Nonlinear Diffusion-based Image Restoration Model

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Abstract: - A novel nonlinear PDE - based image denoising and restoration approach is provided in this article. The proposed anisotropic diffusion based technique is derived from the influential Perona-Malik nonlinear diffusion scheme, representing an enhanced version of that scheme. Our model is based on new versions of the edge-stopping function and the conductance parameter. The proposed PDE-based filtering technique performs an efficient edge-preserving noise removal, outperforming the conventional smoothing approaches and other PDE denoising models.

Key-Words: - PDE-based image denoising, anisotropic diffusion, Perona-Malik scheme, diffusivity function, conductance parameter, edge preservation

1 Introduction

During the past three decades, the mathematical models have been increasingly used in some traditionally engineering domains like image processing, analysis, and computer vision. Since 1980s, the partial differential equations (PDEs) have been successful for solving numerous image processing and computer vision tasks [1].

In recent years, many image processing and analysis techniques making use of PDE-based algorithms and variational calculus have been developed. The variational and partial differential equation based approaches have been widely used and studied in these areas in the past few years because of their modeling flexibility and some advantages of their numerical implementation. Also, numerous classical methods can be reinterpreted as approximations of PDE-based models [2].

Image denoising and restoration with feature preservation represents still a focus in the image processing field, remaining a serious challenge for researchers. The conventional image denoising methods, such as averaging filter, median filter or 2D Gaussian filter are efficient in reducing the amount of noise, but also have the disadvantage of blurring the edges [3]. For this reason, numerous edge preserving techniques based on PDEs have been proposed in the past decades [4].

Numerous diffusion-based noise removal methods have been introduced since the early work of P. Perona and J. Malik in 1987, representing an anisotropic diffusion scheme for image denoising and restoration [5]. Their PDE model was able to filter the processed image while preserving its edges, by encouraging the diffusion within image regions and prohibiting it across strong boundaries.

Many nonlinear diffusion techniques derived from the influential Perona-Malik approach have been proposed in recent years [6-8]. In this article we provide a robust image denoising approach using an anisotropic diffusion based technique derived from the Perona-Malik filter.

Its PDE model is described in the next section, while the corresponding discretization scheme is described in the second. The performed restoration experiments and method comparisons are presented in the fourth section. The paper finalizes with a conclusions section and a list of references.

2 Anisotropic Diffusion based Model

We propose a novel nonlinear anisotropic diffusion based technique performing efficient image noise reduction while preserving successfully the image edges. Our PDE-based image denoising model is given by the following parabolic equation:

$$\begin{cases} \dfrac{\partial u}{\partial t} = div\left(\psi_{K(u)}\left(\left\|\nabla u\right|^2\right) \cdot \nabla u\right), & (x, y) \in \Omega \\ u(0, x, y) = u_0 \end{cases} \qquad (1)$$

where u_0 represents the noised image, its domain being $\Omega \subset R^2$. We propose the following diffusivity (edge-stopping) function $\psi_{K(u)} : [0, \infty) \to [0, \infty)$, for this restoration model:

$$\psi_{K(u)}(s^2) = \begin{cases} \alpha \cdot \left(\dfrac{K(u)}{\beta \cdot s^2 + \eta} \right)^{1/2} + \nu, & \text{for } s \in (0, M) \\ \alpha / \sqrt{\eta}, & \text{for } s = 0 \end{cases} \quad (2)$$

where $M > 0$ is an arbitrarily large but fixed value and the conductance diffusivity depends on the state of the processed image at time *t*. We consider a statistics-based automatic computation of this conductance parameter [7], based on the image noise estimation at each time:

$$K(u) = \frac{\|u\|_F \cdot med(u)}{\varepsilon \cdot n_u}, \quad (3)$$

where $\|u\|_F$ is the Frobenius norm of image *u*, *med*(*u*) represents its median value, n_u is the number of its pixels and the coefficients $\alpha, \beta, \eta, \nu, \varepsilon \in (0, 1]$.

The considered diffusivity function $\psi_{K(u)}$ is properly chosen, satisfying the main conditions related to a edge-stopping function [7]. Obviously, it is always positive and monotonically decreasing, because $\psi_{K(u)}(s_1^2) \le \psi_{K(u)}(s_2^2)$ for $\forall s_1 \ge s_2$. Also, one considers the flux function, defined as $\phi(s) = s \cdot g_{K(u)}(s^2)$, and compute its derivative. Since $\phi'(s) > 0$ for any *s*, our PDE model represents a forward parabolic equation that is stabile and quite likely to have a solution [9].

One can prove the existence and uniqueness of a weak solution in a certain case, related to some values of the parameters of this model. One could be demonstrated that our nonlinear diffusion based scheme converges if $\eta = \alpha^2$. The discretization of this continuous PDE mathematical model for image denoising and restoration is presented in the next section.

3 Numerical Approximation Scheme

We consider an efficient numerical approximation of the solution of the provided differential model. The proposed discretization scheme of equation (1) is based on a 4-NN discretization of the Laplacian operator, Δu [10].

Thus, from the equation (1) one obtains the approximation:

$$u(x, y, t+1) - u(x, y, t) \cong \psi_{K(u)}\left(|\nabla u|^2 \right) \cdot \Delta u \quad (4)$$

which leads to the following numerical approximating scheme [10]:

$$u^{t+1} \cong u^t + \lambda \sum_{q \in N_p} \psi_{K(u)}\left(|\nabla u_{p,q}(t)| \right)^2 \cdot \nabla u_{p,q}(t) \quad (5)$$

where

$$N_p = \{(x-1, y), (x+1, y), (x, y-1), (x, y+1)\} \quad (6)$$

is the set of pixels representing the 4-neighborhood of the image pixel described as a pair of coordinates *p* = [*x*, *y*], $\lambda \in (0, 1)$, and the image gradient magnitude in a particular direction at the iteration *t* is computed as follows:

$$\nabla u_{p,q}(t) = u(q, t) - u(p, t) \quad (7)$$

The obtained image restoration algorithm applies the procedure given by (5) on the processed image for each $t \in \{0, 1, ..., N\}$, where *N* is the maximum number of iterations. Our noise removal technique achieves the desired smoothed image u^N from the degraded image $u^0 = u_0$ in a relatively low number of steps, therefore *N* has to take quite small values. The filtering experiments performed with this algorithm are described in the following section.

4 Experiments

We performed hundreds of image restoration experiments using the described anisotropic diffusion-based algorithm. Our denoising approach has been tested on many images corrupted with various levels of Gaussian noise.

The following parameters of the model provided the best image filtering results: $\alpha = 0.7, \beta = 0.66, \eta = 0.5, \varepsilon = 0.3, \lambda = 0.33, \nu = 0.24$ and *N* = 20. Because $\alpha^2 \cong \eta$ and *N* is low enough, the diffusion equation converges fast to a unique solution. The performance of our restoration scheme was assessed by using the *norm of the error image* measure.

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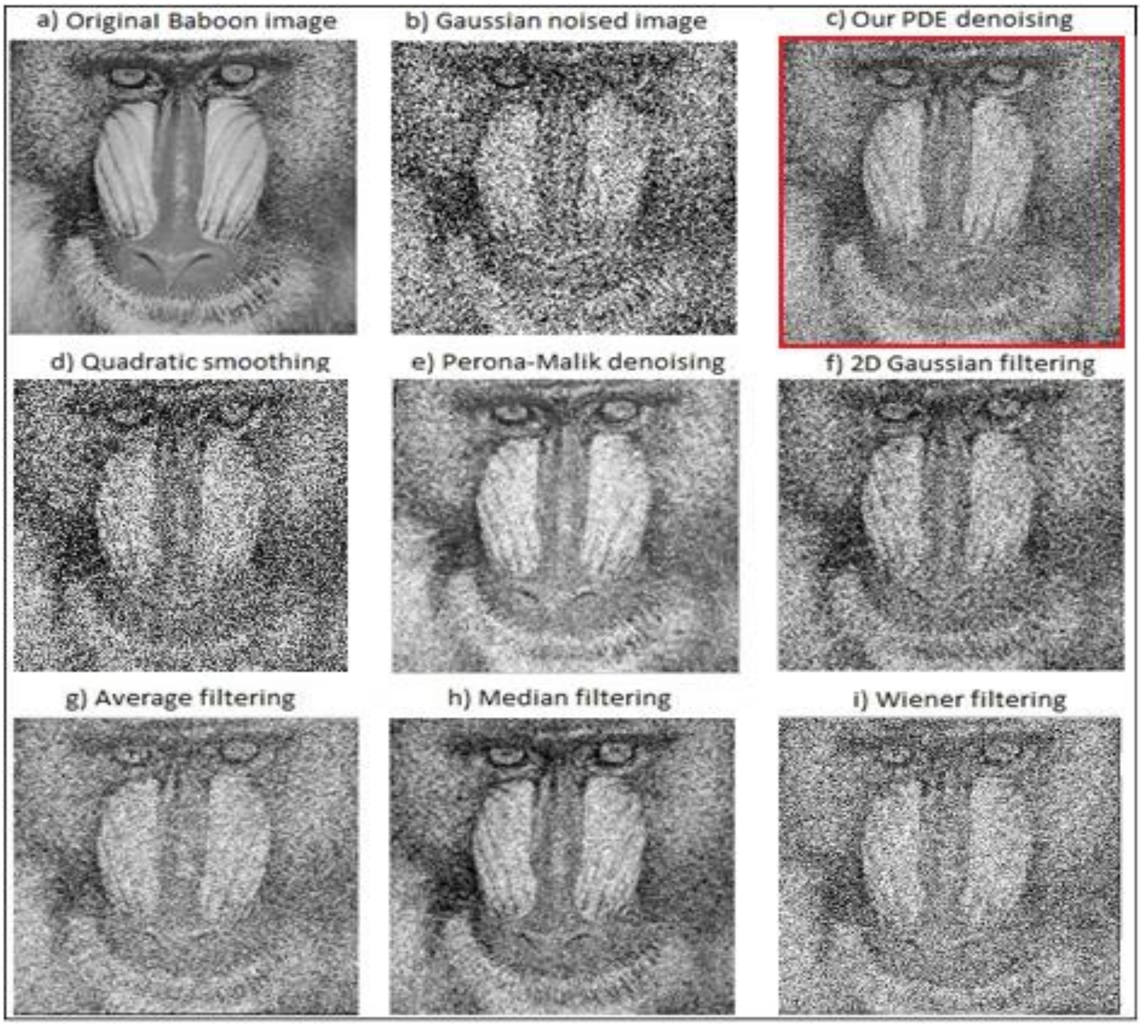

Fig. 1. Method comparison: denoising results produced by various algorithms

Table 1. Norm-of-the-error values for various restoration filters

Our alg.	Quadratic	P-M	Gaussian	Average	Median	Wiener
5.2×10^{3}	6.8×10^{3}	5.9×10^{3}	7.3×10^{3}	6.5×10^{3}	5.8×10^{3}	7.1×10^{3}

Our PDE-based technique outperforms many other noise removal methods, obtaining a much better edge-preserving image smoothing, as resulting from the method comparison described in the above figure and table. In Fig. 1, one can see: a) the original [512 x 512] *Baboon* image; b) image corrupted with Gaussian noise given by $\mu = 0.211$ and *var* = 0.023; c) the image restored by our PDE model; d) quadratic denoising [11]; e) Perona-Malik filtering; f) – i) denoising results achieved by the 2D Gaussian, average, median and Wiener $[3 \times 3]$ filter kernels. The corresponding norm of the error values are displayed in Table 1, the scheme proposed here having the minimum value of this measure.

5 Conclusions

We have proposed a PDE denoising technique based on anisotropic diffusion in this paper. This technique performs an efficient noise removal and also preserves the image edges.

The proposed models for the edge-stopping function and its conductance parameter represent the major contributions of this work. The mathematical discussion about the proper choice of the diffusivity function and the convergence of our PDE scheme, and the obtained iterative algorithm represents also important contributions of this paper.

The performed experiments, providing satisfactory noise reduction and method comparison

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results, prove the effectiveness of the technique developed by us.

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