Two-mass mathematical model of magnetoelectric converter for reversible scanning device

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Abstract: - The performance of reversible scanning device primarily depends on the control system, which performance is provided in turn by reliability of the real object mathematical model used for the calculations. The article describes a mathematical model of the two-mass reversible scanning mechanism and its comparison with the real system.

Key-Words: - two-mass mechanism, magnetoelectric converter, reversible scanning device, mathematical model.

1 Introduction

Along with widespread of the irreversible scanning devices, which are continuous rotation scanning item [1], in modern technology commonly used reversible scanning devices in which the scanning element can perform fairly complex of reciprocating linear or rotational movement. Such devices are used, for example, at 3D buildings and structures printing, structural components printing, multi-axis processing centers, scanning thermal imager systems. The application of reverse scanning devices isn’t restricted by delivered examples. The characteristic feature of these devices is the large responsibility for the reciprocating motion elements inertia.

An example of such irreversibly scanning device is contactless magnetoelectric converter (MEC), based on PMSM with high coercivity permanent magnets. MEC proportionally converts an input electrical signal (voltage) to angular movement of its rotor.

Such scanning devices performance is mostly provided by the performance of the control system, which in turn is provided by reliability of the mathematical model of the real object used for the calculations.

Static and dynamic characteristics of MEC with a limited angle rotation for the scan drive are analyzed in [2], synthesis techniques, mathematical models and simulation results for the one-mass speed control system MEC are described in [3,4]

However, the approach described in [2-4] is applicable only if the stiffness coefficient of the object is high enough. Otherwise, implementation of a control system synthesized this way may provoke fluctuations related with unaccounted slackness and as a result, the inability to achieve the required scan accuracy.

Lack of papers devoted to electric drive control system for scanning devices represented by two-mass models and a small number of papers devoted to direct-drive system lead us to the study of such systems.

2 The "MEC-mechanism" two - mass model

According with [5] the dynamics of the MEC can be described by equations:

$$u_y(t) = R \cdot i(t) + L\frac{di}{dt} + Ke \cdot \frac{d\alpha}{dt} \qquad (1, a)$$

$$J\frac{d^2\alpha}{dt^2} + f\frac{d\alpha}{dt} + K_\alpha \cdot \alpha + M_c = K_I \cdot i(t) \qquad (1, b)$$

Where $u_y(t)$ is the control winding of MEC voltage, $i(t)$ is the control winding circuit, R is the control winding resistance, L is the control winding inductance, K_e is the slope of the back-EMF coefficient, J is the total scanning axis inertial torque, f is the inner damping coefficient, M_c is the coulomb friction torque, $K_\alpha = \frac{dM}{d\alpha}$ is the stiffness of mechanical characteristic or «magnetic spring» stiffness, $K_I = \frac{dM}{dI}$ is the stiffness of traction characteristic or electric circuit sensitivity, α is the rotation angle.

Applying Laplace transform to (1, a) and expressing circuit $i(t)$ from (1, a) follows to (2)

$$i(t) = (u_y(t) - K_e \cdot p \cdot \alpha) \cdot \frac{1/R}{T_e \cdot p + 1} \qquad (2)$$

where $T_e = L/R$ – electrical time constant

Applying Laplace transform to (1, b) follows to (3):

$$J \cdot p^2\alpha + f \cdot p \cdot \alpha + K_\alpha \cdot \alpha = M - M_c \qquad (3)$$

where $M = K_i \cdot i(t)$ is the torque provided by MEC

Accepting $d\alpha / dt = p \cdot \alpha = \Omega$ -the angular speed, equation (3) transforms into (4)

$$J\frac{d\Omega}{dt} + f \cdot \Omega + K_\alpha \cdot \alpha = M - M_c \qquad (4)$$

Hence:

$$\frac{d\Omega}{dt} = (M - M_c - f \cdot \Omega - K_\alpha \cdot \alpha) \cdot \frac{1}{J} \qquad (5)$$

Therefore the two-mass system presented at figure 1 is obtained by combining of block diagram given in [2], equation (5) (where the coordinates of the position α and velocity Ω are considered as speed and position of the first mass), and well-known transfer functions and block diagram of the two-mass mechanism [6].

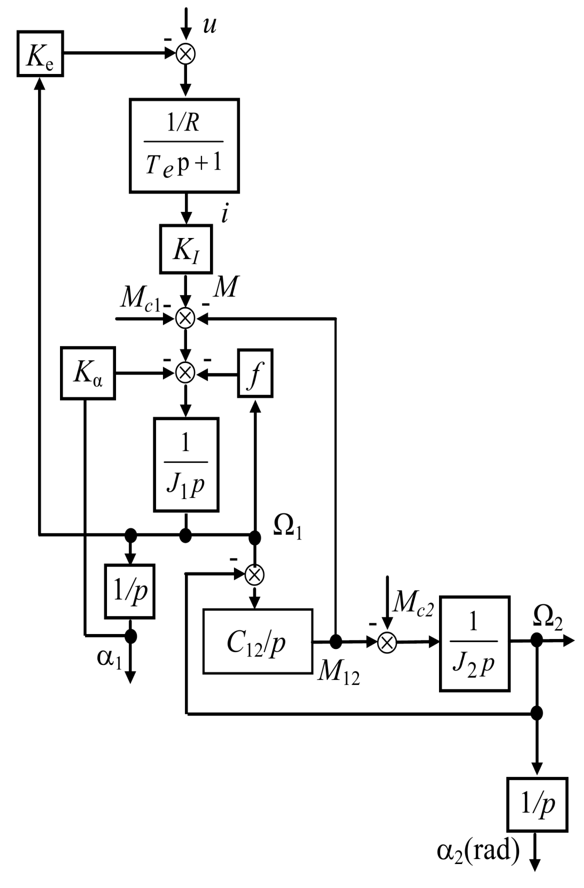

Fig. 1. "MEC-mechanism" two - mass block diagram model

Research of static and dynamic two-mass system characteristics has shown that the transfer functions describing the static properties of the object are not fundamentally different from the transfer functions describing the single-mass model of the system [2]. The differences are only in the order of the numerator and denominator, which is expected taking into account the introduction of additional model integrators.

It should be noted, what there are several state constrains in the system: $u_{\max}(t) \in [-48,48]$, $\alpha(t) \in [-0.5,0.5]$, $\omega_2(t) \in [-1.08,1.08]$.[7] This constraints are natural feature for many scanning systems. The difference are only in values of given states.

Figure 2 a) shows the Ω_{1m} and α_{1m} first mass angular velocity and rotation angle (curves 1 and 3) and figure 2 b) shows Ω_{2m} and α_{2m} second mass angular velocity and rotation angle (curves 2 and 4)

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of a two-mass MEC - mechanism model during the transient of 10 V surge control voltage $u_y(t)$ simulation results. The simulation was performed on the basis of the structure depicted on fig. 1 using the parameters of DB600-100-D3043 drive manufactured at the National University "Lviv Polytechnic". In the simulation was used MatLab, Simulink.

Inverter parameters: $K_\alpha = 4500$ N·m/rad; $K_I = 85$ N·m/A; $f = 0$; $K_e = 1.5$ V·s/rad.; $L = 0.6$ H; $R = 14$ Ohm; $J = 236$ kg·m2, $M_c = 25$ N·m; $U_{y\_max} = 48$ V (maximum voltage of the control winding which is equal to the maximum value of the output voltage of the inverter.). Parameters of the mechanism $C_{12} = 3.3 \cdot 10^6$ N·m/rad, $J_1 = 18$ kg·m^2, $J_2 = 218$ kg·m^2 are obtained from the developers of telescope rotation device.

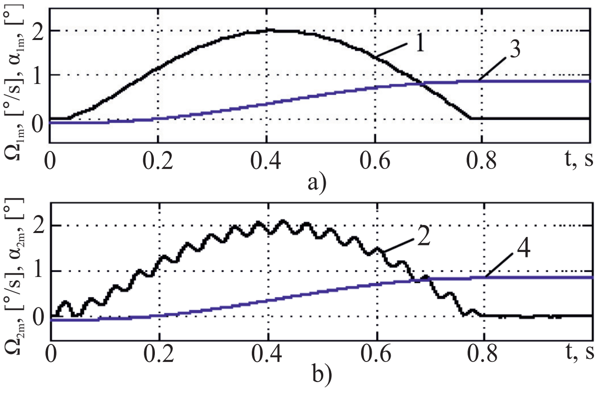

Fig. 2. Electromechanical transients in two-mass model "MEC-mechanism"

3 Comparison with the real object

During the study the comparative analysis of the obtained model and the real object was carried out. The research focuces on the triaxial infrared telescope scan axis. Before the simulation, some adjustments of considered model are made to improve the compliance with the real system. Real object speed circuit loop operates in degrees, not radians, as considered above, so that the outputs of the speed and position must be multiplied by a scaling coefficient of 180/π. At the input of the system scaling coefficient U_{y_max} should be placed due to the fact that regulator microcontroller output provides the value of γ - voltage inverter switch cyclic duration, so the value of the output of the regulator must be multiplied by the maximum voltage of the inverter $U_{y\_\max}$. Thus, we obtain the scheme depicted on figure 3.

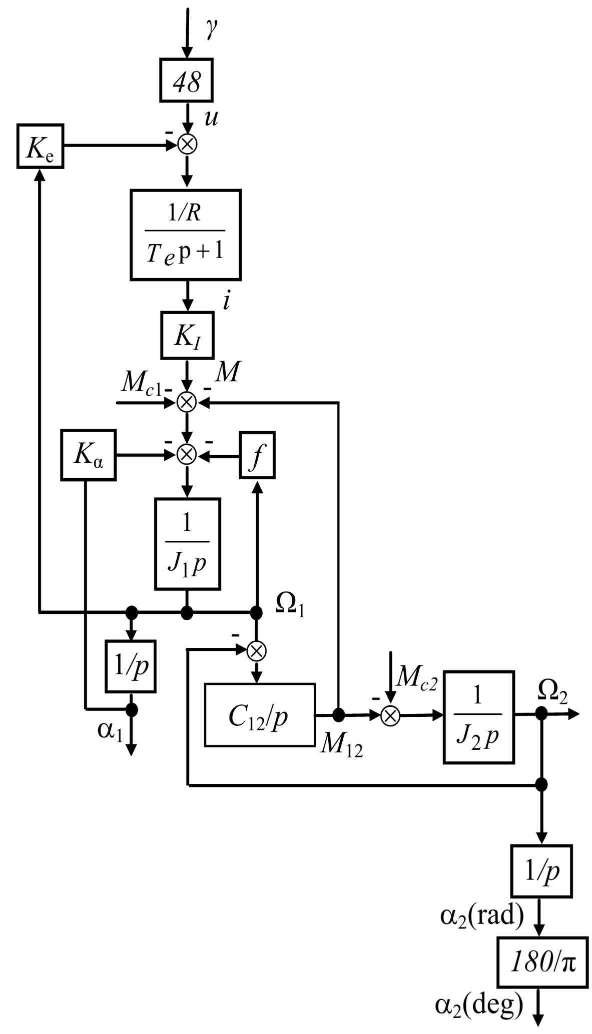

Fig. 3. " Adjusted mathematical "MEC-mechanism" system model

For the experiment, we used the method of direct coupling simulation package MATLAB with the control object. [8]. Figure 4 shows the real object and the mathematical model output values comparing results.

During the work it was found that the values of $C_{12} = 3.3 \cdot 10^6$ N·m/rad and $M_c = 25$ N·m does not correspond to reality. The actual values obtained during the experiment are: $C_{12} = 3.1 \cdot 10^5$ N·m/rad and $M_c = 50$ N·m. In the simulation process the simplest model of friction was used.

Figure 4 shows the real object and previously discussed mathematical model output coordinates reaction comparing result for periodic input square wave with 0.4 γ amplitude and3 sec period duration. On Figure: 1 - real object speed curve Ω_1, 2 – two-mass model first mass speed curve Ω_{1m}, 3 - real object angle curve α_1, 4 - two-mass model first mass angle curve α_{1m}. The second mass speed and angle coordinates aren't presented on the chart because only first mass coordinates can be obtained from the real object.

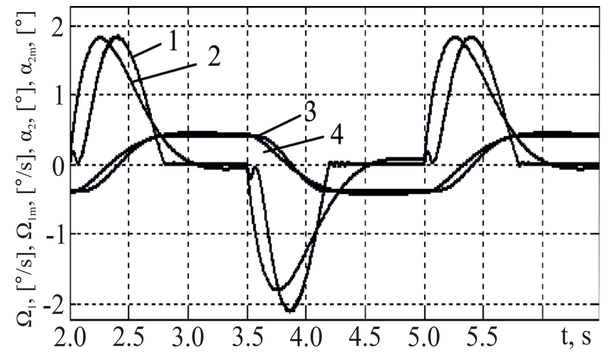

Fig. 4. Electromechanical transients in two-mass model "MEC-mechanism"

Comparative analysis wasn't limited by presented graph, showed that the obtained two-mass mathematical model accurately reflects transient and quasi-stationary modes of the real "MEC - Mechanism" system and can be used for further control system synthesis.

4 Conclusion

During the research two-mass "MEC - mechanism" mathematical model was obtained and simulation of the transient and quasi-stationary modes of the studied system with high accuracy are made. A comparative analysis of the obtained model and the real object was carried out, which allowed to improve model and its structure affirming parameters.

The resulting mathematical model with noted state constraints allows to describe studied system more accurately and give reasonable approach to the electric scanning devices control system synthesis, that take into account the two-mass object characteristics, by using, for example, optimal control theory methods [9-10].

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