is the modifying factor, which corresponds to the formation of second hinge. The cumulative live load factor for the formation of the second hinge is given by:

\[\alpha_{\text{second}} = \alpha_1 + \alpha_2\]

(19)

The final moments and axial forces after the formation of the second hinge become

\[M_2 = M_1 + \alpha_2 M_{LL}\]

(20)
\[P_2 = P_1 + \alpha_2 P_{LL}\]

(21)

Similarly for third hinge and forth hinge \(\alpha_3\) and \(\alpha_4\) are determined. The cumulative live load factor required for the formation of the fourth hinge, thus, becomes:

\[\alpha_{\text{fourth}} = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4\]

(22)

The final moments and axial forces after the formation of the fourth hinge become:

\[M_4 = M_3 + \alpha_4 M_{LL}\]

(23)
\[P_4 = P_3 + \alpha_4 P_{LL}\]

(24)

### 3.4.1 Assumptions of Method

The method is based on the assumptions that:

- At the point of hinge the axial force and shear force resisting capacity is not impaired and it continues to resist the axial force and shear force.
- The effect of the shear forces on moment-axial force interaction envelope has been ignored.
- The point where a hinge is formed will continue to rotate.

### 3.4.2 Idealisation of the Arch

In the proposed procedure, the arch ring having unit width is analysed. The centerline of a masonry arch bridge can be idealized by a number of straight beam elements all along the circumference as shown in Figure 4.
3.4.3 Properties of the Masonry

The predicted response of any masonry arch bridge, by and large, would depend upon the correct evaluation and input of material properties, geometry of the arch, the cross-sectional area and moment of inertia based on unit (1 m) width etc. The compressive strength of the masonry is the most important parameter that determines the load carrying capacity. A value obtained after testing cores taken from the bridge to be analysed gives the most reliable estimate of the compressive strength. In the present investigation, the masonry properties taken from the proposed guideline by the Author given in Table 1 are used.

Table 1. Brick Masonry strength recommendations for Indigenous Constructions

<table>
<thead>
<tr>
<th>Masonry</th>
<th>Modulus of Elasticity (GPa)</th>
<th>Compressive Strength (Mpa)</th>
<th>Tensile Strength (Mpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brick Masonry</td>
<td>1-5</td>
<td>3-7</td>
<td>0.15-0.35</td>
</tr>
</tbody>
</table>

4 Prediction of Ultimate Load Carrying Capacity of Bhaleth Bridge using Proposed Method

The applicability of the method has been proven for the load rating of the masonry arch bridges built elsewhere. The method has been applied to some bridges, that have been tested on full scale by Transport Research Laboratory [8, 9, 10, 11, and 12] and whose results were well documented.

4.1 Bhaleth Bridge

The bridge seen in Fig. 5 is in Service Bridge on State highway Hamirpur Sujanpur in Himachal Pradesh state of India. The bridge is a single lane bridge, without any skew built in brick masonry in 1931. The clear span of the bridge is 34.268 m. The geometry of the bridge is segmental with a central rise of 6.181 m above springing level. The springing are abutting against a strong rock on both ends of the arch. The height of spandrels and fill on the left as well as right springing is 8.1 m to the surface of road. The width of the roadway is 4.5 m. The roadway is supported on the arch having a width of 3.44m. The additional width is provided by the steel joist placed at regular spacing of 1.5 m over the fill and the spandrel walls. The thickness of spandrel walls is roughly 0.4m. The arch ring has 10 courses of brick headers placed vertically at central location. The number of courses increased to 11 near the centre of 2nd circular opening from centre and 13 beyond the centre of 3rd oval opening from the centre of the arch span. The arch had 30 cm fill height at crown and 7.63 m near the springing. The close of the ring and opening are shown in Figure 6.

![Fig. 5 Side view of Bhaleth Bridge](image)

![Fig. 6 Close up of arch ring of Bhaleth Bridge](image)

The masonry arch bridge has been analysed using the proposed mechanism method. The material properties used in the analysis are given in Table 2.

Table 2 Properties of brick masonry used in the analysis

<table>
<thead>
<tr>
<th>Material</th>
<th>Property</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brick Masonry</td>
<td>Modulus of Elasticity</td>
<td>3723</td>
<td>N/mm²</td>
</tr>
<tr>
<td></td>
<td>Compressive Strength</td>
<td>5.84</td>
<td>N/mm²</td>
</tr>
<tr>
<td></td>
<td>Tensile Strength</td>
<td>0.229</td>
<td>N/mm²</td>
</tr>
</tbody>
</table>

The bridge fails by converting into mechanism on formation of four hinges. The first hinge forms at left hand springing at an load of 61.20 kN, 2nd hinge forms at right hand springing at a load of 282.44 kN, 3rd hinge
forms at the quarter point where load is applied at a load of 861.11 kN and fourth hinge forms near the opposite quarter point at a load of 1156.64 kN. The ultimate load carrying capacity is predicted as 1156.64 kN. The bridge was also analyzed using Archie-M [13] which predicted the collapse load at 1275.3 kN, with formation of four hinges at identical locations.

5 Conclusions
An attempt has been made to present some means of assessment of masonry arch bridges and their relative merits and demerits. The work carried out by various researchers has also been reviewed. It has been brought out that many analytical and experimental investigations are required for developing a rational procedure for assessing masonry arch bridges.

Masonry arch bridges have performed very well for centuries and these bridges must be monitored carefully for any signs of distress. For proper examination, before taking the decision to demolish, a rational survey of such bridges is desirable to be carried out. There is an acute need to evolve a rating procedure to rate existing masonry arch bridges of Indian highway and railway network.

The axial force-moment interaction can be effectively used for the prediction of load carrying capacity of the masonry arch bridges. In the proposed method the experimentally determined axial force-moment interaction has been verified and implemented successfully to predict the collapse load.

The proposed interaction, accounts for some minimum tensile strength of the masonry. The proposed method can predict the collapse load on the basis of formation of adequate number of hinges leading to conversion to a mechanism.

The frame analysis program automated for the formation of the hinges and further leading to failure on formation of the mechanism provides a sufficiently quick and simple method for determination of the load carrying capacity of the masonry arches assuming a unit width of the arch ring.

References: