Boundary Layer Flow of a Nanofluid and Heat Transfer over an Exponentially Shrinking Sheet: Copper-Water

NUR LIYANA ALENG, NORFIFAH BACHOK & NORIHAN MD. ARIFIN
Department of Mathematics and Institute for Mathematical Research,
Universiti Putra Malaysia,
43400 UPM Serdang, Selangor,
MALAYSIA
norfifah78@yahoo.com http://www.upm.edu.my

Abstract: - An analysis is carried out to investigate the steady two-dimensional boundary layer flow of a nanofluid and heat transfer over an exponentially shrinking sheet in a water-based copper (Cu). The model used for the nanofluid incorporates the effect of the nanoparticles volume fraction. The governing partial differential equations are converted into a system of nonlinear ordinary differential equations using a similarity transformation, before being solved numerically by using a shooting method. Results for the skin friction coefficient, local Nusselt number, velocity as well as the temperature are presented for different values of the governing parameters. It is found that when the mass suction parameters exceeds a certain critical value, steady flow is possible. Dual solutions for the velocity and temperature distributions are obtained. With increasing values of the nanoparticles volume fraction, the skin friction and heat transfer coefficient increase.
Key-Words: - Boundary layer, Exponentially shrinking sheet, Heat transfer, Dual solutions, Nanofluid

1 Introduction

The study of convective heat transfer in nanofluids has achieved great success in various industry processes. It is worth mentioning that the enhanced thermal behavior of nanofluids could provide a basis for an enormous innovation for heat transfer intensification, which is of major importance to a number of industrial sectors including transportation, power generation, micro-manufacturing, thermal therapy for cancer treatment, chemical and metallurgical sectors, as well as heating, cooling, ventilation and air-conditioning [1]. The theory of nanofluids has presented several fundamental properties with the large enhancement in thermal conductivity as compared to the base fluid [2]. A large number of experimental and theoretical studies have been carried out by numerous researchers on thermal conductivity of nanofluids [3-5]. A comprehensive survey of convective transport in nanofluids was made by Buongiorno [6] and Tiwari and Das [7]. Buongiorno [6] noted that the nanoparticle absolute velocity can be viewed as the sum of the base fluid velocity and a relative velocity and very recently used by several researchers such as, among others, Khan and Pop [8], Bachok et. al [9,10], Mansur and Ishak [11], Nadeem and Lee [12], etc. On the other hand, the Tiwari and Das [7] model analyzes the behavior of nanofluids taking into account the solid volume fraction of the nanofluid. In the present paper, we study the flow and heat transfer characteristics over an exponentially shrinking sheet immersed in copper-water by using the model of Tiwari and Das [7]. This model was used in many recent papers, for instance; Arifin et. al [13], Bachok et. al [14,15], Rohni et. al [16], Kameswaran et. al [17], and Das [18].

The boundary layer flow induced by an exponentially stretching/shrinking sheet is not studied much, though it is very important and realistic flow frequently appears in many engineering process. There are two conditions that the flow towards shrinking sheet is likely to exist, whether an adequate suction on a boundary is imposed [19] or a stagnation flow is considered [20]. It seems that the paper by Bhattacharyya and Vajravelu [21] is the first considered the problem over an exponentially shrinking sheet. Later, Bachok et. al [22] considered this exponentially shrinking sheet problem in a nanofluid. Thus, the aim of the present paper is to extend the problem of an exponentially shrinking sheet considered by Bhattarcharyya [23] to the case of nanofluid using the model of Tiwari and Das. The governing partial differential equations are transformed into set ordinary differential equations using a similarity transformation,

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before being solved numerically by a shooting method. The results obtained are presented graphically and discussed.

2 Problem Formulation

Consider the steady two-dimensional boundary layer flow of a nanofluid and heat transfer over a shrinking sheet. The governing equation of motion and the energy equation may be written in usual notation as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2} \quad (3)$$

where u and v are the velocity components along the $x-$ and $y-$ axes, respectively, T is the temperature of the nanofluid, μ_{nf} is the viscosity of the nanofluid, α_{nf} is the thermal diffusivity of the nanofluid and ρ_{nf} is the density of the nanofluid, which are given by Oztop and Abu Nada [24].

$$\begin{aligned} &\alpha_{nf}=\frac{k_{nf}}{\left(\rho C_p\right)_{nf}},\ \rho_{nf}=(1-\varphi)\rho_f+\varphi\rho_s,\\ &\mu_{nf}=\frac{\mu_f}{(1-\varphi)^{2.5}},\\ &\left(\rho C_p\right)_{nf}=(1-\varphi)\left(\rho C_p\right)_f+\varphi\left(\rho C_p\right)_s,\\ &\frac{k_{nf}}{k_f}=\frac{\left(k_s+2k_f\right)-2\varphi\left(k_f-k_s\right)}{\left(k_s+2k_f\right)+\varphi\left(k_f-k_s\right)}. \end{aligned} \quad (4)$$

Here, φ is the nanoparticle volume fraction, $\left(\rho C_p\right)_{nf}$ is the heat capacity of the nanofluid, k_{nf} is the thermal conductivity of the nanofluid, k_f and k_s are the thermal conductivities of the fluid and of the solid fractions, respectively, and ρ_f and ρ_s are the densities of the fluid and of the solid fractions, respectively. It should be mentioned that the use of the above expression for k_{nf} is restricted to spherical nanoparticles where it does not account for other shapes of nanoparticles (Abu-Nada [25]). Also, the viscosity of the nanofluid μ_{nf} has been approximated by Brinkman [26] as viscosity of a base fluid μ_f containing dilute suspension of fine spherical particles.

The boundary conditions are given by

$$\begin{aligned} &u=-u_w(x),\ v=v_w=v_0e^{x/2L},\\ &T=T_w=T_\infty+T_0e^{x/2L} \end{aligned} \quad \text{at } y=0$$

$$u\to 0,\quad T\to T_\infty \text{ as } y\to\infty \quad (5)$$

The shrinking velocity u_w is given by

$$u_w(x)=ce^{x/L}, \quad (6)$$

where $c>0$ is shrinking constant.

The governing Eqs. (1) – (3) subject to the boundary conditions (5) can be expressed in a simpler form by introducing the following transformation:

$$\begin{aligned} &\eta=y\left(\frac{c}{2\nu_f L}\right)^{1/2}e^{x/2L},\\ &\psi=\left(2\nu_f Lc\right)^{1/2}e^{x/2L}f(\eta),\\ &\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \end{aligned} \quad (7)$$

where η is the similarity variable and ψ is the stream function defined as $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$, which identically satisfies Eq. (1). Employing the similarity variables (7), Eqs. (2) and (3) reduce to the following ordinary differential equations:

$$\frac{1}{(1-\varphi)^{2.5}(1-\varphi+\varphi\rho_s/\rho_f)}f'''+ff''-2f'^2=0 \quad (8)$$

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$$\frac{1}{Pr}\frac{k_{nf}/k_f}{\left[1-\varphi+\varphi(\rho C_p)_s/(\rho C_p)_f\right]}\theta''+f\theta'-f'\theta=0$$
(9)

subjected to the boundary conditions (5) which become

$$f(0)=S,\quad f'(0)=-1,\quad \theta(0)=1,\\ f'(\infty)\to 0,\quad \theta(\infty)\to 0, \qquad (10)$$

where $\Pr(=\nu_f/\alpha_f)$ is the Prandtl number and $S=-v_w\Big/\sqrt{\frac{\nu_f c}{2L}}$ is the mass transfer parameter with $S>0$ ($v_w<0$) corresponds mass suction and $S<0$ ($v_w>0$) corresponds to the mass injection.

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x, which are defined as

$$C_f=\frac{\tau_w}{\rho_f u_w^2},\quad Nu_x=\frac{xq_w}{k_f(T_w-T_\infty)}, \qquad (11)$$

where the surface shear stress τ_w and the surface heat flux q_w are given by

$$\tau_w=\mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0},\\ q_w=-k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0}, \qquad (12)$$

with μ_{nf} and k_{nf} being the dynamic viscosity and thermal conductivity of the nanofluids, respectively. Using the similarity variables (7), we obtain

$$C_f Re_x^{1/2}=\frac{1}{(1-\varphi)^{2.5}}f''(0), \qquad (13)$$

$$Nu_x/Re_x^{1/2}=-\frac{k_{nf}}{k_f}\theta'(0), \qquad (14)$$

where $Re_x=u_w x/\nu_f$ is the local Reynolds number.

3 Results and Discussion

Numerical solutions to the governing ordinary differential equations (8) and (9) with the boundary conditions (10) were obtained using a shooting method. The dual solutions were obtained by setting different initial guesses for the missing values of $f''(0)$ and $-\theta(0)$, where all the profiles satisfy the boundary conditions (10) asymptotically but with different shapes are illustrated in graphs. Following Oztop and Abu-Nada [24], the value of Prandtl number Pr is taken as 6.2 (water) and the volume fraction of nanoparticle is from 0 to 0.2 ($0\le\varphi\le 0.2$) in which φ = 0 corresponds to the regular Newtonian fluid. It is worth mentioning that we have used the data related to the thermophysical properties of the fluid and the nanoparticle of copper as listed in Table 1. For the validation of the numerical results obtained, the case φ = 0 (viscous flow) has also been considered and compared with Bhattacharya [23]. The quantitative comparison are shown in Table 2 and found to be in favorable agreement.

Table 1. Thermophysical properties of fluid and nanoparticles (Oztop and Abu-Nada, [24]).

Physical properties	Fluid phase (water)	Cu
C_p (J/kg K)	4179	385
ρ (kg/m^3)	997.1	8933
k (W/m K)	0.613	400

Table 2. Values of S_c for nanoparticle and values of φ when Pr = 6.2

Nanoparticle	φ	Bhattacharya [23]	Present
	0	2.266684	2.266681
Cu	0.1		1.929508
	0.2		1.860896

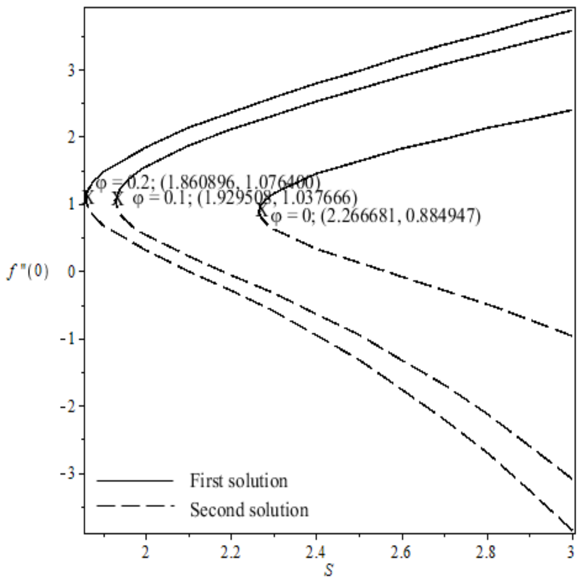

Fig. 1. $f''(0)$ for various values of *S*.

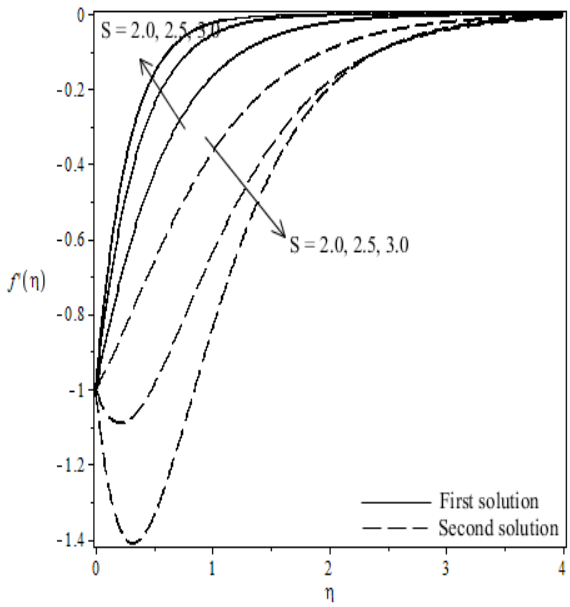

Fig. 3. Dual velocity profiles $f'(\eta)$ for various values of *S* with φ = 0.1.

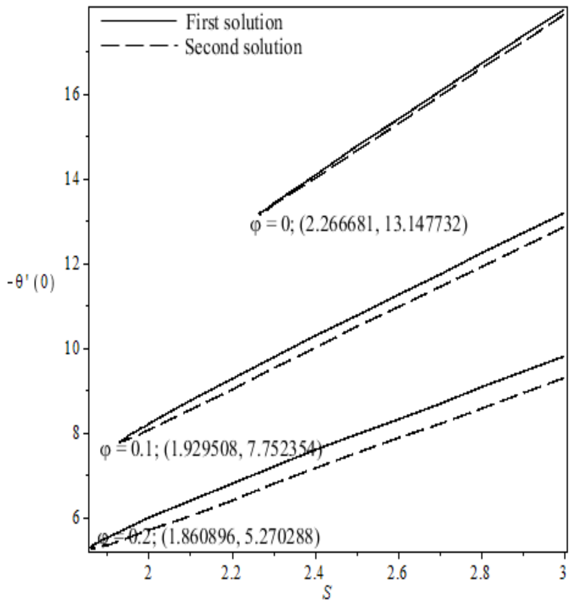

Fig. 2. $-\theta'(0)$ for various values of *S*.

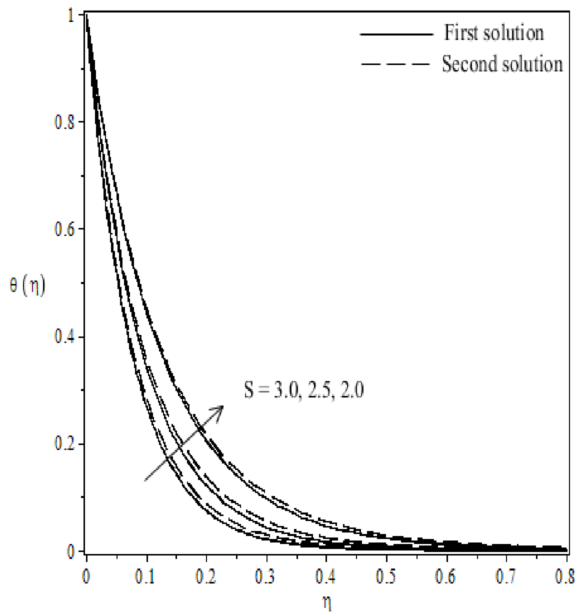

Fig. 4. Dual temperature profiles $\theta(\eta)$ for various values of *S* with φ = 0.1.

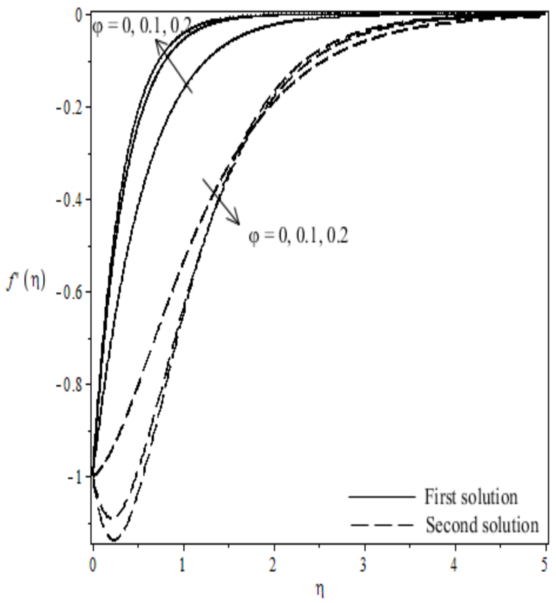

Fig. 5. Dual velocity profile $f'(\eta)$ for various values of φ with S = 2.5.

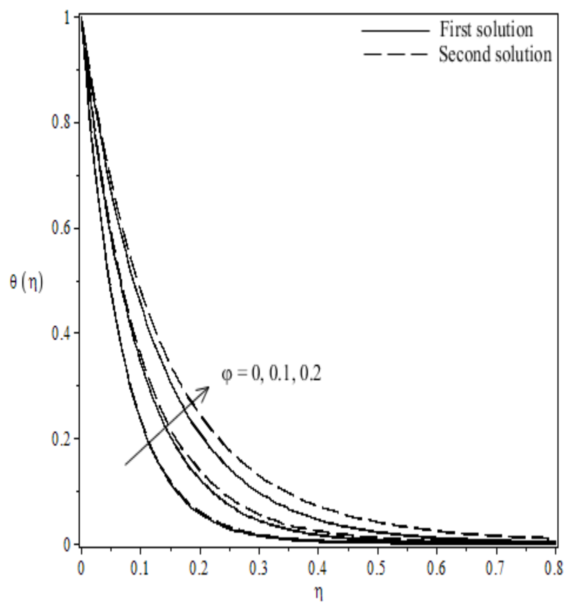

Fig. 6. Dual temperature profile $\theta(\eta)$ for various values of φ with S = 2.5.

Variation of the $f''(0)$ and $-\theta(0)$ for various values of S is shown in Figs. 1 and 2, respectively. It is seen that the dual similarity solutions are obtained for $S \geq S_c$. The similarity solutions exists when the mass suction parameter S satisfies the condition $S \geq S_c$ and consequently for $S < S_c$ the flow has no similarity solution. The conditions for the existence of the steady boundary layer due to shrinking of the sheet and it is found that when the mass suction parameter S exceeds a certain critical value, say S_c, steady flow is possible. Based on our computations, the critical values of S, are 2.266681, 1.929508 and 1.860896 for φ = 0, φ = 0.1 and φ = 0.2 respectively. Hence, the nanoparticle volume fraction parameter φ widens the range of S for which the solution exists. Figs. 1 and 2 also shows that the $f''(0)$ increases for the first solution but decreases for the second solution with increasing the values of S, but the values of $-\theta(0)$ increase with S for the first and the second solutions.

Figs. 3 to 6 displays the variations of velocity and temperature profiles for different values of mass suction parameter S with φ = 0.1 and different values of φ with S = 2.0 respectively, which indicate that all curves approach the far field boundary conditions asymptotically, and thus support the validity of the numerical results obtained. Moreover, these velocity and temperature profiles support the existence of dual nature of the solutions displayed in Figs. 1 and 2.

4 Conclusions

The boundary layer flow of a nanofluid and heat transfer over an exponentially shrinking sheet in a copper-water was investigated. The study revealed that the steady boundary layer flow due to shrinking of the sheet is possible only when the mass suction parameter exceeds a certain value, i.e. $S \geq S_c$, and dual similarity solutions for velocity and temperature fields were found. In addition, for the first solution, the velocity increases with mass suction but decreases for the second solutions.

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References:

[1] N. Bachok, A. Ishak, I. Pop, Flow and heat transfer over a rotating porous disk

in nanofluids, *Physica B*, 406, 2011, pp. 1767-1772.

[2] J. Fan, L. Wang, Heat conduction in nanofluids: structure-property correlation, *Int J Heat Mass Transf*, 54, 2011, pp. 4349-4359.

[3] N. Bachok, A. Ishak, I. Pop, Boundary layer flow of nanofluid over a moving surface in a flowing fluid, *Int J Therm Sci*, 49, 2010, pp. 1663-1668.

[4] S.U.S. Choi, Enhancing thermal conductivity of fluids with nanoparticle, *ASME FED*, 66, 1995, pp. 99-105.

[5] O.D. Makinde, A. Aziz, Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition, *Int J Therm Sci*, 50, 2011, pp. 1326-1332.

[6] J. Buongiorno, Convective transport in nanofluids, *ASME J. Heat Transf.*, 128, 2006, pp. 240-250.

[7] R.K. Tiwari, M.K. Das, Heat transfer augmentation in a two-sided lid-driven differentlially heated cavity utilizing nanofluids, *Int. J. Heat Mass Transf.*, 50, 2007, pp. 2002-2018.

[8] A.V. Khan, I. Pop, Boundary layer flow of a nanofluid past a stretching sheet, *Int. J. Heat Mass Transf.*, 53, 2010, pp. 2477-2483.

[9] N. Bachok, A. Ishak, I. Pop, Unsteady boundary layer flow of a nanofluid over a permeable stretching/shrinking sheet, *Int. J. Heat Mass Transf.*, 55, 2012, pp. 2102-2109.

[10] N. Bachok, A. Ishak, I. Pop, Boundary layer stagnation-point flow toward a stretching/shrinking sheet in a nanofluid, *J. of Heat Transf.*, 135, 2013, pp. 054501.

[11] S. Mansur, A. Ishak, The flow and heat transfer of a nanofluid past a tretching/shrinking sheet with a convective boundary layer, *Hindawi Publishing Corporation*, 2013, pp. 350647.

[12] S. Nadeem, C. Lee, Boundary layer flow of a nanofluid over an exponentially stretching surface, *Boundary Value Prob*, 7, 2012, pp. 94-99.

[13] N.M.D. Arifin, R. Nazar, I. Pop, Viscous flow due to a permeable stretching/shrinking sheet in a nanofluid, *Sains Malaysia*, 40, 2011, pp. 1359-1367.

[14] N. Bachok, A. Ishak, I. Pop, Stagnation-point flow over a stretching/shrinking sheet in a nanofluid, *Nanoscale Research Letters*, 6, 2011, pp. 623-632.

[15] N. Bachok, A. Ishak, I. Pop, Stagnation-point flow over a permeable stretching/shrinking sheet in a copper-water nanofluid, *Boundary Value Prob*, 39, 2013, pp. 1-10.

[16] A.M. Rohni, S. Ahmad, A.I M.D. Ismail, I. Pop, Flow over an unsteady shrinking sheet with suction in a nanofluid, *Int. J. of Modern Phys.*, 9, 2012, pp. 512-519.

[17] P.K Kameswaran, M. Narayana, P. Sibanda, P.V.S.N. Murthy, *Int. J. Heat Mass Transf.*, 55, 2012, pp. 7587-7595.

[18] M.K. Das, Nanofluid flow over a shrinking sheet with surface slip, *Microfluid Nanofluid*, 16, 2014, pp. 391-401.

[19] M. Miklavčič, C.Y. Wang, Viscous flow due to a shrinking sheet. *Quart Appl Math*, 64, 2006, pp. 283–290.

[20] C.Y. Wang, Stagnation flow towards a shrinking sheet, *Int. J. of Non-Lin. Mech.*, 43, 2008, pp. 377-382.

[21] K. Bhattacharyya, K. Vajravelu, Stagnation point flow and heat transfer over an exponentially shrinking sheet, *Commun Nonlinear Sci Numer Simulat*, 17, 2012, pp. 2728-2734.

[22] N. Bachok, A. Ishak, I. Pop, Boundary layer stagnation-point flow and heat transfer over an exponentially stretching/shrinking sheet in a nanofluid, *Int. J. Heat Mass Transf.*, 55, 2012, pp. 8122-8128.

[23] K. Bhattacharyya, Boundary Layer Flow and Heat Transfer over an exponentially shrinking sheet, *Chin. Phys. Lett.*, 28, 2011, pp. 074701.

[24] H.F. Oztop, E. Abu-Nada, Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids, *Int J Heat Fluid Flow*, 29, 2008, pp. 1326-1336.

[25] E. Abu-Nada, Application of nanofluids for heat transfer enhancement of separated flow encountered in a backward facing step, *Int J Heat Fluid Flow*, 29, 2008, pp. 242-249.

[26] H.C. Brinkman, The viscosity of concentrated suspensions and solutions, *J Chem Phys*, 20, 1952, pp. 571-581.