Abstract: - Many universities are realizing that modeling and simulation is becoming an important tool in finding the strategy for solving and understanding numerous and diverse problems. The paper we first briefly introduce the theory of system approach, modeling and simulation as a method of multidisciplinary and system approach to education. Secondly, the two case studies illustrating step by step process of development of modeling, computer simulation, visualization and animation will be introduced. The cases studies will present possibility of simulation of static as well as dynamic systems in MS Excel worksheet and chart.

Key-Words: - Animation, Computer simulation, Dynamic system, Mathematical modeling, Static system, System approach, Visualization

1 Introduction
The term “developing of theoretically acquired knowledge” can be connected with the terms “system”, “model”, “computer simulation”, “dynamic visualization”, “animation” or “multidisciplinary approach”, which are important in current approach to not only scientific, technological and professional practice but also in high school education of science subject. The cases studies will present possibility of simulation of static as well as dynamic systems in MS Excel worksheet and chart. Presented case studies are used in the learning of programming instudy program Physical Measurement and Computer Science Departments of Physics and Department of Informatics, Faculty of Science University of Hradec Králové.

2 Problem Formulation
System approach is closely related to the concept of system. The system in general terms can be based on e.g. understood as a defined set of elements and relationships that creates an integrated whole. The system is also part of external reality and communicates with its environment through its inputs and outputs. System with no links to the surroundings is closed system.

In connection with the concepts of system and system approach is necessary to mention the other term that is commonly used – interdisciplinarity. This concept can be understood as a method of linking and active cooperation between different sciences in order to achieve integrated and synergistic results in theoretical and practical professional activities, science and research.

The systems approach rather than interdisciplinarity talk about multidisciplinarity. A multidisciplinary approach is used mainly in the method of solving problems and creating technically functional units, such as the creation of the computer program – see e.g. [1]. The main reason for the introduction of multidisciplinarity in connection with the system and a systems approach is that the research system as a whole requires a high degree of highly specialized knowledge in different disciplines and these specializations has to be closely linked by mutual connection. Only a multidisciplinary approach will ensure that the whole system will be studied systematically and comprehensively.

Although the interdisciplinary approach in the context of learning process is frequently discussed, the concept of system approach in educational practice is not sufficiently specific and widely implemented.

2.1. System approach and education
System approach, scientific modeling and computer simulation are the terms which are closely related. System approach enables description of the real processes, without which mathematical model and computer simulation model cannot be realized.
From the education point of view the system approach can be applied:
- In the field of sciences - in case of models and simulations of science processes and phenomena;
- In the technical field - in case of the process control of machines and simple robots;
- In the humanities and social studies - in case of the processes and phenomena associated with this issue;
- In management - in case of the management processes and quality control processes.

The other information on system approach in education can be found e.g. in [2], [3].

2.2 Modeling
The main goal of modeling is not only describing the content, structure and behavior of the real system representing a part of the reality but also describing the processes. The process can be understood as series of transformations that changes the input values to output values. From the system point of view the process is dynamic system in which the values of the characteristic of the system elements are changed under the influence of the external elements. The first step in the process of computer simulation is creation of conceptual model of the studied real system / real process. Conceptual model can be represented in different way. The most used representations are:

- Mathematical equitations establishes mathematical model of the studied real system. The model can be obtained either theoretically based on basic physical properties of the system, or numerically by means of the measured values. Determination of parameters of theoretical model developed from empirical data is called system identification.
- Scatter charts (scatter plot) is a type of mathematical diagram using Cartesian coordinates to display values for two variables for a set of data in 2D types of the chart. The scatter plot takes two sets scalar variables and uses them for two axes in 2D space. Time depending variables create dynamic scatter chart representing time depending conceptual model of the dependency of system outputs on its inputs.

Modeling of the static system of real phenomena will be shown in the first case study of this paper.
Modeling of the dynamic system of real process will be shown in the second case studies of this paper.

2.3 Simulation
The process of modeling is closely related to the simulation. Simulation can be understood as process of executing the model. Simulation enables representation of the modeled real system or real process and its behavior in real time by means of computer. The simulation enables also visualization and editing of the model.

A typical simulation model can be written both through specialized programming languages that were designed specifically for the requirements of simulations, or the simulation model can be created in standard programming languages.

From the above considerations, it is clear that simulation is a process that runs on the computer. In some publications, therefore, can be found the term ”computer simulation“. It generally is valid that computer simulation is a computer-implemented method used for exploring, testing and analysis of properties of the conceptual (mathematical or process) models that describe the behavior of the real systems or real process which cannot be solved using standard analytical tools, which is discussed e.g. in [4], [5].

Simulation of the static system of real phenomena via 2D scatter chart created in MS Excel program will be shown in the first case study of this paper.
Simulation of the dynamic system of real process via 2D scatter chart created in MS Excel program will be shown in the second case studies of this paper.

Simulation has from the scientific point of view several functions [1]. From educational point of view the simulation, simulation model and visualization of simulation results on the screen help students better understanding the basic features of the processes and systems and develop their intuition. It is also essential that the teaching by means of simulation is much cheaper and faster than the teaching carried by real experiment. In some cases providing the real experiment cannot be feasible.

3 Case Study 1
- Simulation of Static System
Case study of the simulation of static system in MS Excel chart will be demonstrated by creation of rainbow function. Before starting the discussion on simulation model we analyze the problem from mathematical and physical point of view.

3.1 Mathematical model
The system is based on principles of reflection and refraction of light. In reflection and refraction the reflected (refracted) ray remains in the plane of impact. The plane of impact is determined by two straight lines – an impact ray and a perpendicular to the optical boundary in the place of impact. Moreover for the reflection is valid that angle of reflection is equal to the angle of impact – i.e.

\[ \alpha = \alpha' \]  \hspace{1cm} (1)

For refraction is valid that the proportion of the sine of angle of impact to the sine of angle of refraction is equal to the relative index of refraction – i.e.

\[ \frac{\sin \alpha}{\sin \beta} = n. \]  \hspace{1cm} (2)

Other well know condition that the sum of the inner angles of triangle is always 180º, in tetragon it is 360º, in pentagon it is 540º, and that the correspondent top angles are equal.

With the use of the Figure 1 it is relatively easy to derive right formula for rainbow function.

\[ \gamma = 360^\circ - 2\alpha - 2(180^\circ - 2\beta) = 4\beta - 2\alpha = 4 \arcsin \left( \frac{h}{n} \right) - 2 \arcsin h \]

Because sum of angles in right top corner in figure 1 is the full angle, so we can write:

\[ \gamma = 4\beta - 2\alpha. \]  \hspace{1cm} (3)

Angle \( \alpha \) is the angle of incidence, so \( \sin \alpha = h \), therefore \( \alpha = \arcsin h \). Similarly angle \( \beta \) is the angle of refraction, so \( \sin \alpha = n \sin \beta \), therefore \( h = n \sin \beta \) and \( \sin \beta = \frac{h}{n} \), therefore \( \beta = \arcsin \frac{h}{n} \).

That for the resulting angle of refraction \( \gamma \) on a drop of water is valid this relation:

\[ \gamma = 4 \arcsin \left( \frac{h}{n} \right) - 2 \arcsin(h) \]. \hspace{1cm} (4)

where \( h \) is a relative distance of the dropping ray from the parallel ray crossing the centre of the drop (the real number from 0 to 1). The \( n \) is the relative index of refraction for the transition of the ray from air to water. Because the absolute index of air refraction is normally very close to 1 (1.00026), it is possible to substitute the absolute index of the water refraction for \( n \).

### 3.2 Simulation model

Rainbow function \( \gamma \)is real function of one real variable \( h \) (see equitation (4)), where index of refraction\( n \) that depends on a wave length (and also on the colour) of the dropping light is a parameter. The process of function for three different amounts of refraction index \( n_R = 1.330 \) (red light), \( n_G = 1.334 \) (green light), and \( n_B = 1.337 \) (blue light) can be calculated and depicted as a function type X-Y in the spreadsheet.

Construction of the simulation model in MS Excel is as follows.
In the spreadsheet MS Excel we prepare amounts of the independent variable $h$ from 0 to 1 with the step of 0.01 in the column A and for this column we define a title $h$ in the bookmark “Formula” by the function “Define title…”.

Columns B, C and D we name RED, GREEN and BLUE. In the second line of the chart we state the amounts of indexes of refraction and choose for them a title $n$.

Then we can write the formula:

$$=(4*\text{ARCSIN}(h/n)-2*(\text{ARCSIN}(h))*180/\pi())$$


to the top left cell (B4). The formula figures out the amount of the function $\gamma$ in radians and by multiplying it with $180/\pi$ it transfers it to the angle degrees. Then we copy the formula to the right and down to the whole area of the outputting amounts.

We use graph X-Y with the smoothed connectors and with the grid for more precise deduction of amounts. We re-colour the colours of individual lines so that they correspond to the three chosen colours of the Sun light spectrum.

Simulation model is shown on the Figure 4. Detailed information on creation of rainbow functions can be found in [6].

![Fig.2 Static simulation model of rainbow function](image)

**4 Case Study 2**  
**- Simulation of Dynamic System**

Case study of the simulation of dynamic system in MS Excel chart will be demonstrated by creation of animation of ballistic trajectory. Before starting the discussion on simulation model we analyze the problem from mathematical and physical point of view.

**4.1 Mathematical model**

Ballistic trajectory is trajectory of the flight projectile moving in the air. The main forces acting on the projectile are gravity, and air resistance. Gravity imparts a downward acceleration on the projectile, causing it to drop from the line of sight. Air resistance, decelerates the projectile with a force proportional to the square of the velocity.

During flight, gravity and air resistance have a major impact on the path of the projectile, and must be accounted for when predicting how the projectile will travel.

The ballistic trajectory is given by equitations of motion. The equitations of motion in the $x$, resp. $y$ directions are:

$$ma_x = -kv_x$$  \hspace{1cm} (5)

$$ma_y = -mg - kv_x$$  \hspace{1cm} (6)

where $m$ is mass of the projectile, $k$ is coefficient of the air resistance, $a_x$, resp. $a_y$ are accelerations, $v_x$ resp. $v_y$ are velocities and $g$ is gravity acceleration.

The equitations (5) and (6) can be solved either analytically or numerically. We will show in the paper numeric solution.
Numeric solution is based on splitting the trajectory of the ballistic motion to number of the small time elements $dt$. The solution is as follows:

\begin{align*}
    t_{n+1} &= t_n + dt, \\
    (a_x)_n &= -k(v_x^2)_n, \\
    (a_y)_n &= -k(v_y^2)_n - g, \\
    (v_x)_{n+1} &= (v_x)_n + (a_x)_n dt, \\
    (v_y)_{n+1} &= (v_y)_n + (a_y)_n dt, \\
    (x)_{n+1} &= (x)_n + (v_x)_n dt, \\
    (y)_{n+1} &= (y)_n + (v_y)_n dt,
\end{align*}

with initial conditions:

\begin{align*}
    (v_x)_0 &= v_0 \cos \alpha, \\
    (v_y)_0 &= v_0 \sin \alpha, \\
    (x)_0 &= 0, \\
    (y)_0 &= h,
\end{align*}

where $v_0$ is initial velocity of the projectile shoot, $\alpha$ is angle of the shoot and $h$ is initial height of the shoot.

### 4.2 Simulation model

Simulation model of ballistic curve is based on animation of ballistic trajectory.

Animation in general means a way how a static image can be seemingly set in motion. The principle of an animation is recording of a sequence of images which slightly differ.

The principle of the animation of the scatter chart in MS Excel is based on the change of one or more parameters of the charts input data of the dynamic objects or curves. The objects or curves step by step change its position in relation to the static baselines which are time dependence function of one or more parameters. A change of these parameters can be achieved by using so-called iterative calculation of values in the cell— for more detail see [3], [4], [5].

Main parameter responsible for iterative recalculation is time $t$. The trajectory will be drawn step by step from time $t=0$ with the time increment $dt$.

The value of the actual time $t$ is saved in e.g. cell D2. The value of time increment $dt$ is saved e.g. in cell D10. The circular reference formula $= D2 + D10$ is entered into cell D2. This formula expresses that the value of time in D2 after one iterative recalculation step increases by the increment time $dt$- iteration (animation) step.

By repeated pressings, eventually by holding down the key F9, the value of cell D2 will step by step increase by the value of the animation step – value in cell D10.

If only the formula $= D2 + D10$ was entered to the cell D2, the value of D2 would after each iteration step still increase. It is therefore necessary to determine the terminating condition, i.e. minimum and maximum time limits.

Minimum time is $t = 0$ s. Maximum animation time $t_{max}$ is given by time, when $y$ coordinate of the projectile is less than 0.

This can be done by adding the following condition to formula in cell D2:

$$= IF (MIN($S:$S)<0;0;D2+D10)$$

Principle of the iterative recalculation of the cell D2 and setting of initial conditions is shown on Figure 3.

![Fig. 3 Principle of iterative recalculation of D2](image)

The solution of simulation model is shown on the Figure 4. Green line is ballistic trajectory, red line is trajectory of projectile with no air resistance.

Detailed derivation of the mathematical model and detailed creation of the simulation model can be found in [5] and [7].

Of
4 Conclusion
The paper presents possibility of the creation of static as well as dynamic simulation model of the real physical processes. MS Excel spreadsheet and MS Excel chart has been presented as appropriate tools for creation of such simulation model.

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