Distillation Control: type-1 and type-2 fuzzy control application

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Abstract: - Two advanced controllers were developed in this paper, i.e., a type-1 and a type-2 fuzzy logic controller and their performances compared in simulation for a case study, i.e., a binary distillation column, which is characterized by high nonlinearities and parameter uncertainties in the underlying mathematical model. Priority was given to the type-2 fuzzy controller, with special attention to input fuzzy sets and type-2 Gaussian membership functions. All the simulation results confirmed the robustness and the effectiveness of the fuzzy control action, with evident advantages for the type-2 fuzzy controller.

Key-Words: - Type-2 fuzzy logic controller; feedback control; non-linear dynamical system; mathematical modeling; parameter uncertainty; distillation column.

1 Introduction

Many industrial chemical processes characterized by high nonlinear dynamics and parameter uncertainties make use of conventional controls with a non-satisfactory response in terms of robustness and velocity. Traditional fuzzy logic controllers (type-1 FLCs) have been reported to be successfully used (Zadeh, 1965) for a number of complex and nonlinear processes: Çetinkaya et al. (2006) employed a fuzzy-relational models-dynamics matrix control for an optimal temperature control in a batch polymerization reactor; Shahraz and Bozorgmehry Boozarjomehry (2009) proposed a fuzzy sliding mode control approach for nonlinear chemical processes; moreover, Lima et al. (2009, 2010) adopted not only a fuzzy controller, but also a model-based predictive control system using fuzzy logic for polymerization processes.

Despite their popularity, scientific research has recently shown that in many cases type-1 FLCs can have difficulties in minimizing the effect of uncertainties in the plant model, which are often responsible for the degradation of the actual process control. Therefore, the choice of type-1 FLCs and then of type-1 fuzzy sets (Zadeh, 1975) may not always represent the optimal solution to a control problem, whereas a possible alternative is using type-2 fuzzy logic controllers (type-2 FLCs) that make use of type-2 fuzzy sets (Mendel, 1999; 2000) characterized by a larger number of parameters and, as a consequence, of freedom degrees. The type-2 fuzzy logic control has been applied in various different fields like anesthesia control (Castillo, 2005), level control (Wu and Tan, 2006), marine diesel engine control (Lynch et al., 2006) and more recently vehicle non-linear active suspension systems (Cao et al., 2008), autonomous mobile robot control (Martinez et al., 2009), cable-driven parallel mechanism (Li et al., 2010), Kundur test system (Robandi and Kharisma, 2009), biochemical reactor control (Galluzzo et al. 2008; Galluzzo and Cosenza, 2009a, 2009b, 2012), continuous stirred tank reactor (Galluzzo and Cosenza, 2011) and inverted pendulum model (Li and Sun, 2012).

Fuzzy logic has been applied to control of a distillation column, too. Luo et al. (1995) developed a fuzzy-neural net-based inferential control for a high purity distillation column, while a neuro-fuzzy modeling and control of a batch process involving simultaneous reaction and distillation was developed by Wilson and Martinez (1997). More recently Fabro et al. (2005) proposed a startup technique of a distillation column driven by an intelligent control based on fuzzy sets optimized by genetic algorithms; Worapradya and Pratishthananda (2005) developed a real-time control of a binary distillation column using HGA fuzzy supervisory PI controllers; an adaptive
neurofuzzy control using soft sensors to continuous distillation was applied by Fernandez et al. (2008); the control of binary distillation column using PI controllers was proposed by Javadi and Hosseini (2009); while Barceló-Rico et al. (2011) carried out modeling and control of a continuous distillation tower through fuzzy technique.

The main aim of the fuzzy PI control method developed in this paper is to have the head concentration $x_1$ in binary distillation (starting from a given initial condition) tracking the set point, despite uncertainty in system parameters, with special attention to the relative volatility $\alpha$.

2 Material and methods
2.1. Distillation column model
The system model considers the separation of a liquid stream containing two components (binary distillation) and refers to a conventional plate distillation column.

![Distillation Column Diagram](image)

Fig. 1. Distillation Column (courtesy of University of California edu.docdat.com (2014))

The feed, i.e., the mixture to be separated, enters the central section of the column (see Fig.1) as either a liquid or a liquid-vapor mixture or a vapor. The “stripping section” is the part of the column below the feed section where the concentration of the more volatile component decreases. The “enriching” or “rectifying section” is the name given to the part of the column above the feed section where the concentration of the more volatile component increases. On the top of the column, the overhead vapor, which is mostly made of the more volatile component, moves to a heat exchanger (“condenser”) and the condensate is collected in a tank from which the distillate and reflux streams are taken. The reflux is fed back to the head of the column to provide liquid flow above the feed point and it moves down the column in countercurrent flow with the vapor flowing up the column. On the bottom there is another heat exchanger (“reboiler”). The residue is taken from the bottom as a liquid stream, then it is partly vaporized in the reboiler and re-introduced in the column where flows up the column in countercurrent with the liquid moving down the column.

The dynamical mass balance on the component in the generic $i$-th stage, excluding the feed stage, the condenser and the reboiler, is as follows:

$$\frac{dM_i}{dt} = L_{i-1}x_{i-1} + V_{i+1}y_{i+1} - L_i x_i - V_i y_i$$  \hspace{1cm} (1)$$

The steady state assumption holds for the overall mass balance; therefore, it is:

$$V_i = V_{i+1}$$  \hspace{1cm} (2)$$
$$L_i = L_{i+1}$$  \hspace{1cm} (3)$$

The overall mass balances for the feed tray are:

$$V_{af} = V_{af+1} + F(1-q)$$  \hspace{1cm} (4)$$
$$L_{af} = L_{af+1} + Fq$$  \hspace{1cm} (5)$$

where $q$ is the rate (percentage) of liquid present in the feed and is expressed as:

$$q = \frac{L_{af} - L_{af-1}}{F}$$  \hspace{1cm} (6)$$

whereas its complement to 1 $(1-q)$ represents the rate of vapor present in the feed.

The overall mass balance in the condenser is:

$$L_o + D = V_2$$  \hspace{1cm} (7)$$

where $L_o$ and $D$ are the molar flow rates for the reflux and distillate respectively.

The overall mass balance in the reboiler is:

$$B = L_{NS-1} - V_{reboiler}$$  \hspace{1cm} (8)$$

where $L_{NS-1}$ is the total amount of the liquid entering the reboiler, $B$ is the bottom product flow rate and $V$ is the flow rate of vapor from the reboiler that is re-injected into the column. The component balance in the condenser is:
\[
\frac{dx_i}{dt} = \frac{[V_R(x_{i-1} - x_i)]}{M_D} \tag{9}
\]

In the rectifying section:
\[
\frac{dx_i}{dt} = \frac{[L_R x_{i-1} + V_R y_{i-1} - L_R x_i - V_R y_i]}{M_T} \tag{10}
\]
(from \(i = 2\) to \(i = nf-1\)).

In the feed tray:
\[
\frac{dx_{nf}}{dt} = \frac{[L_R x_{nf-1} + V_{nf} y_{nf-1} + Fz_f - L_R x_{nf} - V_R y_{nf}]}{M_T} \tag{11}
\]

in the stripping section (from \(i = nf+1\) to \(i = ns-1\)):
\[
\frac{dx_i}{dt} = \frac{[L_s x_{i-1} + V_s y_{i-1} - L_s x_i - V_s y_i]}{M_T} \tag{12}
\]

and in the reboiler:
\[
\frac{dx_{ns}}{dt} = \frac{[L_s x_{ns-1} + Bx_{ns} - V_s y_{ns}]}{M_B} \tag{13}
\]

The vapor-liquid thermodynamic equilibrium relationship for each component is:
\[
y_i = \frac{\alpha x_i}{1 + (\alpha - 1)x_i} \tag{14}
\]

The feed conditions here considered are: \(F=1\) mol/min, \(z_f=0.5\), \(q=1\), whereas the model parameters are: \(\alpha=1.5\), \(ns=41\), \(nf=21\), \(V_{reboiler}=3.2\) mol/min as the vapor flow rate. The reflux flow rate is the system manipulation variable as it is usual in distillation control (Javad and Hosseini, 2009).

### 3 Type-2 Fuzzy Logic

#### 3.1. Interval Type-2 Fuzzy Sets

The main characteristic of type-2 fuzzy sets (Zadeh, 1975) is to incorporate uncertainty about the membership function in to fuzzy set theory. Type-2 fuzzy logic shows in fact all its potential only in environments full of uncertainties. It is well known that uncertainty is an inherent part in the real control systems (Klir and Wierman, 1998). The measurement noise, the coarse estimation of process parameters, the system parameter changes, even the ambiguity in the meaning of words used in the FLC rules are all source of uncertainties (Klir and Wierman, 1998; Mendel, 2000; Zadeh, 2005).

\[\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x, u) / h(x, u) \tag{15}\]

with \(x \in X\), \(u \in J_x \subseteq [0,1]\) and \(\mu_{\tilde{A}}(x, u) \in [0,1]\) is a type-2 membership function and is a secondary grade while the primary membership of \(x\) is the domain of the secondary membership function (Fig. 2a). An interval type-2 fuzzy set \(\tilde{A}_f\) (Fig. 2b) can be instead defined (Mendel, 2000; Liang and Mendel, 2000; Castro et al. 2007) as a particular case of type-2 fuzzy logic and is expressed as:
\[\tilde{A}_f = \int_{x \in X} \int_{u \in J_x \subseteq [0,1]} \tilde{\mu}_{\tilde{A}_f}(x, u) / h(x, u) \tag{16}\]

The main difference with a normal type-2 fuzzy set is that the secondary grade of an interval set \(\mu_{\tilde{A}_f}(x, u)\) can assume only two values: 0 or 1 and not values between 0 and 1 as a normal set. Actually, only interval type-2 fuzzy sets are treated. The mathematics that is needed for such sets is in fact simpler than that required for general type-2 fuzzy sets.

#### 3.2. Type-2 Fuzzy System Structure

Fig. 3 shows a type-2 fuzzy logic system (Mendel, 2000; Liang and Mendel, 2000; Karnik and Mendel, 1998), which is characterized by four components: a Rules-Base, a Fuzzifier, an Inference-Engine and an Output-Processor. The main difference between the...
type-2 fuzzy logic system in Fig. 3 and a type-1 system is represented just by the type-2 output processor. This last component in fact maps a type-2 fuzzy set into a type-1 fuzzy set with the type-reducer block and then transforms the fuzzy output in a crisp output with a defuzzifier block.

The structure of a type-2 fuzzy rule coincides with that of a type-1 fuzzy rule. Considering in fact a type-2 FLS with \( p \) inputs \( x_1, \ldots, x_p \in X \) and one output \( y \in Y \), and assuming that there are only \( M \) rules, the \( l \)th rule of the rule base has the following form (Mendel, 2000):

\[
R_l : \text{IF } x_1 \text{ is } \tilde{F}^l_1 \text{ and } \ldots \text{ and } x_p \text{ is } \tilde{F}^l_p, \text{ THEN } y \text{ is } \tilde{G}^l \quad l = 1, \ldots, M
\]  

(17)

4 PID Fuzzy control

4.1. Control strategy

In Fig. 5 the approach to steady state in the “knee” region of Fig. 4 at Reflux = 2.706 is shown as a time series chart in the time domain.

A similar result can be seen in Fig. 6, but this time the reboiler vapor flow rate increase (from 3.1 to 3.2 and from 3.2 to 3.5 mol/min) has negative effects on the controlled variable \( x_i \). A high performance controller must be able to master the
system, driving it towards the desired set-point value, minimizing the negative effects on the system parameter variation and preventing the system to work in undesirable operating conditions. Type-1 and above all type-2 fuzzy logic controllers represent a good and valid choice to handle such uncertainty that is often not taken into account in the design of the controller. The regulation of the head composition using the reflux flow rate as manipulated variable is actually used in industrial practice and represents the fastest way to control this composition.

### 4.2. Fuzzy controllers

It is well known that FLC (fuzzy logic control) and PID (proportional integral derivative) control algorithms have been and continue to be an important research field (Jantzen, 1998, 2007). Experimental results confirm that fuzzy PID controller is able to perform better than the conventional PID controller (Natsheh and Buragga, 2010). The synergic fusion of these two control algorithms is therefore a useful and effective way to control systems characterized by complexity.

The tuning method for type-1 and type-2 fuzzy controllers is that of Jantzen (2007) and each fuzzy controller is a function of three inputs: error, derivative error, and integral error.

\[
U = f(GE * \text{error}, GCE * \text{derivative error}) + GIE * \text{integral error} \]

(18)

The advantages to use derivative error and integral error are those to reduce overshoot and to remove steady state error (“offset”), respectively. It is common (Fig. 7) to separate the integral action in the fuzzy PD+I (FDP+I) controller (Jantzen, 1998), in order to avoid a rule base with three inputs (e.g., rather big and complex).

![Fig. 7. Fuzzy Proportional Integral Derivative Controller Structure.](image)

Each fuzzy controller developed in this work makes use of Sugeno inference method (Takagi and Sugeno, 1985) and Gaussian membership functions and is characterized by two inputs: error and derivative error; and one output (manipulation variable).

### Table 1

#### Type-1 fuzzy sets

<table>
<thead>
<tr>
<th>Error fuzzy sets</th>
<th>Integral error fuzzy sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative = \exp(-0.5*(error+0.5)/0.3)^2;</td>
<td>Negative’ = \exp(-0.5*(delta_error+0.5)/0.3)^2;</td>
</tr>
<tr>
<td>Zero = \exp(-0.5*(error)/0.3)^2;</td>
<td>Zero’ = \exp(-0.5*(delta_error)/0.3)^2;</td>
</tr>
<tr>
<td>Positive = \exp(-0.5*(error-0.5)/0.3)^2;</td>
<td>Positive’ = \exp(-0.5*(delta_error-0.5)/0.3)^2;</td>
</tr>
</tbody>
</table>

**output** = [-2; -1; 0; 1; 2]

### Table 2

#### Type-2 fuzzy sets

<table>
<thead>
<tr>
<th>Error fuzzy sets</th>
<th>Integral error fuzzy sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>NegativeLower = \exp(-0.5*((error+0.5)/0.34)^2);</td>
<td>NegativeLower’(1) = \exp(-0.5*((delta_error+0.5)/0.18)^2);</td>
</tr>
<tr>
<td>NegativeUpper = \exp(-0.5*((error+0.5)/0.38)^2);</td>
<td>NegativeUpper’(1) = \exp(-0.5*((delta_error+0.5)/0.30)^2);</td>
</tr>
<tr>
<td>ZeroLower = \exp(-0.5*(error)/0.3)^2;</td>
<td>ZeroLower’(2) = \exp(-0.5*(delta_error)/0.18)^2;</td>
</tr>
<tr>
<td>ZeroUpper = \exp(-0.5*(error)/0.345)^2;</td>
<td>ZeroUpper’(2) = \exp(-0.5*(delta_error)/0.30)^2;</td>
</tr>
<tr>
<td>PositiveLower(3) = \exp(-0.5*(error-0.5)/0.345)^2;</td>
<td>PositiveLower’(3) = \exp(-0.5*(delta_error-0.5)/0.18)^2;</td>
</tr>
<tr>
<td>PositiveUpper(3) = \exp(-0.5*(error-0.5)/0.39)^2;</td>
<td>PositiveUpper’(3) = \exp(-0.5*(delta_error-0.5)/0.39)^2;</td>
</tr>
</tbody>
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**output** = [-2; -1; 0; 1; 2]

### Table 3

<table>
<thead>
<tr>
<th>Rule base matrix</th>
<th>Negative</th>
<th>Zero</th>
<th>Positive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative’</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Zero’</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>Positive’</td>
<td>0</td>
<td>-1</td>
<td>-2</td>
</tr>
</tbody>
</table>

Type-1 and type-2 fuzzy logic PID controllers have same inputs, same output, and same rule base. The main differences are just in the fuzzy sets. In Tables 1-2 all type-1 and type-2 fuzzy sets are shown. In Table 3 the rule base matrix constituted by 9 rules and equal for type-1 and type-2 FLCs is revealed. The center of type-2 Gaussian membership functions is the same of type-1 Gaussian membership functions. The amplitude values of type-2 fuzzy sets are, on the contrary, chosen by
minimizing an objective function (i.e., IAE) (Galluzzo and Cosenza, 2011; 2012).

5 Simulations and results

In the simulation results reported in this work (Figs. 8 through 16), the controlled variable $x_1$ was always considered to have an initial condition equal to 0.98. In Fig. 8, with an imposed set-point $x_{1,SP}=0.99$, the performance of the system controlled by type-1 and type-2 FLCs is shown by disturbing the system with a linear variation of the parameter $\alpha$ from 1.5 to 1.3 during the entire simulation, i.e., in a time as long as 50 min; further, a step in the vapor flow from 3.2 to 3.5 mol/min is superimposed at time $t=30$ min.

Fig. 8. Time trajectory of the system $x_1$ concentration for set point tracking ($x_{1,SP}=0.99$) under a disturbance in the parameter $\alpha$ (linear variation from 1.5 to 1.3 throughout the simulation time) and another disturbance in the vapor flow rate (step from 3.2 to 3.5 mol/min at $t=30$ min).

Similarly, Fig. 11 shows the behavior of the manipulation variable (reflux flow rate).

Fig. 11. Time trajectory of the system manipulation variable (reflux flow rate) for set point tracking ($x_{1,SP}=0.99$) under a disturbance in the parameter $\alpha$ (linear variation from 1.5 to 2 throughout the simulation time).

Analogous results are shown in Figs 12-14 for set point tracking and disturbance rejection. In this case, the set-point was changed stepwise from 0.99 to 0.995 at $t=28$ min and, later on, $\alpha$ was stepped from 1.56 to 1.8 at $t=35$ min. The zoom of Fig. 12 shown in Fig. 13 confirms the previous statement.
beginning of the simulation, is located at one extreme of its range revealing a condition of saturation (see the black curve in the leftmost part of Figs 15 and 16). Later on, the error decreases because of the loop controlling action, the first input moves toward the center of its range and, after some time, the system reaches the set point and the error approaches zero.

For the type-2 FLC, on the contrary, the presence of a lower and an upper membership function as the extremes of the first input range allows for more error handling. It was observed that the type-2 FLC does not reach the saturation condition of its type-1 counterpart and the error moves toward the center of its range (see the shaded band in the central part of Fig. 15) in a shorter time, thus ensuring faster and a more effective control action.

In order to give a more sound explanation of the advantages provided by a type-2 FLC in the above results, the attention was focused on the control loop error, i.e., the first input to both type-1 and type-2 FLCs. By tracking the error during each simulation run, it was observed that the earliest value of the error, i.e., the first input to the type-1 FLC at the beginning of the simulation, is located at one extreme of its range revealing a condition of saturation (see the black curve in the leftmost part of Figs 15 and 16). Later on, the error decreases because of the loop controlling action, the first input moves toward the center of its range and, after some time, the system reaches the set point and the error approaches zero.

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6 Conclusions

The application of type-2 and type-1 FLCs to a complex and nonlinear dynamical system, like a
distillation column presenting parameter uncertainty, has been analyzed by simulations. All the simulation results demonstrate the effectiveness of type-2 FLC in achieving a very high control performance and allowing a faster and more precise control of the process, both for set point tracking and disturbance rejection, with less amount of overshoot as compared to the type-1 counterpart controller. Moreover, the type-2 FLC, because of the built-in presence of a lower and an upper membership function as the extremes of its first input range, allows for a smoother error handling and prevents the occurrence of saturation conditions. Type-2 FLCs therefore represent an effective solution for the control of non-linear processes in which uncertainty is present, as usually occurs in the real world.

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