Nash and social welfare impact in international trade

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Abstract: We study the international trade model where, in the first stage, the governments maximize competitively (Nash) or cooperatively (social) welfares; and, in the second stage, firms maximize competitively (Nash) profits. Let the maximal tariff of a government be such that the other country is unable to export. If the maximal tariffs of both governments are similar, then the governments face a prisoner's dilemma; but if the maximal tariffs are too different then the governments deal with a lose-win dilemma.

Key–Words: Game Theory; International Duopoly; Prisoner's Dilemma

1 Introduction

We consider a usual duopoly international trade model with complete information, where there are two countries and a firm in each country that sells in its own country and exports to the other one (see [1, 2, 4, 5, 11, 12, 13, 15]).

The international trade model has two stages: in the first stage, the governments simultaneously choose simultaneously their tariff rates to maximize competitively (Nash) or cooperatively their (social) welfares; and, in the second stage, the firms observe the tariff rates and simultaneously choose their quantities for home consumption and for export to maximize competitively their profits.

We show that there is a social optimum and a Nash equilibrium. Furthermore, we find only the prisoner's dilemma (PD) and lose-win social strategies (LW) games outcomes in this model.

2 Strategic tariffs

Let $W_i(t_i, t_j)$ and $W_j(t_i, t_j)$ be the welfares of the countries X_i and X_j depending only upon the tariffs t_i and t_j imposed by the governments of the two countries. We are going to interpret $W_i(t_i, t_j)$ and $W_j(t_i, t_j)$ as the utilities of a game where the players are the governments of the countries and their actions are the tariffs (t_i, t_j).

The quantity $t_i^{BR}(t_j) \equiv t_i^{BR}(t_j; W)$ is the *best response* of the country X_i for the utility W_i, if for all tariffs t_i,

$$W_i(t_i^{BR}(t_j), t_j) \geq W_i(t_i, t_j) \ .$$

A pair of tariffs $(t_i^N, t_j^N) \equiv (t_i^N(W), t_j^N(W))$ is a *Nash equilibrium* or a *global strategic optimum*, if for all tariffs t_i

$$W_i(t_i^N, t_j^N) \geq W_i(t_i, t_j^N)$$

and for all tariffs t_j

$$W_j(t_i^N, t_j^N) \geq W_i(t_i^N, t_j) \ .$$

In other words, a pair of tariffs (t_i^N, t_j^N) is a Nash equilibrium, if

$$t_i^N = t_i^{BR}(t_j^N) \qquad \text{and} \qquad t_j^N = t_j^{BR}(t_i^N) \ .$$

A pair of tariffs $(t_i^P, t_j^P) \equiv (t_i^P(W), t_j^P(W))$ is a *Pareto optimum*, if there is no pair (t_i, t_j) of tariffs such that

$$W_i(t_i, t_j) \geq W_i(t_i^P, t_j^P) \qquad \text{for all } \ i, j \in \{1, 2\},$$

and at least for one country X_i, $i \in \{1, 2\}$ gets a better payoff with (t_i, t_j) than with (t_i^P, t_j^P), i.e.

$$W_i(t_i, t_j) > W_i(t_i^P, t_j^P) \ .$$

The *social utility* u_S is given by

$$W_S(t_i, t_j) = W_i(t_i, t_j) + W_j(t_i, t_j) \ .$$

The quantity $t_i^{SR}(t_j) \equiv t_i^{SR}(t_j; W)$ is the *social best response*, if for all tariffs t_i

$$W_S(t_i^{SR}(t_j), t_j) \geq W_S(t_i, t_j) \ .$$

ISBN: 978-960-474-394-0

A pair of tariffs $(t_i^S, t_j^S) \equiv (t_i^S(W), t_j^S(W))$ is a *social optimum*, if for all tariffs t_i

$$W_S(t_i^S, t_j^S) \geq W_S(t_i, t_j^S) \ ,$$

and for all tariffs t_j

$$W_S(t_i^S, t_j^S) \geq W_S(t_i^S, t_j) \ .$$

In other words, a pair of tariffs (t_i^S, t_j^S) is a social optimum, if

$$t_i^S = t_i^{SR}(t_j^S) \qquad \text{and} \qquad t_j^S = t_j^{SR}(t_i^S) \ .$$

(SE) **Social equilibrium**: When the social optimum coincides with the Nash equilibrium

$$(t_i^S, t_j^S) = (t_i^N, t_j^N)$$

and the social optimum is the only Pareto optimum. In this case, the individualist Nash choice of the tariffs by the governments lead to a social equilibrium. Hence, a priori there is no need of a trade agreement between the two governments of the two countries.

(PD) **Prisoner's dilemma**: When the social optimum (t_i^S, t_j^S) is different from the Nash equilibrium

$$t_i^S \neq t_i^N \qquad \text{or} \qquad t_j^S \neq t_j^N$$

and both utilities are bigger in the social optimum than in the Nash equilibrium,

$$W_i(t_i^S, t_j^S) > W_i(t_i^N, t_j^N)$$

and

$$W_j(t_i^S, t_j^S) > W_j(t_i^N, t_j^N) \ .$$

In this case, the game is like the Prisoner's dilemma, where the Nash strategy leads to a lower outcome for both countries than if they would agree among therein (through a trade agreement) in opting for the social optimum.

(LW) **Lose-win social strategies**: When the social optimum (t_i^S, t_j^S) is different from the Nash equilibrium

$$t_i^S \neq t_i^N \qquad \text{or} \qquad t_j^S \neq t_j^N$$

and one of the utilities is bigger in the social optimum and the other utility is bigger in the Nash equilibrium,

$$W_i(t_i^S, t_j^S) < W_i(t_i^N, t_j^N)$$

and

$$W_j(t_i^S, t_j^S) > W_j(t_i^N, t_j^N) \ .$$

In this case, the governments can implement an external mechanism (trade agreement) that will force them to opt by the social optimum in such a way that the country with the advantage in its utility compensates the loss in the utility of the other country and can also give some extra benefit to persuade the other country to implement the social optimum.

3 International duopoly model

In this section, we introduce the relevant economic quantities of the international duopoly model.

The *home consumption* h_i is the quantity produced by the firm F_i and consumed in its own country X_i. The *export* e_i is the quantity produced by the firm F_i and consumed in the country X_j of the other firm F_j, where $i, j \in \{1, 2\}$ with $i \neq j$. The *tariff rate* t_i is determined by the government of country X_i on the import quantity e_j. The *inverse demand* p_i in the country X_i is

$$p_i \equiv p_i(h_i, e_j) = \alpha - (h_i + e_j) \ ,$$

where $\alpha \geq 0$ is the *demand intercept*. The *payoff* π_i of firm F_i is

$$\begin{aligned} \pi_i &\equiv \pi_i(h_i, e_i, h_j, e_j; t_i, t_j) \\ &= (p_i - c_i)h_i + (p_j - c_i)e_i - t_j e_i \ , \end{aligned}$$

where $c_i \geq 0$ is the firm F_i's *unitary production cost*. The *custom revenue* CR_i of the country X_i is given by

$$CR_i \equiv CR_i(e_j; t_i) = t_i e_j \ .$$

The *consumer surplus* CS_i in the country X_i is given by

$$CS_i \equiv CS_i(h_i, e_j) = \frac{1}{2}(h_i + e_j)^2 \ .$$

The *welfare* W_i of the country X_i is

$$W_i \equiv W_i(h_i, e_i, h_j, e_j; t_i, t_j) = CR_i + CS_i + \pi_i \ .$$

4 Second stage Nash equilibrium

In this section, we give a presentation of the Nash equilibrium of the second subgame in the case of complete information, i.e. when both firms have full information on their and others utility functions (see [12]).

Let $i, j \in \{1, 2\}$ with $i \neq j$ and $\alpha_i := \alpha - c_i$. Define the *maximal tariffs* T_i and T_j of the governments of countries X_i and X_j, respectively, by

$$T_i \equiv T_i(\alpha_i, \alpha_j) = \alpha_j - \alpha_i/2 \ ,$$

$$T_j \equiv T_j(\alpha_i, \alpha_j) = \alpha_i - \alpha_j/2 \ .$$

ISBN: 978-960-474-394-0

Assumption (A1): For all $i \in \{1,2\}$, $T_i > 0$ and

$$0 \le t_i \le T_i \ .$$

The *best response* $(h_i^{BR}(e_j), e_i^{BR}(h_j;t_j))$ of the firm F_i is the solution of

$$(h_i^{BR}(e_j), e_i^{BR}(h_j;t_j)) \\ = \arg\max_{(h_i,e_i)} \pi_i(h_i,e_i,h_j,e_j;t_i,t_j) \ .$$

Hence,

$$\begin{cases} h_i^{BR}(e_j) = \frac{\alpha - e_j}{2} \\ e_i^{BR}(h_j;t_j) = \frac{\alpha - h_j - t_j}{2} \ . \end{cases}$$

The *Nash equilibrium*

$$(h_i^N(t_i), e_i^N(t_j); h_j^N(t_j), e_j^N(t_i))$$

is the solution of

$$\begin{cases} \left(h_i^N(t_i), e_i^N(t_j)\right) = \left(h_i^{BR}(e_j^N(t_i)), e_i^{BR}(h_j^N(t_j))\right) \\ \left(h_j^N(t_j), e_j^N(t_i)\right) = \left(h_j^{BR}(e_i^N(t_j)), e_j^{BR}(h_i^N(t_i))\right) \ . \end{cases}$$

Under assumption (A1), for every $t_i \in [0, T_i]$ and every $t_j \in [0, T_j]$, the home $h_i^N(t_i)$ and export $e_i^N(t_j)$ quantities for the firms at the Nash equilibrium (see [12]) are

$$h_i^N(t_i) \equiv h_i^N(c_i, c_j; t_i) = \frac{2T_j + t_i}{3} \ ,$$

$$e_i^N(t_j) \equiv e_i^N(c_i, c_j; t_j) = \frac{2(T_j - t_j)}{3} \ .$$

We observe that the export quantity $e_i^N(t_j)$ is positive if, and only if, assumption (A1) holds.

The price $p_i^N(t_i)$ of the firm F_i is

$$p_i^N(t_i) \equiv p_i^N(c_i, c_j; t_i) = \frac{\alpha + c_i + c_j + t_i}{3} \ .$$

The profit $\pi_i^N(t_i, t_j)$ of the firm F_i is

$$\pi_i^N(t_i,t_j) \equiv \pi_i^N(c_i,c_j;t_i,t_j) \\ = \frac{1}{9}[(2T_j + t_i)^2 + 4(T_j - t_j)^2] \ .$$

The custom revenue $CR_i^N(t_i)$ is

$$CR_i^N(t_i) \equiv CR_i^N(c_i, c_j; t_i) = \frac{2t_i(T_i - t_i)}{3} \ .$$

The consumer surplus $CS_i^N(t_i)$ is

$$CS_i^N(t_i) \equiv CS_i^N(c_i, c_j; t_i) = \frac{\left(2(T_i + T_j) - t_i\right)^2}{18} \ .$$

5 Nash and social welfares

In this section, we will find which of the three typical games occurs depending upon the production costs: social equilibrium (SE), prisoner's dilemma (PD), or lose-win social strategies (LW).

Recall that the welfare $W_i^N(t_i, t_j)$ of the country X_i is

$$W_i^N(t_i,t_j) = \pi_i^N(t_i,t_j) + CR_i^N(t_i) + CS_i^N(t_i) \\ = \frac{1}{9}\Big[10T_j^2 + 2T_i^2 + 4T_iT_j + (4T_i + 2T_j)t_i \\ - 8T_jt_j + 4t_j^2 - \frac{9t_i^2}{2}\Big] \ .$$

The maximum point of the polynomial $W_i^N(t_i, t_j)$ is

$$A_{W,i} = \frac{2(T_j + 2T_i)}{9} \ .$$

The social utility $W_S^N(t_i, t_j)$ is

$$W_S^N(t_i,t_j) = W_i^N(t_i,t_j) + W_j^N(t_i,t_j) \ .$$

Hence, if $0.63\ldots T_j < T_i < 1.57\ldots T_j$ and $0.63\ldots T_i < T_j < 1.57\ldots T_i$ then the game is of the type PD, otherwise the game is of the type LW.

6 Conclusion

For every pair of tariffs (t_i, t_j), we found the Nash equilibrium for the second subgame, i.e. the home and export quantities such that the firms maximize strategically their profits. Then, using the Nash equilibrium for the home and export quantities, we found the tariffs that lead to a Nash equilibria or to a social optimum for the welfares of both countries.

Acknowledgements: We acknowledge the financial support of LIAAD-INESC TEC through Strategic Project - LA 14 - 2013-2014 with reference PEst-C/EEI/LA001 4/2013, USP-UP project, IJUP, Centro de Matemática da Universidade do Porto, Faculty of Sciences, University of Porto, Calouste Gulbenkian Foundation, FEDER, POCI 2010 and COMPETE Programmes and Fundação para a Ciência e a Tecnologia (FCT) through Project "Dynamics and Applications", with reference PTDC/MAT/121107 /2010 (FCOMP-01-0124-FEDER-020871). M. Choubdar research was supported by FCT - Fundação para a Ciência e Tecnologia grant with reference SFRH / BD / 51173 / 2010. J.P. Zubelli was supported by CNPq grants 302161/2003-1 and 474085/2003-1 and by FAPERJ through the programs *Cientistas do Nosso Estado* and *Pensa Rio*.

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