The new method of investigation of the problem of synchronization electrical machine with power network.

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Abstract: - Connecting a generator to the network, either in a normal regime or under a significant load change, is possible with greater (than commonly accepted now) initial differences between the rotor speed and the frequency of the stator current, OR, possible with small frequency differences and any misalignment angle between the magnetic axis of spinning rotor and the rotary electro-magnetic field of the stator.

Key-Words: -synchronous electric machines, powerful network, transient processes, Routh's structure of asymptotically simplified equations

1. Introduction

In modern turbine-generators, the control system which maintains voltage at the output is based on feedback control (or, direct control based on the given voltage). The modern high power machines use the rotary field, with another low-power excitation generator mounted on the same shaft as the main generator. The embodiment often includes a high-frequency electromechanical excitation generator based on non-contact inductor generators, as well as thyristor brushless excitation systems with a rotating transformer, et al. This control system can be simplified or enhanced based on this study.

The study investigated the regimes of direct synchronization of a turbine-generator unit with the power network. "Direct" here implies that a voltage at the stator windings is set (predetermined) in the form of harmonic functions of a given amplitude and frequency (50 or 60 Hz), the voltage at the windings is ~ 10 kV and current ~ 20 kA, and therefore, the power is in the range 100-300 MW. It is assumed that the output voltage of the generator is set (predetermined), although the practice its modification is possible.

The problem of direct synchronization (without control) is modeled as a transition process when the generator is connected to the network under the given initial conditions, which include: the misalignment angle between the magnetic axis of spinning rotor and of the electro-magnetic field of the stator, which rotate with frequency of 50/60 Hz (the last rotate due to distribution of the stator three-phase winding), and the initial difference in the rotational speeds of the rotor and the stator field (slip).

The basics of the mathematical model of synchronous machine are well known (see, for example [4], [5], [7], [8]) and the transient processes are under consideration we can make the main following assumptions:

a) Only the first harmonic with the spatial period equal to the double pole division is taken into account when the spatial distribution of the fields of self-induction of windings of the rotor and stator is considered;

b) A real damper winding (for explicit-pole machines) or the rotor body acting as a damper winding (for implicit-pole machines) is replaced by two equivalent damper contours located in longitudinal (contour t) and transversal (contour k) axes of the machine. Longitudinal axis of the rotor d passes through the mid-pint of the rotor pole and has the direction coinciding with the direction of the magnetic field of the excitation winding. The direction of the current in the longitudinal damper contour is positive if the direction of magnetic flux due to this current is coincident with the positive direction of axis d. The transverse axis of rotor q is located between the neighbouring magnetic poles of rotor and forms the angle of 90 electric degree to axis d. The positive direction of axis q is that for which this axis is behind axis d. The positive direction of current in contour k coincides with that in contour t;

c) The phase windings of the stator (anchor) are spools distributed over the stator circumferences and connected to each other by means of the

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star-connection or delta connection. The magnetic axes of the three-phase system of spools a, b, c are shifted to each other on 120 electric degree. Without loss of generality, in what follows we consider the model of implicit-pole synchronous machine with a three-phase stator winding. The location of the contours of the idealized model of synchronous machine is shown in Fig.1

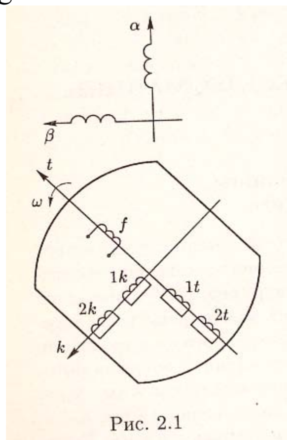

Fig.1

2. Problem formulation

The problem of reducing equations of the synchronous machine in the case of powerful network is considered in most books on transient processes in synchronous machines (see [5], [8]]). The full description of this model used Park-Gorev transformation [1] you can find, for example in [5]. These equations have Lagrange-Maxwell form for electromechanical systems in quasi stationary electromagnetic field

$$\frac{d}{dt}[Li_a + Mcos\frac{2\pi}{3}(i_a + i_b) + (M_f i_f + M_t i_t)cos\vartheta - M_k i_k sin\vartheta] + R_a i_a + R_0 i_0 = Ucos\Omega_0 t, \quad (a,b,c)$$

$$\frac{d}{dt}\{M_f\left[i_a cos\vartheta + i_b\cos\left(\vartheta - \frac{2\pi}{3}\right) + i_c\cos\left(\vartheta + \frac{2\pi}{3}\right)\right] + L_f i_f + M_{ft} i_t\} + R_f i_f = 0,$$

$$\frac{d}{dt}\{M_t\left[i_a cos\vartheta + i_b\cos\left(\vartheta - \frac{2\pi}{3}\right) i_c\cos\left(\vartheta + \frac{2\pi}{3}\right)\right] + L_t i_t + M_{ft} i_f\} + R_t i_t = 0,$$

$$\frac{d}{dt}\left\{-M_k\left[i_a sin\vartheta + i_b\sin\left(\vartheta - \frac{2\pi}{3}\right) + i_c\sin\left(\vartheta + \frac{2\pi}{3}\right)\right] + L_k i_k\right\} + R_k i_k = 0,$$

$$J\frac{d^2\vartheta}{dt^2} = -\{(M_f i_f + M_t i_t)[i_a sin\vartheta + i_b\sin\left(\vartheta - \frac{2\pi}{3}\right) + i_c\sin(\vartheta + \frac{2\pi}{3})] + M_k i_k[i_a cos\vartheta + i_b\cos\left(\vartheta - \frac{2\pi}{3}\right) + i_c\cos(\vartheta + \frac{2\pi}{3})]\} + M_m,$$

$$\frac{d\vartheta}{dt} = \Omega \qquad (1)$$

It is also possible to pass on to the introduced system of relative units from any known system of relative units, say from the system x_{ad} or from the system of equal magnetomotive forces (mmf) [5]. After what we introduce non-dimensional currents and flux linkages in axes d. The final expressions for the flux linkages in the introduced system of relative units are as follows

$$\Psi_d = (1 + \sigma_d)i_d + i_f + i_t,$$
$$\Psi_f = i_d + (1 + \varepsilon_r\sigma_f)i_f + i_t,$$
$$\Psi_t = i_d + i_f + (1 + \varepsilon_r\sigma_t)i_t, \qquad (2)$$
$$\Psi_q = (1 + \sigma_q)i_q + i_k,$$
$$\Psi_k = i_q + i_k,$$

where

$$1 + \sigma_d = \frac{L_d}{L_{ad}}, 1 + \sigma_q = \frac{L_q}{L_{aq}}, 1 + \varepsilon_r\sigma_f = \frac{L_q}{L_{af}},$$
$$1 + \varepsilon_r\sigma_t = \frac{L_t}{L_{at}},$$
$$L_{ad} = \frac{3}{2}\frac{M_f M_t}{M_{ft}}, L_{af} = \frac{M_f M_{ft}}{M_t}, \ L_{at} = \frac{M_t M_{ft}}{M_f}$$

more detail about introducing non-dimensional variables you can see in [8]. It's important that small parameters $\varepsilon_r\sigma_f$ and $\varepsilon_r\sigma_t$ characterize the dissipation between contours $t$and f. Subtracting expression for Ψ_t from expression forΨ_f in equation (2) we obtain

$$\varepsilon_r\Psi_r = \Psi_f - \Psi_t = \varepsilon_r(\sigma_f i_f - \sigma_t i_t),$$

where the quantity Ψ_r is proportional to the flux linkage of the dissipation between the excitation winding and the damper contour in longitudinal axis. Parameter ε_r is small because this dissipation is small as compared with the main flux. However parameter ε_r is not small for hydrogenerators and some types of turbogenerators, in this case the asymptotic transformation of equations of synchronous machine is carried out in [12].

When modelling the initial non-linear system (using Park-Gorev equations), threeimportant small parameters are introduced: it is assumed that the network period (0.02 sec) is more small with respect to the decay time (0.4-0.5 sec) of the transient processes in the damper circuits (such as the rotor body wholly as a steel cask), and the ratio of the network period to the mechanical time constant, which determined as ratio of amplitude kinetic energy of rotation and machine power. These expressions the next:

$$\varepsilon = \frac{1}{\omega_* T_k}, \varepsilon_f\nu_f = \frac{R_a T_k}{L_{ad}}, \varepsilon_f e_f = \frac{E_f T_k}{L_{af} i_{d*}},$$

$\varepsilon_\omega = \frac{3}{2}\frac{L_{ad} i_{d*}^2 T_k}{J\omega_*}$,

where$T_k = \frac{L_k}{R_k}$ denotes the time constant of the damper contour in axis k. The main small parameter $\varepsilon = 1/\omega_* T_k$, where ω_* denotes the synchronous frequency, is given by ratio of the period of network voltage to the time constant of damper contour. This parameter is small for all synchronous machines (for turbogenerators its characteristic value is 0.02).

3. Problem Solution

Before we proceed to derivation of the asymptotically simplified equations for synchronous machine let us explain a necessity of using axes $\alpha, \beta, 0$ for separation of slow processes in the stator circuits. As follows from equations for the stator circuits in axes d, q, in the transient regime variables Ψ_d, Ψ_q have a fast oscillating component. In the case of the small sliding these equations are quasi-linear, therefore transient to the axes α, β (Fig.1) is somehow equivalent to the Van-der-Pol replacement in the theory of nonlinear oscillations.

After that the asymptotic averaging method for analysis of transient process at connecting a generator to the network is used. The asymptotically simplified equations of the transient processes can be written down in the form

$$\begin{aligned} &\dot{\Psi}_f + \frac{r_f}{l}\left(\Psi_f - \gamma cos\delta\right) = e_f, \\ &\dot{\Psi}_k + \frac{r_k}{l}(\Psi_k - \gamma sin\delta) = 0, \qquad (3) \\ &\kappa\ddot{\delta} + \frac{\gamma}{l}\left(\Psi_f sin\delta + \Psi_k cos\delta\right) = m. \end{aligned}$$

The new variable $\delta = \vartheta - \vartheta_0$ is an angle of rotor rotation "relative to the network", where $\vartheta_0 = \omega_* t$ and $s = \dot{\delta}$ is a sliding. Additionally, another dimensionless "slow" time $\tau = \sqrt{\varepsilon\varepsilon_\omega/\sigma_d}\,\omega_* t$ is introduced. Variable δ formally is fast (the rate of change is of the order of unity), but we consider only the motions under which sliding is small and δ changes slowly. This corresponds to analysis of particular solutions of the system with two fast phases in the case of principle resonance. Here dimensionless parameters have expressions

$$r_f = \sqrt{\frac{\varepsilon\sigma_d}{\varepsilon_\omega}}\frac{\varepsilon_f \nu_f \nu_t}{\left(\nu_f + \nu_t\right)}, r_k = \sqrt{\frac{\varepsilon\sigma_d}{\varepsilon_\omega}}\nu_k \kappa = \frac{1}{\sigma_d},$$

$$l = \frac{\sigma_d}{1+\sigma_d}, \gamma = \frac{u}{\sigma_d}, \nu_n = \frac{R_a T_k}{L_{at}}, \nu_t = \frac{R_t T_k}{L_{at}}.$$

Equations (3) have a structure of Routh's equations in which Ψ_f and Ψ_k are the quasi-cyclic generalized momenta and δ is the positional generalized coordinate. This allows one to make use Lagrange equations which are more comfortable in this case. Let us introduce the currents (generalized velocities)

$$I_f = \frac{1}{l}\left(\Psi_f - \gamma cos\delta - \frac{le_f}{r_f}\right), I_k = \frac{1}{l}(\Psi_k + \gamma sin\delta)$$

We obtain the equations

$$\begin{aligned} &l\dot{I}_f - \gamma\dot{\delta} sin\delta + r_f I_f = 0, \\ &l\dot{I}_k - \gamma\dot{\delta} cos\delta + r_k I_k = 0, \qquad (4) \\ &\kappa\ddot{\delta} + \gamma\left(I_f sin\delta + I_k cos\delta\right) + \frac{\gamma e_f}{r_f} sin\delta = m. \end{aligned}$$

It is remarkable that the averaged equations has a more smaller dimension and preserve the Lagrangian structure (they described a pendulum with a damping conducting circuit, rotating together with the pendulum in a constant magnetic field, Fig. 1[1])

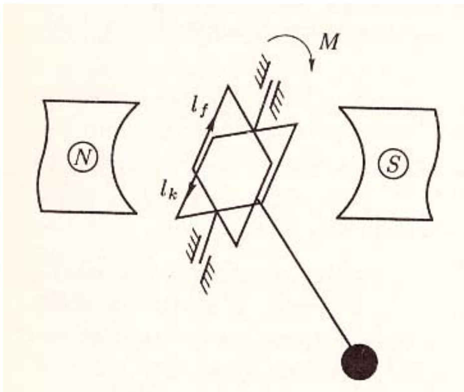

Fig.2

These equations (4) describe both swinging of the rotor of synchronous machine and the motions of pendulum with the magneto-electric extinguishers [13] subjected to gravity and external moment m with the only difference that for the pendulum the factor γe_f must be replaced by another dimensionless parameter. In this case the extinguishers have a single contour with currents I_f and I_k and these contours' planes being mutually perpendicular and the coefficient of mutual induction between the contours being equal to zero. The contours are placed in the homogeneous magnetic field and exhibit angular vibra-

[1]Non-Linear Electromechanics"by D. Skubov, K.Khodzhaev, Springer, 2008

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tions or revolve when the pendulum vibrates and rotates, see Fig.2.
In the case in which the inductance l is negligible in eq.(4) and the resistances are equal to $r_f = r_k = r$, this system reduces to the well-known Tricomi equation [2], [7], [11]

$$\ddot{\delta} + \beta\dot{\delta} + sin\delta = \widetilde{m}, \quad (5)$$

where the differentiation is performed with respect to dimensionless time $\tau = \Omega t$ and $\Omega^2 = \frac{\gamma e_f}{r\kappa}, \beta = = \frac{\gamma^2}{r\Omega}, \widetilde{m} = \frac{m}{\kappa\Omega^2}$. If$\widetilde{m} < 1$, eq.(4) has two equilibrium position $\delta_* = arcsin\frac{m}{\gamma e_f}$ and $\pi - \delta_*$ for the zero currents I_k, I_f and sliding $\dot{\delta} = 0$. For the pendulum without extinguishers the first equilibrium is stable and the second one is unstable. The extinguishers strengthen stability, i.e. a stable equilibrium becomes asymptotically stable whereas an unstable equilibrium remains unstable. If we are changing initial value of sliding so for stationary solution of system (4) we can obtain as stable equilibrium position or rotatory motions of the pendulum which correspond periodic motion in $\dot{\delta}$with a constant mean value after the sliding period $\dot{\delta} > 0$ (for $m > 0$) what for synchronous machine is equivalent to asynchronous motion. Our task consists in finding as themselves rotations so relation of parameters determining the type of transient process. Moreover the considerable interest has a solution of the problem of investigation the transient processes at switching of load of synchronous electric motor namely the possible level of jump of load. Rotatory motions of the equivalent pendulum are analysed withthe help of method of harmonic balance. Used the replacement

$$x_1 = I_f sin\delta + I_k cos\delta,$$
$$x_2 = I_f cos\delta - I_k sin\delta$$

and introducing the independent variable δ instead t in eq. (4) yields

$$\begin{aligned} \omega x_1' - \omega x_2 - \gamma\omega + r x_1 = 0, \\ \omega x_2' + \omega x_1 + r x_2 = 0, \\ \omega\omega' + \gamma x_1 + e_f sin\delta + \beta\omega = m \end{aligned} \quad (6)$$

Approximate rotatory solutions of eq. (6) is sought in the form

$$\begin{aligned} \omega = \omega_0 + \omega_s sin\delta + \omega_c cos\delta, \\ x_1 = x_{10} + x_{1s} sin\delta + x_{1c} cos\delta, \\ x_2 = x_{20} + x_{2s} cos\delta - x_{2c} sin\delta \end{aligned} \quad (7)$$

After substitution (7) into (6) we obtain cubic equation for constant component of ω

$$\beta\omega_0^3 - m\omega_0^2 + (r^2\beta + \gamma^2 r)\omega_0 - mr^2 = 0 \quad (8)$$

and the constant components of fluxes

$$x_{10} = \frac{\gamma r\omega_0}{\omega_0^2 + r^2}, \quad x_{20} = -\frac{\gamma\omega_0^2}{\omega_0^2 + r^2} \quad (9)$$

The Descartes rule gives that equation (8) has three real positive roots. For parameters, corresponding turbogenerator TVV-200 MWT the graph of eq.(8) is shown on Fig.3

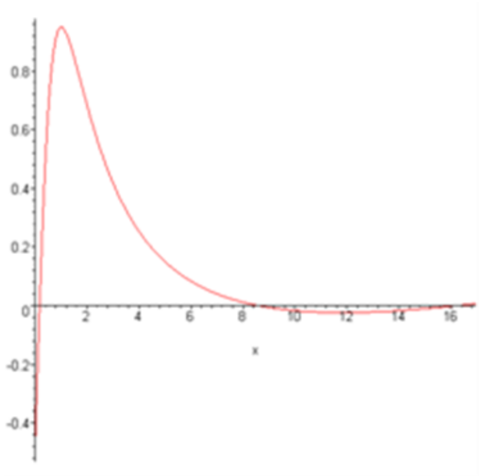
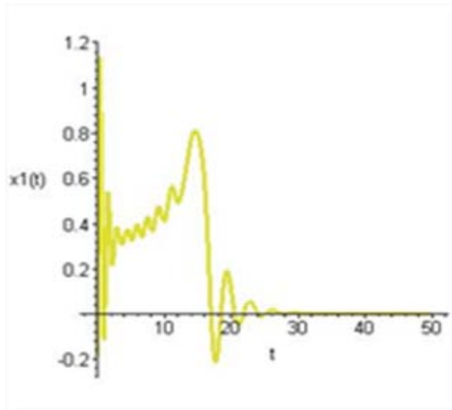


Fig.3

Two last roots completely coincide with results of calculation of Cauchy problem for system (6) and middle mode is unstable and mode with greater root is stable. The question founding the attractive field of the mode with first lesser value of ω_0 is more difficult and now have not solution.
The graphs of transient processes at different initial values of sliding $s(0)$ are shown on Fig.4. The initial fluxes are determined from condition of switching of machinewith power network at initial misalignmentangle between the magnetic axis of spinning rotor and of the electromagnetic field of the stator, which in our caseequal $\delta_0 = \pi/3$. At these Fig.4 next designations are input$x4(t) = \dot{\delta}$and$x1(t) = x_1, x2(t) = x_2$.



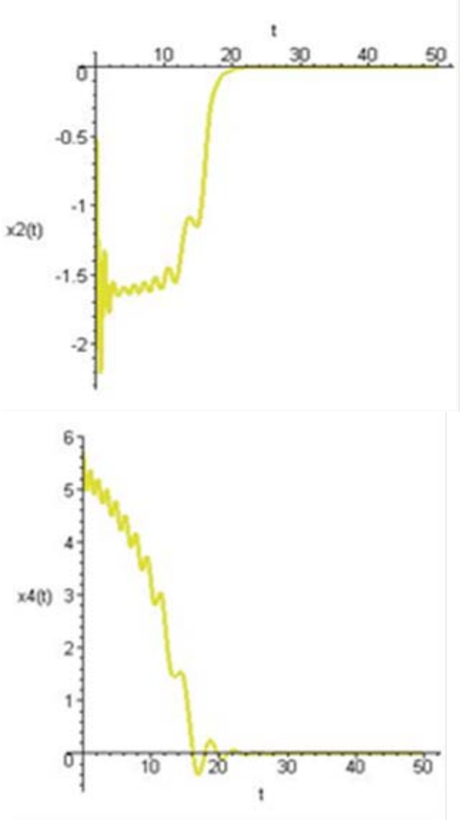


Fig.4

At these graphs we can see the some processes of synchronization with network at substantial values of initial sliding, which can achieved $S(0) = 2 \div 3$ Hz (the last graph at Fig.4)Undoubtedly, that so large values of initial sliding gives us the possibility the more simple method to connection of synchronous electric machine in power network.

4. Conclusion

The mathematical model realized by numerical-analytic method used for simplifying the equations of the transient processes at connecting a generator to power network gives the possibility more reliable (rough) method of its synchronization with power network.

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