Abstract: Orthogonal frequency-division multiplexing (OFDM) is a method of encoding digital data on multiple carrier frequencies. This scheme uses a number of sub-carriers and converts a frequency selective fading channel to a set of parallel flat fading channels. The OFDM signal has a drawback that some signal values can be much higher than the average value. This results in high Squared Crest Factor (SCF). This paper analyzes the Tone Reservation (TR) technique for SCF reduction. In this scheme, a small number of unused subcarriers called Peak Reduction Carriers (PRCs) are reserved to reduce the SCF and the goal of the TR scheme is to find the optimal values of the PRCs that minimize the SCF of the transmitted OFDM signal. Similarly, the Bit Error Rate also gets lowered. In contrast to previous methods, the TR method with Customized Convex Optimization implies a very quick convergence of minimum SCF and the bit error rate solution at a lower computational cost. The proposed method is compared with the SCF reduction techniques like Differential Scaling. Also, the effects of oversampling at $l = 1, 2, 4, 8$ on the performance of the proposed OFDM system is discussed.

Key-Words: Convex Optimization, Complementary Cumulative Distribution Function (CCDF), Orthogonal Frequency Division Multiplexing (OFDM), Squared Crest Factor (SCF), Tone Reservation (TR), Peak Reduction Carriers (PRCs).

1 Introduction
Give the increasing demand for ubiquitous multimedia service future wireless communication is expected to provide higher data rates and robustness to the changing wireless environment. To achieve these goals, a new modulation scheme is required. In recent years, Orthogonal Frequency Division Multiplexing (OFDM) has attracted considerable attention for its potential to increase spectral efficiency and to resist multipath fading. One of the main practical issues of OFDM is the high Squared Crest Factor of the transmit signal. There are many SCF reduction techniques in which side information is either needed or not needed. Depending on the necessity of side information the SCF reduction technique can be classified into two groups. The Partial Transmit Sequence (PTS) [1],[10] and the Selected Mapping (SLM) [2],[9] modify frequency domain symbols and Time domain symbols respectively. To prevent the OFDM system from burst errors, the transmitter delivers side information to the receiver. The other group working with no side information includes Clipping and Filtering (CF) [3], Peak Windowing (PW) [4], Active Constellation Extension (ACE) [5], and Tone Reservation [6], [7]. The TR method reserves a small number of subcarriers and assigns appropriate values to reduce SCF. This approach is also iterative like the ACE, but it does not change data symbols at all.

2 Tone Reservation
In the TR approach, a small number of unused subcarriers are reserved by the transmitter for SCF control. These unused subcarriers are known as Peak Reduction Carriers (PRCs) which are designed to be orthogonal with the subcarriers containing data thereby causing no distortion. At the receiver side, the contents of the PRCs will be ignored since it does not contain useful information and also no side information needs to be transmitted. In general, there are two schemes to reduce the SCF in the TR scheme. The first is to select the PRC set for the TR technique for the improvement of the SCF performance of OFDM
signals. The second is to design the proper values on these PRCs to generate an optimal peak-canceling signal that minimizes the SCF of a transmitted OFDM signal.

3 Differential Scaling
Differential Scaling is another technique which is used for the reduction of SCF. Since different ranges of the amplitudes of the signal are scaled in a different manner, it is called Differential Scaling. We have considered three types of scaling as described below.

3.1 Scale Up
In this method, the higher amplitudes are scaled up by a factor of $x$. This leads to increase in average value and the peak value does not get affected. The resulting Squared Crest Factor gets reduced. The Squared Crest Factor reduction function can be defined as

$$h(x) = \begin{cases} \alpha x_p, & \text{if } x > \alpha x_p \\ \beta x, & \text{if } x < A \\ x, & \text{if } A \leq x \leq \alpha x_p \end{cases}$$

where $x_p$ be the amplitude peak value occurring in an OFDM symbol block, is the factor deciding the clipping threshold in terms of percentage of the peak value and $\alpha$ is the scaling factor in the range $[A, \alpha x_p]$ whose value is greater than one. The values of the parameters used are mentioned at the end of this section.

3.2 Scale Down
In this method, the higher amplitudes of the signal are scaled down by a factor of $x$. This leads to decrease in the peak value. Although the average value would also fall down, the resulting Squared Crest Factor reduces. Because the reduction in average power is lesser than the reduction in the peak power. The Squared Crest Factor reduction function can be defined as

$$h(x) = \begin{cases} \alpha x_p, & \text{if } x > \alpha x_p \\ \gamma x, & \text{if } B \leq x \leq \alpha x_p \\ \beta x, & \text{if } x < A \\ x, & \text{if } A \leq x \leq B \end{cases}$$

where $x_p$ be the amplitude peak value occurring in an OFDM symbol block, is the factor deciding the clipping threshold in terms of percentage of the peak value and $\alpha$ is the scaling factor for the range $[B, \alpha x_p]$. In order to make all these scaling techniques realizable, a marker needs to be used.

A marker, basically a small set of signal values that is embedded onto the transmitted information used to identify the combining (inversion) sequence at the receiver. The marker may be transmitted on different frequency orthogonal to the carriers or accommodated like the pilot carriers.

4 Tone Reservation with Customized Convex Optimization
The OFDM signal is generated and the QPSK modulation is performed. IFFT converts the frequency domain information back to time domain and the Tone Reservation technique is applied. Then again the information is converted back to frequency domain using FFT. The signal is demodulated so that the original signal is obtained and the CCDF characteristics are plotted. The process flow diagram for the Tone Reservation technique is given below.
4.1 Customized Convex Optimization

Convex minimization, a subfield of optimization, studies the problem of minimizing convex functions over convex sets. Convex Optimization [8] is used widely in the estimation of signals, communication and networking and for designing electronic circuits. Condition for convexity:

\[ f(ax + by) \leq a f(x) + b f(y) \]  

(4)

General form of optimization problem:

Minimize \( f(x) \) (objective function)
Subject to \( g(x) \leq b \) (constraint function)  

(5)

where \( x \) is the optimization variable and the vector \( x^* \) is called an optimal solution if it has the smallest objective value among all the vectors that satisfy the constraints. The SCF problem is treated as a convex optimization problem so that minimum SCF is obtained. The steps involved in convex optimization are described in the flowchart below.

The OFDM signal used here is with 48 data, 4 pilot symbols and 12 free subcarriers and 128 symbols. The length of the Cyclic Prefix is 16. The subcarriers (data/pilot) were modulated using 16-quadrature amplitude modulation (QAM). The baseband OFDM signal was generated by dividing the data into many data streams and was modulated by passing each to a subcarrier. The modulated data streams were sent in parallel on the orthogonal
subcarriers. The frequency constellation was then time domain transformed through IFFT. The signal is then demodulated and is received by the receiver. For Customized convex optimization instead of generating a new signal the tone reserved signal is given as input and the SCF characteristics are obtained.

5 Simulation Results

Simulation and analysis of the proposed method are carried out for 128 sub-carrier OFDM system with QPSK modulation scheme. The figures 3 and 4 shown below shows the generation of OFDM signals with 128 sub carriers and the QPSK modulation for the generated signal respectively.

Figure 5 shown above describes the CCDF vs SCF plot for Tone Reservation and Differential Scaling technique. The performance of Squared Crest Factor (SCF) is better in Differential Scaling when compared to Tone Reservation. The below table shows the SCF performance for different sampling rates and subcarriers.

<table>
<thead>
<tr>
<th>Sampling Rate</th>
<th>No of Subcarriers</th>
<th>SCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>L=2</td>
<td>128</td>
<td>3.984</td>
</tr>
<tr>
<td></td>
<td>256</td>
<td>4.342</td>
</tr>
<tr>
<td>L=4</td>
<td>128</td>
<td>4.01</td>
</tr>
<tr>
<td></td>
<td>256</td>
<td>4.346</td>
</tr>
<tr>
<td>L=8</td>
<td>128</td>
<td>4.019</td>
</tr>
<tr>
<td></td>
<td>256</td>
<td>4.363</td>
</tr>
</tbody>
</table>

Table 1 SCF performance for different sampling rate and different subcarriers

The below figure represents a general OFDM signal with and without optimization.

- **Fig.3** Transmitted data phase representation
- **Fig.4** Transmitted OFDM signal
- **Fig.5** CCDF vs. Squared Crest Factor
- **Fig.6** OFDM signal with and without Optimization
- **Fig.7** SCF Performance with Convex Optimized TR, TR without optimization and Differential scaling
From the figure 6, it is clear that the SCF is reduced while applying customized convex optimization. The SCF is about 10.9 when no optimization is applied and if optimization technique is applied, it reduces to 5.7. So there will be a reduction of 5.2 dB the optimization technique is applied.

From the figure 7 it is clear that after applying the customized convex optimization in the tone reserved signal the SCF gets reduced more when compared with Differential Scaling technique.

The below table shows the SCF performance for Differential Scaling and Tone Reservation with and without optimization.

<table>
<thead>
<tr>
<th>CCDF</th>
<th>SCF</th>
<th>Performance Measure</th>
<th>Normalized Frequency 0.4</th>
<th>Normalized Frequency 0.6</th>
<th>Normalized Frequency 0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>2.7</td>
<td>Without TR Technique</td>
<td>-0.2499 dB</td>
<td>-0.4758 dB</td>
<td>-0.8984 dB</td>
</tr>
<tr>
<td>0.25</td>
<td>3.0</td>
<td>With TR Technique</td>
<td>-1.328 dB</td>
<td>-3.374 dB</td>
<td>-3.262 dB</td>
</tr>
<tr>
<td>0.2</td>
<td>3.4</td>
<td>Reduction</td>
<td>1.0781 dB</td>
<td>2.8982 dB</td>
<td>2.3636 dB</td>
</tr>
<tr>
<td>0.15</td>
<td>3.65</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>3.84</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>3.98</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>4.19</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2 SCF performance

![Fig.8 PSD plot for the proposed technique](image)

From the Figure 8, the Power Spectral Density is reduced in TR technique with respect to Frequency. This shows that the Out of band radiation is minimized in TR Technique.

### 6 Conclusion

The proposed Tone reservation technique changes the response of the filter and hence produces better reduction in Squared Crest Factor. The SCF reduction is achieved more while applying Customized Convex Optimization. Simulation results clearly shows that this method offers less distortion in the processed OFDM symbols with better out-of-band radiation. The algorithm was defined for QPSK but it equally applies for other modulation techniques as well.
7 Acknowledgments
Our thanks to the experts who have contributed towards the development of this paper.

References: