MHD Boundary Layer Slip Flow over a Stretching Sheet in a Darcy-Forchheimer Porous Medium with Radiation and Ohmic Dissipation

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Abstract: - A mathematical model is presented for analyzing the MHD boundary layer slip flow of an incompressible fluid over a stretching sheet in Darcy-Forchheimer porous medium. Analysis has been carried out in the presence of thermal radiation and ohmic dissipation. Velocity and thermal slips are considered instead of no-slip conditions at the boundary. The governing partial differential equations are first transformed into ordinary differential equations before they are solved numerically using shooting method. The effects of governing parameters on the flow and thermal fields are examined. The skin friction and wall temperature gradient with effects of slip parameter are reported graphically for various parametric conditions to show interesting aspect of the numerical solution.

Key-Words: - Boundary layer; heat transfer; stretching surface; suction; injection; slip parameters

1 Introduction
The flow and heat transfer due to a stretching surface in viscous fluid is of great practical interest because it occurs in a number of engineering processes i.e. in the polymer industry when a polymer sheet is extruded continuously from a die, with a tacit assumption that the sheet is inextensible. During its manufacturing process a stretched sheet interacts with the ambient fluid both thermally and mechanically. The thermal interaction is governed by the surface flux. This quantity can either be prescribed or it is the output of a process in which the surface temperature distribution has been prescribed. The cooling of a large metallic plate in the bath (an electrolyte) is another problem belonging to this category. Glass blowing continuous casting and spinning of fibers also involve the flow due to a stretching surface.

However, in real situation one has encounter the boundary layer flow over the stretching sheet. For example, in a melt-spinning process, the extrudate is stretched into a filament or sheet while it is drawn from the die. Finally, this sheet solidifies while it passes through effectively controlled cooling system in order to acquire the top-grade property of the final product. The quality of final product depends on the rate of heat transfer at the stretching surface. The problem of flow due to a stretching sheet has been later extended to many flow situations. Crane [1] was the first to examine the problem of steady two-dimensional boundary layer flow of an incompressible and viscous fluid caused by a stretching sheet whose velocity varies linearly with the distance from a fixed point on the sheet. Pal and Mondal[2] have provided comprehensive discussion on hydromagnetic non-darcy flow and heat transfer over a stretching sheet in presence of thermal radiation and ohmic dissipation. Qasim et al. [3] examined effects of slip conditions on stretching boundary layer heat transfer with thermal radiation. Mukhopadhyay [4] numerically studied mixed convection boundary layer flow along a stretching cylinder in porous medium. Combined effects of non-uniform heat source/sink and thermal radiation on heat transfer over an unsteady stretching-permeable surface was presented by Pal [5].
Recently, Olenrewaju [6] analyzed the effects of internal heat generation on the hydromagnetic non-Darcy flow and heat transfer over a stretching sheet in the presence of thermal radiation and Ohmic dissipation. Therefore, the purpose of the present study is to study the slip effects on MHD Darcy-Forchheimer convective flow over a permeable stretching sheet in the presence of thermal radiation and Ohmic dissipation. It extends, in fact, the papers by Pal and Mondal [2] to the case of slip flow. Numerical results are compared with those of Pal and Mondal [2] for special cases, and are presented in tables.

2 Mathematical Formulation

We investigate the two-dimensional steady incompressible electrically conducting fluid flow over a continuous stretching sheet embedded in a porous medium. The flow region is exposed under uniform transverse magnetic fields \( \mathbf{B} = (0, B_0, 0) \) and uniform electric field \( \mathbf{E} = (0, 0, -E_0) \). Since such imposition of electric and magnetic fields stabilizes the boundary layer flow. It is assumed that the flow is generated by stretching of an elastic boundary sheet from a slit by imposing two equal and opposite forces in such a way that velocity of the boundary sheet is of linear order of the flow direction. We know from Maxwell’s equation that \( \mathbf{E} = 0 \) and \( \mathbf{B} = 0 \). When magnetic field is not so strong then electric field and magnetic field obey Ohm’s Law \( j = \sigma(\mathbf{E} + \mathbf{q} \times \mathbf{B}) \), where \( j \) is the Joule current, \( \sigma \) is the magnetic permeability and \( \mathbf{q} \) is the fluid velocity. We assume that magnetic Reynolds number of the fluid is small so that induced magnetic field and Hall effect may be neglected.

We take into account of magnetic field effect as well as electric field in momentum and thermal boundary layer equations. Under the above stated physical situation, the governing boundary layer equations for momentum and energy under Boussinesq’s approximation and the governing and viscous and Ohmic dissipations are

\[
\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0, \tag{1}
\]

\[
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = v \frac{\partial^2 u}{\partial y^2} + \sigma \left( E_0 B_0 - B_0^2 u \right) - \frac{v}{k} u - Fu^2 \tag{2}
\]

\[
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma}{\rho C_p} \left( u B_0 - E_0 \right)^2 - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} \tag{3}
\]

where \( u \) and \( v \) are the velocity components in the \( x \) and \( y \) directions, respectively; \( v \) is the kinematic viscosity; \( \rho \) is the density of the fluid; \( k \) is the permeability of the porous medium; \( q_r \) is the radiative heat flux in the \( y \)-direction; \( F \) is the empirical constant (Forchheimer number) in the second-order resistance and setting \( F = 0 \) in Eq. (2), the equation is then reduced to the Darcy’s law, \( C_p \) is the specific heat at constant pressure and \( \kappa \) is the thermal conductivity. Thermal boundary layer Eq.(3) takes into account the Joule heating or Ohmic dissipation due to the magnetic as well as electric fields The third and fourth terms on the right hand side of Eq.(2) stand for the first-order (Darcy) resistance and second-order porous inertia resistance, respectively. It is further assumed that the normal stress is of the same order of magnitude as that of the shear stress in addition to usual boundary layer approximations for deriving the momentum boundary layer Eq.(2). Thermal boundary layer Eq. (3) takes into account the joule heating or Ohmic dissipation due to the magnetic as well as electric field and the internal heat generation. The appropriate boundary conditions are put into the following forms

\[
\begin{align*}
T &= T_w + K_0 \frac{\partial T}{\partial y}, \quad \text{aty} = 0, \quad (4) \\
\theta &= T - T_\infty, \quad \text{asy} \to \infty. \quad (5)
\end{align*}
\]

Here \( N_0 \) is the velocity slip factor and \( K_0 \) is the thermal slip factor. We now introduce the following similarity transformation and dimensionless stream function and temperature as follows

\[
\begin{align*}
\theta &= \frac{T - T_\infty}{T_w - T_\infty}, \\
\psi &= x \frac{f'(\eta)}{y}, \quad \eta = \frac{b y}{\sqrt{v}} \quad (6)
\end{align*}
\]

Here, \( f(\eta) \) is the dimensionless stream function and \( \eta \) is the similarity variable. Substitution of Eq.(6) in the Eq.(2) and (3), results in a third-order non-linear ordinary differential equation of the following form

\[
f''' + f'' f' - f'^2 + 2 H a^2 (E_1 - f') - k_1 f'
\]

\[
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where \( k_1 = v/k_b \) is the porous medium parameter, \( Ha = B_0\sqrt{\sigma/pb} \) is Hartmann number, \( E_2 = E_0/B_0b_x \) is the local electric parameter, \( F^* = Fx \) is the local inertia-coefficient, and \( Re_x = U_x/v \), \( Pr = v/\alpha \) is Prandtl number, \( E_c = b^2t^2/A_0C_p \) is Eckert number and \( Nr = 16a^*\lambda^3/3kkk \) is the thermal radiation parameter. The boundary conditions (4) and (5) take the form

\[
\begin{align*}
1 + N_r \frac{Pr}{\theta''(0)} &= f'(0) = 0, \\
-f''(0) &= 0, \\
\end{align*}
\]

(7)

\[
\begin{align*}
\frac{Pr}{\theta''(0)} &= (f'(0) - 2f'(0)) + E_c f''(0) \\
+ E_c Ha^2 (f' - E_1)^2 &= 0, \\
\end{align*}
\]

(8)

where \( \delta \) and \( \beta \) are the velocity slip and thermal slip parameters, respectively.

### 3 Result and discussion

Equations (7) and (8) subjects to the boundary conditions (9) and (10) have been solved numerically by using the shooting method. The results are given to carry out a parametric study showing the influences of the non-dimensional parameters, namely the velocity slip parameter \( \delta \), thermal slip parameter \( \beta \), porous medium parameter \( k_1 \), the inertial parameter \( F^* \)and other physical quantities of interest. In order to validate the present results, we have compared them with those Pal and Mondal [2] for the no-slip case \( \delta = \beta = 0 \) as shown in Table 1. Table 2 gives the values of skin friction \( f''(0) \) and walltemperature gradient \(-\theta'(0)\) for several values of velocity slip parameter \( \delta \) when \( \beta = 0 \) (no-slip) and \( \beta = 0.1 \). It is noticed that as the velocity slip parameter \( \delta \) increases, this leads to significant changes in the values of skin friction \( f''(0) \) and walltemperature gradient \(-\theta'(0)\). The values of walltemperature gradient \(-\theta'(0)\) for several values of thermal slip parameter \( \beta \) when \( \delta = 0 \) (no-slip) and \( \delta = 0.1 \) are presented in Table 3. The values of \(-\theta'(0)\) as shown in Table 3 decrease with the increasing values of velocity slip parameter \( \delta \). The effects of \( \delta \) and \( \beta \) on the walltemperature gradient \(-\theta'(0)\) are displayed in Fig. 1. It is observed that for a particular value of \( \delta \) the walltemperature gradient \(-\theta'(0)\) is decreased as the thermal slip parameter \( \beta \) is increased. The variations of the skin friction \( f''(0) \) and walltemperature gradient \(-\theta'(0)\) for several values of Hartmann number \( Ha \) are presented in Figs 2 and 3, respectively. As can be seen from Fig. 2, for a particular value of \( Ha \), the values of skin friction decreases as the velocity slip parameter \( \delta \) increases. The same behavior is observed in Fig. 3, the walltemperature gradient \(-\theta'(0)\) is decreased with increasing thermal slip parameter. Figs. 4 to 6 present some samples of velocity and temperature profiles for different values of \( \delta \) and \( \beta \). With the increasing values of \( \delta \) and \( \beta \), the fluid velocity increases monotonically. Due to slip condition at the plate the velocity of fluid adjacent to the plate has some positive value and accordingly the thickness of momentum boundary layer decreases. These figures also show that the boundary conditions (9) and (10) are satisfied.

### 4 Conclusion

The present work deals with the the non-Darcy fluid flow and heat transfer over a stretching sheet embedded in porous media with thermal radiation and ohmic dissipation as considered by Pal and Mondal [5]. We have extended the previous work by taking into consideration slip conditions at the boundary. The governing equations are transformed into ordinary differential equations using similarity transformation and are then solved numerically using the shooting method. The effects of the velocity slip and thermal slip parameters and some values of the physical parameters on the flow and heat transfer characteristics are studied. In general, the increase of velocity slip and thermal slip parameters reduces the momentum boundary layer thickness and also enhances the heat transfer from the plate.

### Acknowledgement

The authors gratefully acknowledged the financial support received in the form of a FRGS research grant from the Ministry of Higher Education, Malaysia.

| Table 1: Comparison of wall temperature gradient, \(-\theta'(0)\) for \( Ha=0, \lambda=0 \) and various values of \( Pr \). |
|---|---|---|
| \( Pr \) | Pal and Mondal [2] | Present |

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\( \delta \) and \( \beta \) are the velocity slip and thermal slip parameters, respectively.
Table 2: Values of skin friction $-f''(0)$ and wall temperature gradient $-\theta'(0)$ for various values of $\beta$ and $\delta$ when $Ha = 1.0$, $Ec = 1.0$, $E_1 = 1.0$, $Pr = 3.0$.

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$-f''(0)$</th>
<th>$-\theta'(0)$</th>
<th>$-\theta'(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.656104</td>
<td>2.290967</td>
<td>1.814465</td>
</tr>
<tr>
<td>0.1</td>
<td>0.555343</td>
<td>1.814455</td>
<td>1.816110</td>
</tr>
<tr>
<td>0.2</td>
<td>0.482246</td>
<td>1.502054</td>
<td>1.809633</td>
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<tr>
<td>0.3</td>
<td>0.426887</td>
<td>1.281410</td>
<td>1.800122</td>
</tr>
<tr>
<td>0.4</td>
<td>0.382692</td>
<td>1.117301</td>
<td>1.789724</td>
</tr>
<tr>
<td>0.5</td>
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<td>1.779345</td>
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<td>0.6</td>
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<td>1.743270</td>
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<td>0.631795</td>
<td>1.735824</td>
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</tbody>
</table>

Table 3: Values of wall temperature gradient $-\theta'(0)$ with $Ha = 0.1$, $Ec = 1.0$, $E_1 = 1.0$, $Pr = 3.0$ with various velocity slip parameter $\delta$ against temperature slip $\beta$.

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$-\theta'(0)$</th>
<th>$-\theta'(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta = 0.0$</td>
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<td></td>
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<tr>
<td>0.0</td>
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<td>2.281787</td>
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<tr>
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<td>0.640204</td>
</tr>
</tbody>
</table>

Fig.1: Variation of wall temperature gradient $-\theta'(0)$ with $\beta$ when $Ha = 0.0$, $Ec = 0.0$, $E_1 = 0.0$ and $Pr = 3.0$ for several values of $\delta$.

Fig.2: Variation of skin friction $f''(0)$ with $\delta$ when $\beta = 0.0$, $Ec = 0.0$, $E_1 = 0.0$ and $Pr = 3.0$ for several values of $Ha$.

Fig.3: Variation of wall temperature gradient $-\theta'(0)$ with $\beta$ when $\delta = 0.0$, $Ec = 0.0$, $E_1 = 0.0$ and $Pr = 3.0$ for several values of $Ha$. 
Fig.5: Temperature profiles $\Theta(\eta)$ for different values of $\delta$ when $\beta = 0.1, Ha = 0.1, E_i = 1.0, E_c = 1.0, k_i = 0.1, Nr = 1.0, F^* = 0.3$ and $Pr = 3.0$.

Fig.6: Temperature profiles for different values of $\beta$ when $\delta = 0.1, Ha = 0.1, E_i = 1.0, E_c = 1.0, k_i = 0.1, Nr = 1.0, F^* = 0.3$ and $Pr = 3.0$.

References: