Supersonic Boundary Layer Stability at Mass Transfer through a Wall

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Abstract: - Within the framework of the linear and nonlinear theory the supersonic boundary layer stability is considered in the presence of mass transfer (injection or suction) through a permeable porous wall. The boundary conditions for the disturbances on a permeable wall are derived with account for the gas compressibility in pores and the presence of a suction chamber. At moderate Mach number on suction the velocity profiles become more convex and the boundary layer thickness decreases and the flow is stabilized. Contrariwise on injection the boundary layer is destabilized. At the high Mach number disturbances of two types is considered. The range of existence of the growing second-mode fluctuations is fairly narrow, as compared with the first mode. At high supersonic regimes injection and suction through porous permeable surfaces lead to considerable stabilization of the second-mode disturbances, which can favor the transition delay.

Key-Words: - Supersonic flow, boundary layer, hydrodynamic stability, laminar-turbulent transition, nonlinear theory, porous wall

1 Introduction
In developing flight vehicles much attention is given to the nature of the boundary layer flow near their surfaces. This interest is primarily due to the well-known fact that the friction drag can be reduced by means of laminarizing the flow, thus improving the economic proficiency of flight vehicles. For this reason, it is very important to be able to predict the laminar-turbulent transition position and to control it.

At low intensities of external disturbances the process of the laminar boundary layer transition in the turbulent state can be conditionally subdivided into three stages, namely, wave generation, their build-up in accordance with the linear theory laws, and the nonlinear breakdown of the laminar flow regime. Each of these stages is associated with a characteristic spatial region, as the leading edge of the model is receded. The linear and nonlinear development of the transition process is determined by the properties of the original flow including its stability. There are certain methods of forming stable boundary layer flows; one of these techniques is the mass transfer on the surface in a flow. Using the mass transfer in the form of gas injection or suction for creating and maintaining certain flow patterns belongs to passive methods of the control.

During the suction low-velocity gas masses are removed from wall regions; as a result, the mean velocity profiles become more convex. This leads to an increase in their stability and the critical Reynolds number and transition delay. Boundary layers become thinner and less susceptible to transition into the turbulent state [1, 2] but their sensitivity to various inhomogeneities rises. Contrariwise, during the injection the boundary layers become thicker, while the velocity profiles become less convex, with the tendency to the appearance of local inflections, which intensifies the linear processes of disturbance development and accelerates transition. The injection flow is frequently used for enhancing transfer processes in conjectural regions.

The studies on the stability of compressible boundary layers with mass transfer are few in number. Certain data presented in [3] indicate that the suction can be used for stabilizing disturbances in supersonic flow regimes. The study made in [4] began the systematic modeling of the control of disturbance development using the mass transfer on the surface. The linear wave characteristics and the nonlinear interactions were studied at moderate (M = 2) and high Mach numbers (M = 5.35).

The approximation, in which the radii of the pores through which injection/suction is produced are much smaller than the boundary layer thickness and the velocity disturbances on the permeable surfaces are, as in [3], zero, was considered. In this case, the disturbance parameters are affected only by the variations of the mean flow parameters.
Admittedly, this is a limited case which does not allow for all components of the actual processes on permeable walls.

As shown in [5–10], in flow past porous surfaces the properties of the surfaces themselves should be necessarily taken into account. On these surfaces the destabilization of vortical disturbances and the stabilization of acoustic ones is observable. In this study, this aspect is included in the modeling, which makes it possible to considerably expand the range of the possible applications of the studies and to bring them closer to the actual conditions.

At low external disturbance levels the linear enhancement stage is followed by that of the nonlinear interaction between the waves. One of the most typical and frequently realized initial stages of the nonlinearity is the subharmonic three-wave resonance well studied for subsonic boundary layers [2, 11, 12]. At recent time, the number of the studies devoted to this nonlinearity scenario for supersonic boundary layers has considerably increased [4–7, 13, 14].

At high Mach numbers, apart from natural oscillations or traveling waves, vortical in nature (which are also called Tollmien–Schlichting waves, viscous waves, and first-mode disturbances), there appear inertiadriven disturbances due to the excitation of acoustic modes, of which it is the second mode that is the most growing on the Mach number range considered [3].

In this study, the nonlinear interaction is investigated between vortex waves in nonsymmetric three-wave systems at Mach 2 and between waves, different in nature, namely acoustical and vortical, in symmetric triplets at Mach 5.35.

2 Basic Relations and Methods of Solution

The basic propositions of the nonlinear model of interactions in three-wave resonance systems for compressible boundary layers are presented in detail in [13, 14] and briefly in [4–7]. Here, we will present necessary explanations. We denote the fluctuation field amplitude scale by \( \varepsilon \) \( \ll 1 \) and consider the compressible-gas velocity, density, pressure, and temperature flow of a compressible gas in a boundary layer on a flat plate is taken as an initial undisturbed flow.

\[
\begin{align*}
  u &= U(Y) + \varepsilon w',
  v &= \varepsilon v',
  w &= \varepsilon w',
  \rho_0 &= \rho(Y) + \varepsilon \zeta',
  p_0 &= p(Y) + \varepsilon p',
  T_0 &= T(Y) + \varepsilon \Theta',
  \frac{p'}{P} &= \frac{\zeta'}{\rho} + \frac{\Theta'}{T},
\end{align*}
\]

In a dimensionless Cartesian coordinate system \((X, Y, Z) = (x, y, z) / \delta\), where \( \delta = \infty \), is the characteristic scale length and the subscript refers to the parameter values at the outer edge of the boundary layer. The primed and primeless quantities are the fluctuating and mean components of the corresponding quantities. The nondimensionalizing is made on the flow parameters at the outer edge and the Mach and Reynolds numbers are based on these parameters: \( M = U / a_e \), where \( a_e \) is the speed of sound, and \( Re = (xU / \nu) \).

To determine the mean characteristics of the boundary layer system the dynamic and thermal boundary layers is solved in the self-similar Blasius variables [3]

\[
\begin{align*}
  \frac{d}{dY} \left( \frac{dU}{dY} \right) + g \frac{dU}{dY} &= 0, \\
  \frac{d}{dY} \left( \frac{dT}{dY} \right) + g \frac{dT}{dY} + (\gamma - 1)M_e^2 \mu \left( \frac{dU}{dY} \right)^2 &= 0, \\
  \frac{dg}{dY} &= \frac{U}{2T}.
\end{align*}
\]

The boundary conditions for the dynamic boundary layer are as follows

\[
g = C_q, \quad V = 0 \quad \text{at} \quad Y = 0; \quad T = 1, \quad U = I \quad \text{at} \quad Y = \infty.
\]

Here, \( \sigma = C_p \nu / k \) is the Prandtl number, \( k \) is thermal conductivity, and \( \gamma = C_p / C_v \). The parameter \( C_q = - \Re V_w T_w \) characterizes the surface injection or suction intensity \( (V_w \) is the suction rate and \( T_w \) is the wall temperature); two cases may be considered. In the first case, \( V_w \) varies inverse proportional to \( \sqrt{x} \) along the plate; in this case, \( C_q = \text{const} \). In the second case \( \text{V}_w = \text{const} \) and \( C_q \) varies downstream. In this study we consider the first case at which the similarity condition is satisfied.

The solution for the flow field is constructed using an expansion in the small parameter \( \varepsilon \) and a two-scale expansion of the streamwise coordinate. Apart from the “fast” scale \( X \), a “slow” scale \( \zeta = \varepsilon \) \( X \) is introduced, which is justified by a large difference between the rates of variation of the disturbance phase and amplitude.

The wave solutions are sought in the form:

\[
(Z)_{j} = A_j(\zeta)(Z)^0(Y) \exp (i\theta_j) + \text{c.c.} + \varepsilon (Z)^1_j + ..., \quad j = 1, 2, 3,
\]

where \( Z' = \mu \zeta ' \), \( v', w', p', \Theta' \) are the amplitude eigenfunctions of the longitudinal, transverse, and transversal components of the wave velocity and the pressure and temperature disturbances, \( A \) is the slowly varying amplitude, c.c. stands for complex-conjugate quantities, \( \theta = \alpha X + \beta Z - \omega t , \alpha = \alpha_+\)
\( i \alpha', \alpha' \) is the streamwise growth rate (at \( \alpha' < 0 \)), and \( \omega \) is the real frequency; the wavenumbers \( \alpha \) and \( \beta \) and the frequency \( \omega \) are related by the dispersion equation \( \alpha = \alpha(\omega, \beta) \) in accordance with the linear theory.

From the complete system of the equations of motion and conservation for a compressible gas the following recurrent system for disturbances of the vector function \( Z \) can be obtained within the framework of the weakly-nonlinear theory [13].

\[
\varepsilon \left[ \sum_{j=1}^{k} e^{j\theta} (L(Z_j) + \partial L(Z_j)/\partial \alpha_j \partial / \partial X + 
+ \partial L(Z_j)/\partial \omega_j \partial / \partial t)]A_j + e^{j(\theta_0 + \theta)} [L(Z_{j+1}) + (2) + A_k M L_j(Z_l Z_{l+1} \ldots)] \right] = 0
\]

with the corresponding conditions of disturbance decay at infinity \( \{u, v, w, \Theta \} = 0 \) at \( Y = \infty \). Here, \( L \) is the linear operator.

The crucially important moment is the formulation of the boundary conditions on the permeable wall. For porous coatings with small pore radii \( r \) (at \( \nu \ll 1 \), which corresponds to about ten microns) the condition \( v(0) = 0 \) can be regarded as adequate; then at \( Y = 0 \) we have \( u, v, w, \theta = 0 \) [4]. However, in the problems of the compressible boundary layer stability it should be taken into account that on porous coatings the transverse velocity disturbances are proportional to the pressure disturbances and nonzero. This was theoretically considered in [15–17] and applied in [5–8].

We will consider the compressible boundary layer over a permeable porous surface, \( H \) in thickness, placed above a suction chamber, \( L_t \) in depth. The surface has cylindrical pores, \( r \) in radius. The pressure and the normal velocity in a pore can be written in terms of the tube element (pore) impedance \( Z_t \) and the acoustic conductivity of the gas [17]. The equation relating these components is as follows:

\[
K = \frac{v(0)}{p(0)} = n \left( Z_0 - X_t h(\lambda H) \right) \frac{n}{Z_0 (Z_t h(\lambda H) - X_t)}
\]

where \( n \) is porosity, \( \lambda \) is the propagation constant, and \( Z_0 \) is the characteristic impedance. They can be expressed in terms of \( Z_t \) and the coefficient \( W \) characterizing the compression energy reserve and the losses for heat transfer to the walls

\[
Z_t = iac \frac{I_0(\eta)}{L_0(\eta)} W = -iac M \left[ \gamma + (\gamma - 1) \frac{I_1(\sqrt{\eta})}{I_0(\sqrt{\eta})} \right] \\
\eta = \sqrt{iac Re \varepsilon}, \lambda = \sqrt{Z_t W}, Z_0 = \frac{Z_t}{W},
\]

where \( I_0 \) and \( I_1 \) are the Bessel functions of the zero and second orders.

The value \( X_t \) depends on the particular mode of the interaction of the lower end of the pore with the medium below it. If in the suction chamber the inviscid equation of motion is solved for \( p \) and \( v \) (\( v = 0 \) at the bottom), then \( X_t \) can be represented as follows

\[
X_t = \frac{nic}{T \lambda_t i h(\lambda_t L_t)}, \lambda_t = \sqrt{1 - \frac{c^2 M^2}{T_w}}, c = \frac{\omega}{\alpha}
\]

So the boundary conditions take the form:

\[
u, v, w, \Theta = 0 \text{ at } Y = \infty, \quad (3a)\]

\[
u, w, \Theta = 0, v = K_p \text{ at } Y = 0. \quad (3b)
\]

The linear operator \( L \) for the disturbances (Dan-Lin system) is as follows:

\[
[p Gu + U_v v + iac p / (\gamma M^2) - (\mu / Re) u_p] \exp(i\theta) = 0 \\
[p Gw + i \beta p / (\gamma M^2) - (\mu / Re) w_p] \exp(i\theta) = 0 \\
[p Gp + p_v / (\gamma M^2)] \exp(i\theta) = 0, \quad (3c)
\]

\[
(G \zeta + \rho_0 v + \rho (i c a u + v + i \beta w)) \exp(i\theta) = 0 \\
[p (\Theta / T) v + (\gamma - 1) (i c a u + v + i \beta w) - \mu / (Re \Theta)] \exp(i\theta) = 0
\]

\[
\zeta = \rho (p / P - \Theta / T), \quad G = i (\omega + \alpha U).
\]

The normalization for the amplitude functions is chosen as follows: \( v(Y_J) = 1 \). At \( M = 2 \) the integration was carried out up to \( Y_K = 10 \); for \( M = 5.35 \) \( Y_K = 25 \).

In the first order in \( \varepsilon \) at given \( \beta \), \( \omega \), and \( Re = \sqrt{Re \varepsilon} \) the eigenvalues \( \alpha \) are determined from the homogeneous system (3), together with the amplitude eigen functions of the linear function (1) with an indefinite \( A \).

In the weakly nonlinear theory the above-mentioned parameters are assumed to be given and the nonlinearity has an effect only on the wave amplitude \( A \). Within the framework of this approach we will consider the process of three-wave interaction between the waves under the condition of the synchronization of their phases \( \theta_0 = \theta_t + \theta_t \) (resonance model). In the second order in \( \varepsilon \) the higher-order disturbances \( Z_t \) can be found from the inhomogeneous equations (2) and, using the solvability conditions, the amplitude equations for the waves in the triad can be constructed [12]. We will write down these equations for the three-wave

\[
dA_j / d \xi = -\alpha c A_j + S_{j+1} A_j A_j \exp(i\Delta), \\
dA_k / d \xi = -\alpha c A_j + S_{j+1} A_j A_j \exp(i\Delta),
\]

where

\[
I_0 \text{ and } I_1 \text{ are the Bessel functions of the zero and second orders.}
\]
\[ dA / d\xi = -\alpha_j A_j + S_{ij} A_i A_j \exp(i\Delta), \]
\[ S_{ij} = \int_0^\infty Z_j^0 M_{ij}^0 dY / \int_0^\infty (\partial L_i/Z_j^0) / (\partial \alpha_j) dY, \]
\[ \Delta = \int (\alpha_k + \alpha_l - \alpha_j) \, dX. \]

Here \( \Delta \) is the phase synchronization coefficient which takes account for the possible detuning with respect to the wave numbers, \( Z^0 \) is the solution of the system of equations conjugate with respect to system (3), and \( M^0_{ij} \) are the nonlinear terms of the system of Navier–Stokes equations [4–7, 14]. System (4) was solved for the argument \( \Re \), while \( d/d\Re = 2d/d\xi \).

For any wave mode the dimensionless frequency parameter \( \Re \) related with the frequency by the equation \( \omega = \Re f \) and the reduced dimensionless wavenumber \( b = \beta 10^3 / \Re \) were kept constant. Both plane two-dimensional waves with \( \beta = 0 \) and oblique three-dimensional waves with \( \beta \neq 0 \) were considered. For the vortex disturbances the three-dimensional waves are most growing, their growth rates being greater than those of the two-dimensional waves, whereas for the acoustic disturbances the dependence is opposite: it is the two-dimensional components that are most growing [3–8]. The initial conditions for the traveling wave amplitudes \( A_j \) in Eq. (4) were obtained from the intensities \( I \) of the wave components in the initial section \( \xi_0 \). The relation between \( I \) and \( A \) is expressed in terms of the calculated values of the mass velocity fluctuations \( m \) as follows:
\[ I_j(\xi_0) = A_j(\xi_0)m_j(Y_\infty)\exp(-\alpha_j(\xi_0)). \]

It was assumed that in \( Y_\infty \) the value of the mass velocity fluctuation of the wave component \( m \) of one of the modes is maximum on the harmonic frequency. The value of the mean mass velocity \( \rho U(Y_\infty) \) was numerically calculated and the initial intensities \( I_j \) were expressed in terms of \( \rho U \) (in percents).

3 Discussion of the Results

The numerical modeling was made for the regimes realized in the experiments [9, 10], that is, at constant stagnation temperatures from the 310 to 390 K range, at \( \gamma = 1.4, \sigma = 0.72 \), and \( M = 2 \) and 5.35. The measurements were carried out for \( 3.5 \times 10^3 \leq \Re \leq 5.3 \times 10^3 \), which corresponds to the values \( \Re > 650 \).

3.1 \( M = 2 \) regime.

Mean characteristics. As shown in [4], the mass transfer has primarily the effect on the dynamic boundary layer, while the thermal boundary layer adjusts to the former. On suction the velocity profiles become more convex and the boundary layer thickness decreases in proportion to \( C_q \). Contrariwise, on injection \( (C_q < 0) \) the layer becomes thicker and the concavity of the velocity profiles diminishes. At \( C_q \leq -0.25 \) an inflection point appears in the middle of the layer. At the same time, the stagnation temperature \( T_w \) varies only slightly: it decreases on injection and increases on suction, as compared with the impermeable wall; the signs of the gradients \( T(Y) \) do not change.

We will present the linear wave stability characteristics in the \( r \ll q \) regime, when, due to the smallness of the pore radii, the pressure fluctuations within them are not capable of inducing the normal velocity fluctuations on the wall. In Fig. 1 the linear growth rates of the two-dimensional vortex waves are plotted against \( C_q \). The frequencies \( F \) are taken in different positions with respect to the neutral curve, namely, on the lower branch, in the middle of the instability region, and above the upper branch. It is established that everywhere injection destabilizes the disturbances: the growth rates increase, the effect being enhanced on high frequencies. Contrariwise, on suction both the growing and the decaying waves are stabilized. There exist the limiting values of \( C_q \), above which the dependences are reversed and on the damping oscillation range the stability margin of the two-dimensional waves diminishes. The nature of the reverse is poorly understood; apparently, this can be due to high gradients of the mean parameters.

In [4] it was shown how the three-wave interaction intensities change: they increase, as compared with the case of the impermeable wall, on injection, while on suction they are not realized at all on the Reynolds number range considered, since the linear waves become damping.

![Fig. 1. Dependence of the linear growth rates −2α of the two-dimensional vortex first-mode waves on Cq in the r ≪ q regime at Re= 600, M = 2, and the frequency parameter values F × 10^4=(0.19, 0.38, 0.57, 0.76) (curves (1–4)).](image)
Permeable wall effect. In [5–7] the effect of the porous coating parameters (the pore radius \( r \), the thickness \( H \), and the porosity \( n \)) on the linear characteristics of vortex waves was considered. We will illustrate them in this study supplementing them with the dependence of the growth rate on the suction chamber depth \( L_1 \).

In Fig. 2a the effect of \( r \) on the growth rates of the three-dimensional vortex waves is shown. The dimensionless value of the pore radius is measured on the abscissa axis in the form of the pore Reynolds number \( R = r/\text{Re} \) or \( R = r \text{ Re}_1 \). Using the unit Reynolds number \( \text{Re}_1 \approx 10^51/\text{m} \) typical of these regimes, we can readily obtain the dimensional values of \( r \). On the permeable wall the vortex waves are destabilized in the presence of both injection and suction. An analysis of the disturbance characteristics for small \( r \) made it possible to establish the range of applicability of the condition \( \nu(0)=0 \) as \( 0 \leq r \leq 2 \) microns. Maximum growth rates are in the vicinity of \( r \approx 0.75 \text{ mm} \), although at large radii the tendency to destabilization changes only slightly. The dependences of the growth rates on the porous wall thickness \( H \) presented in Fig. 2b also do not promise any hopes. A maximum destabilization is found to exist at thicknesses of \( 1 \) to \( 5 \text{ mm} \), though at smaller \( H \) so sharp increase in the growth rates can be avoided. Finally, Figure 2c gives an idea on the influence of the chamber depth \( L \) on the linear growth rates in the case of suction. The destabilizing effect of \( L \) increases with increase in the thickness \( H \) and can be reduced only at very small values of \( L \).

Linear and nonlinear stability of the vortex waves. With increase in porosity \( n \) the above-listed features manifest themselves more clearly. This is shown in Fig. 3 which demonstrates the porosity effect on the growth rates of the three-dimensional waves at different \( C_q \). The values corresponding to \( n = 0 \) give the growth rates for the regime with \( \nu(0)=0 \); in going over to the regime with \( \nu(0)=Kp(0) \) the disturbance destabilization can be observable. This tendency enhances with increase in \( C_q \) and a decrease in the boundary layer thickness (the sensitivity of thin boundary layers was noted above).

Thus, for the linear vortex waves the destabilizing porosity effect can suppress very rapidly the stabilizing suction influence. In the case of injection the disturbances grow more intensely.

It is established that the greatest growth rates are typical of the three-dimensional waves with \( b \approx 0.13 \) to \( 0.17 \) (the angle between the wave vector and the flow velocity direction of about 50°). The reverse of \( a_i \) was also found for the three-dimensional waves both at large values of \( C_q \) and in the \( \nu(0)=0 \) regime (Fig. 1).

![Figure 2](image_url)

Fig. 2. Dependence of the linear growth rates\( -2\alpha_i \) of the three-dimensional vortex waves \( (b = 0.13) \) at Re \( = 600, M = 2, F = 0.19 \times 10^{-4} \), and \( n = 0 \). \( 5 \) on the radius \( R \) for \( H = 10^3, L_1 = 10^3 \), and \( C_q = -0.25, 0 \), and 0.25 \((1-3) \) (a); the thickness \( H \) for \( C_q = 0.1, L = 10^3 \), and \( R \times 10^{-1} = 0.5 \) and \( I(1 \text{ and } 2) \); (b), and the depth \( L_1 \) for \( C_q = 0.1, R = 10^3 \), and \( H \times 10^{-2} = 0.5, 1, 2.5 \), and \( 5 \ (1-4) \) (c).

The nonlinear interactions were also studied in nonsymmetric triplets relating the three-dimensional waves on the harmonic \( G \) and subharmonic \( S \) frequencies. Previously it was established that these triplets are most representative, since they produce a maximum level of the three-wave interaction. On the harmonic frequency \( F_1 \) a component with the maximum growth rate was taken. This determined the value of the azimuthal wavenumber \( \beta_i \). At half frequency \( F_2 \) the first three-dimensional subharmonic component \( S_1 \), also having a maximum growth rate, was taken. The third wave was so chosen that the exact phase equality with respect to the wavenumbers was fulfilled. This second subharmonic \( S_2 \) usually shows the greatest nonlinear growth rate. It has the least linear growth rate, which is most easily compensated by the nonlinear terms in the amplitude equations (4). The initial subharmonic amplitudes were the same, being by an
order lower than the intensity of the harmonic, which usually amounted to 0.6% of the mean mass velocity. Figure 4 illustrates the intensity of the nonlinear processes in these triplets in the cases of injection and suction through a permeable wall in comparison with the nonlinear interactions in the $r \ll \delta$ regime. In the case of injection the distinctive feature of the harmonic component evolution is its stable growth, because of which this wave remains a pumping wave of higher frequency on the whole Re range considered. The harmonic develops in accordance with the linear laws and its intensity does not almost differ from the linear values. Both the linear and nonlinear growth of both subharmonics are intensified and increase when the compressibility in the pore is taken into account. Here, we present the subharmonic $S_2$ dynamics in the nonsymmetric triplet for the case of suction. Clearly that the nonlinear intensity on the porous wall considerably increases compared with the $r \ll \delta$ regime. This intensification of the linear and nonlinear growth on a porous wall is typical of all vortex disturbances at $M=2$.

![Fig. 3. Dependence of the linear growth rates $-2\alpha_i$ of the three-dimensional vortex waves ($b=0.13$) on $n$ at Re= 600, $M = 2$, $F = 0$, $19 \times 10^{-4}$, $R = 500$, and $H = 10^3$ for $C_q = -0.25$, $0$, $0.1$, $0.25$, $0.5$, and $0.7$ (1–6).](image)

3.2 Results for $M=5.35$.

The profiles of the mean velocity $U$ and the temperature $T$ at different intensities of injection and suction from the surface are deformed similarly to the $M=2$ regime. At $M=5.35$ on the thermally-insulated wall the temperature factor $T_w = 5.857$. In the case of injection the value $C_q = -0.1$ turned out to be limiting value at which the boundary value problem for the mean profiles could be satisfied.

At the high Mach number disturbances of two types can be observable. The disturbances of the first type are the vortex disturbances which in hypersonic or high supersonic regimes are classified as the first mode waves. The first-mode disturbances are due to viscosity by their genesis. Precisely viscosity generates such phase shift of the velocity fluctuation components in the wall region that there arise the Reynolds stresses ensuring the fluctuation energy generation at the expense of the mean flow energy. At the same time, viscosity is also responsible for energy dissipation. The second-type, or second-mode, inertia-driven waves are frequently called acoustic; these are connected with the presence of a generalized inflection point in the mean parameter profiles.

The waves of these modes differ in the phase velocities, the locations of maximum values of the mass velocity fluctuations (for the second mode the maximum is located nearer to the outer edge), and suppresses the stabilizing suction effect. In this case the nonlinear processes preceding transition to the turbulent regime are considerably intensified.

![Fig. 4. Nonlinear intensities $I_{nl}$ of the second subharmonic $S_2$ in the cases of injection $C_q = -0.25$ (1) and suction $C_q = 0.1$ (2) for $M=2$ and $F = 0$, $19 \times 10^{-4}$; (1) the $r \ll \delta$ regime and (2) the regime on the permeable wall at $R = 500$, $L_1 = H = 10^3$, and $n = 0.5$.](image)
the more intense asymptotic decay of the first-mode waves in the external field.

![Graph showing the dependence of the linear growth rates $-2\alpha$ on $L$ at $M = 5.35$, $Re = 10^3$, $F = 1.5 \times 10^{-4}$ and $n = 0.5$ for $C_q = 0.1$, $R = 500$, and $H \times 10^{-3} = (0.1, 0.5, 1.2, 10)$ (1–5).]

Fig. 5. Dependence of the linear growth rates $-2\alpha$ of the two-dimensional ($b = 0$) vortex second-mode waves on $L$ at $M = 5.35$, $Re = 10^3$, $F = 1.5 \times 10^{-4}$ and $n = 0.5$ for $C_q = 0.1$, $R = 500$, and $H \times 10^{-3} = (0.1, 0.5, 1.2, 10)$ (1–5).

At $M = 5.35$ the dependences of the growth rates on $R$ and $H$ are similar with those presented in Fig. 2, though these have no clearly expressed maxima. Only the depth $L_1$ effect should be considered in more detail. Figure 5 gives an idea on this. At small $H$ and $L_1$ the waves are reflected from the chamber bottom. With increase in $H$ and $L_1$ this process is replaced by the wave decay and even for the depth of about 0.5 mm the behavior of the dependences is similar with that observable for the vortex waves at moderate $M$ (Fig. 2c). It is also obvious that the thicker the porous coating the greater this effect.

The linear wave growth waves as functions of the frequency parameter are presented in Fig. 6. The frequency ranges of the first and second modes are deliberately separated in order to reveal the obvious differences in their behavior. Two regimes are compared, namely, the mass transfer in the $r \ll \delta$ regime and with account for the porous wall properties. For the first mode waves (Fig. 6a) the mass transfer effect is analogous to that considered above for $M = 2$. The most growing disturbances are three-dimensional, while for all $Re$ the maximum growth rate is in the vicinity of the frequency $F = 0.3 \times 10^{-4}$.

On the range of large $F$ maximum growth rates correspond to the inertia-driven second-mode waves (Fig. 6b). The linear growth rates of these waves are considerably higher than those of the first mode. In the $r \ll \delta$ regime the dependences of the streamwise growth rates on the mass transfer direction are the same for the waves of different modes; in the linear region injection destabilizes both the first and the second-mode disturbances, whereas suction stabilizes them. This can be seen in the figure in comparison with the $C_q = 0$ case. The range of existence of the growing second-mode fluctuations is fairly narrow, as compared with the first mode; in the presence of mass transfer the frequency scatter and a strong $C_q$-dependence of the frequency corresponding to the greatest growth rate can be observed. It is also confirmed that the acoustic waves have more growing two-dimensional components.

![Graph showing the dependence of the linear growth rates $-2\alpha$ on $F$ at $M = 5.35$ and $Re = 10^3$ for three-dimensional ($b = 0.13$) first-mode waves at $C_q = -0.05, 0, 0.1$, and $0.5$ (1–4) (a) and two-dimensional ($b = 0$) second-mode waves at $C_q = -0.075, 0, 0.1$, and $0.25$ (1–4) (b); $I$ is the $r \ll \delta$ regime and $II$ is the permeable wall at $R = 500$, $H = 10^3$, $L = 10^3$, and $n = 0.5$.]

Fig. 6. Dependence of the linear growth rates $-2\alpha$ on the frequency parameter $F$ at $M = 5.35$ and $Re = 10^3$ for three-dimensional ($b = 0.13$) first-mode waves at $C_q = -0.05, 0, 0.1$, and $0.5$ (1–4) (a) and two-dimensional ($b = 0$) second-mode waves at $C_q = -0.075, 0, 0.1$, and $0.25$ (1–4) (b); $I$ is the $r \ll \delta$ regime and $II$ is the permeable wall at $R = 500$, $H = 10^3$, $L = 10^3$, and $n = 0.5$.

The differences start to manifest themselves when the porous wall properties are taken into account. As shown in [5–10], on this wall the second-mode waves are stabilized due to absorption. In this case the linear growth rates considerably diminish both on injection and on suction. This can be clearly seen by comparing curves $I$ and $II$ in Fig. 6b. The components having a maximum growth rate are considerably displaced toward the lower frequencies. The values of the growth rates of the
first and second-mode waves come closer to each other. This tendency increases with increase in porosity $n$.

The distinctive features of the linear wave evolution presented above have undoubtedly an effect on the nonlinear interactions of disturbances different in nature. For $M = 5.35$ we consider the interaction in symmetric triplets consisting of a two-dimensional acoustic harmonic and two three-dimensional vortex subharmonics. The main features, characteristic of this interaction, are described and studied in detail in [4–7]. It was established that it depends on the acoustic harmonic intensity. If its initial value is not small, the interaction is realized in the parametric pumping regime. In this case, the disturbance of the primary unstable harmonic develops in accordance with its own linear law. When its amplitude (or intensity) reaches a threshold value, the subharmonic components start to be excited.

![Graph showing linear and nonlinear intensities of harmonics and subharmonics on Reynolds's number in a symmetric triplet for $M=5.35$ at $C_q = -0.1; 0; 0.1$ (1-3). Dashed lines are designated linear intensity of subharmonics.]

Fig. 7. Dependences of linear and nonlinear intensities of harmonics and subharmonics on Reynolds's number in a symmetric triplet for $M=5.35$ at $C_q = -0.1; 0; 0.1$ (1-3). Dashed lines are designated linear intensity of subharmonics.

If the initial intensities of the harmonic components are lower than the threshold values, then the nonlinear interaction is almost absent and all the waves develop in accordance with the linear law. When the threshold values are slightly exceeded, weak interactions begin but lead to only slight excess over the linear values of the subharmonics. The further increase in the harmonic intensity, as a result of the nonlinear interaction, leads to an almost blow-up growth of the subharmonics. In Fig. 7 the different initial harmonic levels make it possible to observe all the three stages. When the surface properties are taken into account, due to a decrease in the growth rates of the pumping wave, the intensities of the subharmonic growth are reduced. It only remains for us to add that at low initial intensities of the acoustic component, due to the fact that the linear growth rates of the waves come closer, the parametric pumping regime can go over into the weak redistribution regime, in which the nonlinear process intensity and direction are completely determined by the values of the nonlinear terms in Eqs. (4).

Summing up the above-said it can be concluded that in high supersonic regimes injection and suction through porous permeable surfaces lead to considerable stabilization of the second-mode disturbances, which can favor the transition delay. The behavior of the vortex disturbances is similar with that described above for the $M = 2$ case.

### 4 Conclusion

The study augments the information field for studying the passive techniques of controlling the supersonic boundary layer instability. That the effects of the factors considered on different-in-nature disturbances can be different in direction provides fairly wide possibilities of controlling the flow regimes in the required direction.

At the same time, the severity of the problem of organizing distributed mass transfer on permeable surfaces should be noted, which does not allow us to recommend this control techniques as easy-to-realize in practice.

### References:


