Abstract: - In this paper, an efficient method for minimizing the high Peak to Average Power Ratio (PAPR), based on Clipping and Differential Scaling, in Orthogonal Frequency Division Multiplexing (OFDM) systems is proposed. In this technique, the amplitude of complex OFDM signal is clipped and then scaled in such a way so that the PAPR is reduced without causing much degradation in Bit Error Rate (BER). Convex Optimization technique is implemented to obtain the minimum value of PAPR. PAPR and BER of the system considered using simulations for QPSK constellation are presented. Performance of the proposed PAPR reduction technique with the performance of the existing techniques is also compared.

Keywords: - OFDM, PAPR, BER, ISI, EVM, RMS.

1 Introduction
Orthogonal Frequency Division Multiplexing (OFDM) [3] is a technique of encoding digital data on multiple carrier frequencies and is most commonly used method for transmission of signals over wireless channels. OFDM is a modulation technique and a multiplexing technique. OFDM uses a large number of parallel narrow-band subcarriers [3] other than using a single wide-band carrier to transmit information. OFDM is mainly used to increase the robustness against frequency selective fading or narrowband interference.

Though the principle of OFDM communication was for several years [3], it was only in the last decade that it has been initiated to be used in commercial systems. OFDM is essential for Digital Audio Broadcast (DAB), DVB, WLAN and more recently Wireless Local Loop (WLL). One of the drawbacks in OFDM is high PAPR, which makes system performance very sensitive to distortion which is introduced by nonlinear devices such as power amplifiers (PAs). As PAPR [12] turns to maximum rate, the complexity of the analog-digital and digital-analog converter becomes higher and in turn minimizes the efficiency of the radio frequency power amplifier.

Many methods [5], [12] like amplitude clipping, clipping and filtering [2], [13], coding, Partial Transmit Sequence (PTS), Selected Mapping (SLM) and interleaving are proven to minimize the crest factor. These methods achieve peak power reduction at the expense of transmit signal power increase, BER increase, data rate loss, computational complexity increase, and so on. The process of optimization [9] is done by repeated iteration. By the proposed algorithm a greater reduction in PAPR in a few iterations that is, less processing delay, with minimum BER (Bit Error Rate) and less out-of-band radiation can be obtained. For different oversampling factor, l =1, 2, 4, 8, performance analysis is made.

2 OFDM Scheme
OFDM symbol, x(k) [14] expressed as

\[ x(k) = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} c(i) e^{-j2\pi i k/N} \]  

(1)

Where k = 1, ..., IN is time index.

Let \( c \in C^N \) be the frequency-domain OFDM symbol and \( \{c(i), i = 1, ..., N\} \) be the symbol value carried by the i-th sub-carrier.

The frequency-domain OFDM symbol c is expressed as

\[ c(i) = \frac{1}{\sqrt{IN}} \sum_{k=1}^{IN} x(k) e^{-j2\pi i k/N} \]  

(2)

where i = 1, ..., N. Nowadays, OFDM modulation and demodulating can be implemented via IFFT and FFT, respectively.

The PAR is defined as the ratio of the highest signal peak power to its RMS level.
3 Clipping and Differential Scaling

In this section a new technique called Clipping and Differential Scaling is proposed. The probability distribution of amplitudes of the OFDM signal follows Rayleigh distribution [12] and thus the probability of high peaks is very less. An OFDM system model with PAPR reduction block upper threshold above which the signal amplitudes do not contribute much to the signal is determined as follows. Using simulations, BER for the modified signals are determined. The clipping threshold at which the BER is degraded from $1.5 \times 10^{-3}$ to $3.5 \times 10^{-3}$ at SNR of 10dB and the amplitudes above this clipping threshold are selected and clipped. Instead of clipping the signal further to reduce the PAPR, a reversible process is considered - Differential Scaling which would reduce the PAPR but not deteriorate the BER. Since different ranges of amplitudes of the signal are scaled in a different manner, it is called Differential Scaling.

Three types of scaling are considered and described below.

3.1 Scale Up

In this method, the lower amplitudes of the signal are scale up by a factor of $\beta$. This leads to increase the average value without affecting the peak values. Therefore, the resulting PAPR reduces. The PAPR reduction function can be defined as

$$ h(x) = \alpha x_p, \text{ if } x > \alpha x_p $$
$$ = \beta x, \text{ if } x < A $$
$$ = x, \text{ if } A \leq x \leq \alpha x_p $$

where $x_p$ is the amplitude peak value occurring in an OFDM symbol block, $\alpha$ is the factor deciding the clipping threshold in terms of percentage of the peak value and $\beta$ is the scaling factor for the range $[0,A)$ whose value is greater than one.

3.2 Scale Down

In this method, the higher amplitudes of the signal by a factor of $\gamma$ are scaled down. This leads to decrease the peak value.

The PAPR reduction function can be defined as

$$ h(x) = \alpha x_p, \text{ if } x > \alpha x_p $$
$$ = \gamma x, \text{ if } B \leq x \leq \alpha x_p $$
$$ = \beta x, \text{ if } x < A $$
$$ = x, \text{ if } A \leq x \leq B $$

where $x_p$ is the amplitude peak value occurring in an OFDM symbol block, $\alpha$ is the factor deciding the clipping threshold in terms of percentage of the peak value and $\beta$ is the scaling factor for the range $[0,A)$ and $\gamma$ is the scaling factor for the range $[B,\alpha x_p]$.

4 OFDM PAR in a convex form

This section gives a brief overview on the convex formulation of the OFDM PAR minimization problem and on the used IPM algorithm [10]. A WLAN 802.11a using OFDM signal with 48 data, 4 pilot, 12 free subcarriers and 128 symbols, was used in the validation process. The 52 subcarriers (data/pilot) were modulated using 16-Quadrature Amplitude Modulation (QAM).

The baseband OFDM signal was generated by dividing the information data into multiple data streams, each of was passed to a subcarrier for modulation. The modulated data streams were sent in parallel on the orthogonal subcarriers. The frequency constellation was then time domain transformed through IFFT. Cyclic prefix insertion as well as windowing was then applied for time-spreading handling and InterSymbol Interference (ISI) elimination. The time domain symbols were packed in a series form and sent for In-phase and Quadrature (IQ) modulation [1].

A main constraint in the optimization process is to keep the in-band error, represented by Error Vector Magnitude (EVM), below a specified standard limit [1].

Despite the difference between EVM levels at back-of region, the EVM of the optimized signal is still accepted by the standards.

5 Customized Convex optimization

Conic optimization is a subfield of convex optimization that studies a class of structured convex optimization problems called conic.
optimization problems. A conic optimization problem consists of minimizing a convex function over the intersection of an affine subspace and a convex cone. The customized interior point method involves simple steps such as initialization, computing search direction which involves solving the equations using Newton’s method, updating, iterating till the gap is below the specified value. An overview of this algorithm is presented below. The main benefits of this algorithm over a generic solver include a fast FFT based method of constructing the linear system of equations, an improved update procedure to reduce the number of iterations, and the elimination of all control (if/then) statements. The same algorithm can be used for any OFDM constellation type. For each filtering iteration of Iterative Clipping and Filtering (ICF), we propose to choose a filter that minimizes the current OFDM symbol’s Error Vector Magnitude (EVM) subject to the desired PAPR\(_{\text{max}}\). This designed filter will replace the rectangular filter in the classic ICF method. Moreover, the clipped OFDM symbols obtained by the optimized ICF method have less distortion and lower out-of-band radiation than the existing method.

6 Simulation results

Fig. 6.1 PAPR Comparison

Fig. 6.1 shows the CCDF with Scale up, Scale down and Scale up-down techniques. We have also shown the performance without PAPR reduction technique. It can be seen that all the three techniques significantly reduce the PAPR and the best PAPR reduction is achieved by Scale up-down technique as expected.

The figure also shows the comparison of the output signals obtained from the clipping and differential scaling methods with the original OFDM signal.

Fig. 6.2 BER Comparison

The above figure Fig.6.2 shows the comparison of BER for various techniques. From the figure, it is clearly shown that the Scale-up down method has better BER reduction when compared to the other techniques. Hence the expected output is obtained and shown.

Fig. 6.3 Customized Convex Optimization

Fig.6.3 shows the comparison graph of Differential Scaling Technique with Customized Convex Optimization and without Customized Convex Optimization. After Customized Convex Optimization PAPR is further reduced.
4 Conclusion
Optimizing OFDM signals based on convex optimization algorithm for PAR reduction was evaluated on a RF PA. It showed a limited improvement in power performance near compression; an extra 1 dB in output power and 3.4% in efficiency. While in the backed-off region it had 5.4% PAE improvement and a gain of 5dB in output power. A reason for such behavior is the increase in lower peak density distribution in the signal. Spectral emission was considered to be a draw back in the method which needs further investigations in the algorithm in order to meet the standard requirements for emission. The main cause to the spectral leakage is the absence of the power free-subcarriers. It is advised to adjust the minimization algorithm to consider part of the subcarriers as guard intervals. Even though PAR reduction by convex optimization is a more complicated method in terms of digital signal processing, it is worth to investigate and apply as it is free in terms of hardware adjustments. This result is believed to be of significance in a world where every dB is worth a Billion. Hence the efficiency can be improved in OFDM systems using Customized Convex optimization in Differential Scaling.

References: